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Ann. I. H. Poincaré – AN 30 (2013) 959–960
ANNALES DE L'INSTITUT HENRI POINCARÉ ANALYSE NON LINÉAIRE
www.elsevier.com/locate/anihpc

Erratum

Erratum to "Fading absorption in non-linear elliptic equations" [Ann. I. H. Poincaré – AN 30 (2) (2013) 315–336] ☆

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Received 21 December 2012; accepted 22 December 2012

Available online 23 January 2013

The purpose of this note is to correct an error that occurred in the proof of Theorem 1.2 of the paper 'Fading absorption in non-linear elliptic equations' which appeared in Ann. I. H. Poincaré – AN, 2013.

The theorem itself is correct as stated. However Proposition 3.1 (used in its proof) and relation (3.18) are wrong. We restate Proposition 3.1 and provide a modified argument to replace the part of the proof from (3.18) to the end.

Let U_j, $j = 1, 2, \dots$ be the unique solution of the boundary value problem

$$\begin{aligned} &-\Delta U_j + \bar{h} U_j^q = 0 \quad \text{in } \mathbb{R}^N_+, \\ &U_j(x', 0) = \gamma_j(x') \quad \text{for } x' \in \mathbb{R}^{N-1} \end{aligned} \tag{0.1}$$

dominated by the harmonic function $\int_{\mathbb{R}^{N-1}} P(x, y')\gamma_j(y')\,dy'$. Here $\bar{h}$ and γ_j are given by (1.3) and (3.2) respectively. Proposition 3.1 is replaced by:

Proposition 3.1′. *Under the assumptions of Theorem* 1.2,

$$\lim_{j\to\infty} U_j(0, x_N) = \infty \quad \forall x_N > 0. \tag{0.2}$$

Proof. The proof is based on (3.17) and the inequality $u_j \leqslant U_j$ in Ω_j. This inequality follows from the comparison principle and the fact that $\bar{h} \leqslant a_j$ in Ω_j while $u_j \leqslant U_j$ on $\partial\Omega_j$. This inequality and (3.17) yield

$$u_{j-1}(x', 0) \leqslant U_j(x', \tau_j) \quad \forall j \geqslant j_0,\ |x'| < r_{j-1}.$$

Therefore by the comparison principle applied in Ω_{j-1},

$$u_{j-1}(x', x_N) \leqslant U_j(x', x_N + \tau_j) \quad \forall j \geqslant j_0,\ x \in \Omega_{j-1}. \tag{0.3}$$

Let $j > j_0$ and $0 \leqslant k \leqslant j - j_0$. Using (0.3), (3.17) and induction on k we obtain,

$$u_{j-k-1}(x', x_N) \leqslant U_j\left(x', x_N + \sum_{i=0}^{k} \tau_{j-i}\right) \quad \forall x \in \Omega_{j-k-1}. \tag{0.4}$$

DOI of original article: http://dx.doi.org/10.1016/j.anihpc.2012.08.002.

☆ This research was supported by The Israel Science Foundation grant No. 91/10.

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http://dx.doi.org/10.1016/j.anihpc.2012.12.006

By (1.10) and (3.16) $\sum_{j=0}^{\infty}\tau_j=\infty$ and $\sup_{m\geqslant 0}\tau_m=\bar\tau<\infty$. Therefore, if $b>\bar\tau$ then, for every $j\geqslant j_b$, there exists an integer $\lambda_j=\lambda_j(b)$ such that $0\leqslant b-\sum_{\lambda_j+1}^{j}\tau_m=:\delta_j\leqslant\tau_{\lambda_j}$ and $\lambda_j\to\infty$ as $j\to\infty$. Hence by (0.4)

$$u_{\lambda_j}(x',\delta_j)\leqslant U_j\Bigg(x',\delta_j+\sum_{\lambda_j+1}^{j}\tau_m\Bigg)=U_j(x',b)\quad\forall x'\colon\ |x'|<r_{\lambda_j}. \tag{0.5}$$

Applying the comparison principle to u_{λ_j} in Ω_{λ_j} and using (3.12) we find that the inequality $0\leqslant\delta_j\leqslant\tau_{\lambda_j}$ implies $u_{\lambda_j}(0,\delta_j)\geqslant\frac{1}{4\alpha}u_{\lambda_j}(0,\tau_{\lambda_j})$. Therefore (0.5) and (3.17) imply,

$$\frac{1}{4\alpha}A_{\lambda_j-1}^{-1}\leqslant\frac{1}{4\alpha}\gamma_{\lambda_j-1}(0)\leqslant\frac{1}{4\alpha}u_{\lambda_j}(0,\tau_{\lambda_j})\leqslant U_j(0,b).$$

Finally, as $\lim_{k\to\infty}A_k=0$, this inequality implies (0.2) for $x_N>\bar\tau$. It is easy to see that if (0.2) fails for some $x_N>0$ then it fails for all larger values of x_N. Therefore (0.2) holds for every $x_N>0$. □

Completion of proof of Theorem 1.2. Let v_j be the solution of (3.3) where γ_j is replaced by $\Gamma_j=A_j^{-1}r_j^{N-1}\delta_0$ on $\partial\Omega_j\cap[x_N=0]$. As in Section 3.1, the function $\tilde v_j$ defined by $\tilde v_j(y)=A_jv_j(r_jy)$, $y\in D_0$ satisfies the boundary value problem (3.9) with $\tilde\gamma$ replaced by $\tilde\Gamma=\delta_0$. Denote the solution of this problem by $\tilde v$. Next we apply Lemma 3.1 to $\tilde v$ in $D_0\cap[x_N>b]$ (b a fixed positive number). We conclude that choosing $\beta>0$ sufficiently large $0<c(\beta)\leqslant\frac{\tilde v(y',\beta)}{\phi_1(y')}\leqslant 1$ for $|y'|<1$. Hence, if $\gamma_j'(x'):=v_j(x',r_j\beta)$ then $c(\beta)\leqslant\frac{\gamma_j'(x')}{A_j^{-1}\phi_1(x'/r_j)}\leqslant 1$ in the ball $|x'|<r_j$. Obviously $u_j'(x):=v_j(x',x_N+r_j\beta)$ satisfies (3.3) with γ_j replaced by γ_j'. Proceeding as in Section 3.2 we obtain a sequence $\{\tau_j\}$ satisfying (3.16) and

$$\gamma_{j-1}'(x')\leqslant u_j'(x',\tau_j),\quad |x'|\leqslant r_j,\ j\geqslant j_0.$$

Let U_j' (resp. V_j) be defined in the same way as U_j except that γ_j is replaced by γ_j' (respectively Γ_j) extended by zero for $|x'|\geqslant r_j$. Then Proposition 3.1′ applies to $\{U_j'\}$ so that

$$\lim_{j\to\infty}U_j'(0,x_N)=\infty. \tag{0.6}$$

Furthermore $V_j\geqslant v_j$ in Ω_j so that $V_j(x',r_j\beta)>\gamma_j'(x')$, $|x'|<r_j$. By the comparison principle, $V_j(x',x_N+r_j\beta)\geqslant U_j'(x)$ in $\mathbb{R}_+^N$. Hence

$$\lim_{j\to\infty}V_j(0,x_N)=\infty\quad\forall x_N>0.\qquad\Box$$