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Bifurcation and multiplicity results for nonlinear elliptic problems involving critical Sobolev exponents

by

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ABSTRACT. — In this paper we study the existence of nontrivial solutions for the boundary value problem

$$\begin{cases} -\Delta u - \lambda u - u\,|\,u\,|^{2^*-2} = 0 & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

when $\Omega \subset \mathrm{R}^n$ is a bounded domain, $n \geqslant 3$, $2^* = \dfrac{2n}{n-2}$ is the critical exponent for the Sobolev embedding $\mathrm{H}_0^1(\Omega) \subset \mathrm{L}^p(\Omega)$, λ is a real parameter.

We prove that there is bifurcation from any eigenvalue λ_j of $-\Delta$ and we give an estimate of the left neighbourhoods $]\lambda_j^*, \lambda_j]$ of λ_j, $j \in \mathrm{N}$, in which the bifurcation branch can be extended. Moreover we prove that, if $\lambda \in]\lambda_j^*, \lambda_j[$, the number of nontrivial solutions is at least twice the multiplicity of λ_j.

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The same kind of results holds also when Ω is a compact Riemannian manifold of dimension $n \geqslant 3$, without boundary and Δ is the relative Laplace-Beltrami operator.

Key-words: Boundary value problem, critical Sobolev exponent, bifurcation, critical points, eigenvalue, variational problem, Riemannian manifold.

RÉSUMÉ. — Dans cet article, nous étudions l'existence de solutions non triviales pour le problème aux limites

$$\begin{cases} -\Delta u - \lambda u - u\,|\,u\,|^{2^*-2} = 0 & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

où $\Omega \subset \mathrm{R}^n$ est un domaine borné, $n \geqq 3$, $2^* = \dfrac{2n}{n-2}$ est l'exposant critique pour le plongement de Sobolev $\mathrm{H}_0^1(\Omega) \subset \mathrm{L}^p(\Omega)$, λ est un paramètre réel.

Nous démontrons que toute valeur propre λ_j de $-\Delta$ est une valeur de bifurcation, et nous donnons une estimation des voisinages $[\lambda_j^*, \lambda_j]$ de λ_j où existent des solutions non triviales. Nous montrons en outre que le nombre de celles-ci est au moins le double de la multiplicité de λ_j.

On a les mêmes résultats quand Ω est une variété riemannienne compacte de dimension $n \geqq 3$, et Δ l'opérateur de Laplace-Beltrami.

AMS (MOS) Subject Classifications: 35 A 15, 35 J 20, 58 E 99.

INTRODUCTION

Let Ω be a bounded domain in R^n, $n \geqslant 3$, $2^* = \dfrac{2n}{n-2}$ the critical exponent for the Sobolev embedding $\mathrm{H}_0^1(\Omega) \to \mathrm{L}^q(\Omega)$. For a real parameter $\lambda \in \mathbb{R}$ consider the boundary value problem

$$\text{(0.1)} \qquad \begin{cases} -\Delta u - \lambda u - u\,|\,u\,|^{2^*-2} = 0 & \text{in } \Omega \\ u\,|_{\partial\Omega} = 0 \end{cases}$$

corresponding to the functional $f_\lambda : \mathrm{H}_0^1(\Omega) \to \mathbb{R}$ given by

$$\text{(0.2)} \qquad f_\lambda(u) = 1/2 \int_\Omega (\,|\,\nabla u\,|^2 - \lambda\,|\,u\,|^2)dx - 1/2^* \int_\Omega |\,u\,|^{2^*} dx\,.$$

Since the embedding $\mathrm{H}_0^1(\Omega) \to \mathrm{L}^{2^*}(\Omega)$ is not compact the functional f_λ in general will not satisfy the Palais-Smale condition.

However, recently Brezis and Nirenberg [5] were able to establish the existence of positive solutions of (0.1) for any λ in a certain range $]\lambda^*, \lambda_1[$, where λ_j, $j \in \mathbb{N}$ $(\lambda_1 < \lambda_2 < \ldots)$, denote the eigenvalues of the operator $-\Delta : H_0^1(\Omega) \to H^{-1}(\Omega) = (H_0^1(\Omega))^*$, and $\lambda^* \geqslant 0$ is some constant depending on n and Ω.

In this paper we study the existence of nontrivial solutions for (0.1) also for $\lambda > \lambda_j$ to obtain bifurcation from any eigenvalue λ_j. We give an estimate of the left neighbourhoods $]\lambda_j^*, \lambda_j]$ of λ_j, in which the bifurcation branch « can be extended »; moreover we prove that, if $\lambda \in]\lambda_j^*, \lambda_j[$, the number of nontrivial solutions of (0.1) is at least twice the multiplicity of λ_j (cp. Theorem 1.1).

Our results are based on the observation that although the Palais-Smale condition does not hold globally for f_λ (cp. Remark 2.3) it is satisfied locally in a certain energy range (cp. Lemma 2.1 or [5, Remark 2.2]).

We observe that the tools used in proving the above results do not depend on the shape of Ω and on the dimension n.

With suitable modifications the existence and bifurcation results also apply to problem (0.1) posed on a compact Riemannian manifold without boundary of dimension $n \geqslant 3$ (cp. Theorem 1.3).

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1. RESULTS

Let $\|u\| = \left(\int_\Omega |\nabla u|^2 dx\right)^{1/2}$, $|u|_p = \left(\int_\Omega |u|^p dx\right)^{1/p}$ denote the norms in $H_0^1(\Omega)$, $L^p(\Omega)$, respectively, and let

$$S = \inf\{ \|u\|^2 / |u|_{2^*}^2 : u \in H_0^1(\Omega) \setminus \{0\} \}$$

denote the best constant for the embedding $H_0^1(\Omega) \to L^{2^*}(\Omega)$.

THEOREM 1.1. — *For $\lambda > 0$ let $\lambda_+ = \min\{\lambda_j \mid \lambda < \lambda_j\}$, and suppose*

$$\lambda_+ - \lambda < S\,[\text{meas}\,(\Omega)]^{-2/n}.$$

Let m be the multiplicity of λ_+. Then problem (0.1) *admits at least m pairs of nontrivial solutions*

$$\{u_k(\lambda), -u_k(\lambda)\} \qquad k = 1, \ldots, m$$

such that

$$\|u_k(\lambda)\| \to 0 \qquad as \qquad \lambda \to \lambda_+ .$$

∎

REMARK 1.2. — If Ω is starshaped, it is well known that (0.1) admits only the trivial solution for $\lambda \leqslant 0$ (cp. [5] [8]).

A result analogous to Theorem 1.1 holds for the problem

$$-\Delta_M u - \lambda u - u|u|^{2^*-2} = 0 \tag{1.1}$$

on a compact Riemannian manifold M of dimension $\geqslant 3$ and without boundary. Here Δ_M is the Laplace-Beltrami operator on M, $\lambda \geqslant 0$ a parameter and $2^* = \dfrac{2n}{n-2}$ as before. Denote by $H^1(M)$ the closure of $C^\infty(M)$ with respect to the norm

$$\|u\|_M = \left(\int_M (|\nabla u|^2 + |u|^2)dM\right)^{1/2}$$

which in local coordinates on a covering $\{T_h\}$ of M is given by

$$\|u\|_M = \left(\sum_h \int_{T_h}\left(\sum_{i,j=1}^n g^{ij}\frac{\partial u}{\partial x_i}\frac{\partial u}{\partial x_j} + |u|^2\right)\sqrt{g}dx\right)^{1/2}$$

g^{ij} denoting the metric tensor, and $g = \det(g^{ij})$. Note that the quadratic form $\int_M |\nabla u|^2 dM$ is only positive semidefinite in $H^1(M)$, then the operator

$$-\Delta_M : H^1(M) \to H^{-1}(M) := (H^1(M))'$$

possesses eigenvalues $\mu_1 < \mu_2 < \dots \mu_k < \dots$ which are $\geqslant 0$ (cp. Appendix 1 of [*4*]).

THEOREM 1.3. — *For* $\lambda > 0$ *let* $\mu_+ = \min\{\mu_j \mid \lambda < \mu_j\}$ *and suppose*

$$\mu_+ - \lambda < S\left(\int_M dM\right)^{-2/n}.$$

Let m be the multiplicity of μ_+. *Then problem* (1.1) *admits at least m pairs of nonconstant solutions*

$$\{u_k(\lambda), -u_k(\lambda)\} \qquad k = 1, \dots, m$$

such that

$$\|u_k(\lambda)\|_M \to 0 \qquad as \qquad \lambda \to \mu_+ .$$

■

2. PROOF OF THEOREMS 1.1, 1.3

The proof of Theorem 1.1 requires some lemmata.

LEMMA 2.1. — *For any* $\lambda \in \mathbb{R}$ *the functional* f_λ (*see* (0.2)) *satisfies the Palais-Smale condition in* $\left]-\infty, \frac{1}{n}S^{n/2}\right[$ *in the following sense:*

(P. S.) *If* $c < \frac{1}{n} S^{n/2}$ *and* $\{u_m\}$ *is a sequence in* $H_0^1(\Omega)$ *such that as* $m \to \infty$ $f_\lambda(u_m) \to c$, $df_\lambda(u_m) \to 0$ *strongly in* $H^{-1}(\Omega)$, *then* $\{u_m\}$ *contains a subsequence converging strongly in* $H_0^1(\Omega)$.

REMARK 2.2. — An analogous result has been proved in [*5*]. Nevertheless for completeness we give here a proof of lemma 2.1 which is slightly different from that contained in [*5*].

Proof. — Let $\lambda \in \mathbb{R}$, and suppose $\{u_m\}$ is a sequence in $H_0^1(\Omega)$ such that as $m \to \infty$

$$f_\lambda(u_m) \to c_1 < \frac{1}{n} S^{n/2} \tag{2.1}$$

$$df_\lambda(u_m) \to 0 \text{ strongly in } H^{-1}(\Omega). \tag{2.2}$$

As in [*5*, estimates (2.18)] from (2.1), (2.2) we obtain that

$$\{\|u_m\|\} \text{ is bounded}. \tag{2.3}$$

Hence we may extract a subsequence $\{u_m\}$ (relabeled) such that

$$u_m \to u \text{ weakly in } H_0^1(\Omega) \tag{2.4}$$

$$u_m \to u \text{ strongly in } L^p(\Omega) \text{ for any } p \in [1, 2^*[. \tag{2.5}$$

Moreover u is a solution of (0.1). Indeed, letting $\phi \in C_0^\infty(\Omega)$, by (2.4), (2.5) and (2.2) we deduce that

$$\langle df_\lambda(u), \phi \rangle = \langle df_\lambda(u_m), \phi \rangle + o(1) = o(1).$$

Hence u weakly solves (0.1). But by regularity results (cp. [*5*] [*6*] [*7*] and [*10*]) it follows that

$$u \in L^\infty(\Omega) \tag{2.6}$$

and hence that u is regular and is a solution of (0.1) in the classical sense. To show that $u_m \to u$ strongly in $H_0^1(\Omega)$ as $m \to \infty$, let $v_m = u_m - u$. Testing (2.2) with v_m we obtain

$$\begin{aligned} o(1) &= \langle df_\lambda(u_m), v_m \rangle \\ &= \int_\Omega (\nabla u \nabla v_m + |\nabla v_m|^2 - \lambda(u+v_m)v_m - |u+v_m|^{2^*-2}(u+v_m)v_m)dx. \end{aligned} \tag{2.7}$$

By (2.4) and (2.5) we have

$$\int_\Omega (\nabla u \nabla v_m - \lambda(u + v_m)v_m)dx = o(1). \tag{2.8}$$

Whence from (2.7), (2.8) we deduce that

$$\|v_m\|^2 = \int_\Omega |u + v_m|^{2^*-2}(u + v_m)v_m dx + o(1). \tag{2.9}$$

Now we claim that

$$\| v_m \|^2 = | v_m |_{2^*}^{2^*} + o(1) . \tag{2.10}$$

In fact, by using (2.5) and (2.6), we have

$$\begin{aligned} (2.11)\quad & \left| \int_\Omega (u + v_m) | u + v_m |^{2^*-2} v_m dx - \int_\Omega | v_m |^{2^*} dx \right| \\ & = \left| \int_\Omega \int_0^{u(x)} \frac{\partial}{\partial \xi} [(v_m + \xi) | v_m + \xi |^{2^*-2}] v_m d\xi dx \right| \\ & = \left| (2^* - 1) \int_\Omega \int_0^1 | v_m + tu |^{2^*-2} v_m u dt dx \right| \\ & \leqslant \text{const.} \left[\int_\Omega (| u | | v_m |^{2^*-1} + | v_m | | u |^{2^*-1}) dx \right] = o(1) \end{aligned}$$

and (2.10) easily follows from (2.9) and (2.11).
Since

$$\langle df_\lambda(u_m), u_m \rangle = o(1)$$

we have

$$| u_m |_{2^*}^{2^*} = \int_\Omega (| \nabla u_m |^2 - \lambda | u_m |^2) dx + o(1) .$$

Inserting into the expression for $f_\lambda(u_m)$ we obtain

$$\begin{aligned} (2.12)\quad f_\lambda(u_m) & = \frac{1}{n} \int_\Omega (| \nabla u_m |^2 - \lambda | u_m |^2) dx + o(1) \\ & = \frac{1}{n} \int_\Omega (| \nabla u |^2 - \lambda | u |^2) dx + \frac{1}{n} \int_\Omega | \nabla v_m |^2 dx + o(1) . \end{aligned}$$

Moreover, since u is a solution of (0.1)

$$\int_\Omega (| \nabla u |^2 - \lambda | u |^2) dx - \int_\Omega | u |^{2^*} dx = \langle df_\lambda(u), u \rangle = 0 .$$

Whence in particular

$$\int_\Omega (| \nabla u |^2 - \lambda | u |^2) dx \geqslant 0 . \tag{2.13}$$

From (2.12) and (2.13) we now infer

$$\| v_m \|^2 \leqslant n f_\lambda(u_m) + o(1) .$$

Then, by (2.1), for m sufficiently large we obtain

$$\| v_m \|^2 \leqslant c_2 < S^{n/2} . \tag{2.14}$$

Now, by (2.10)

$$\| v_m \|^2 \leqslant S^{-2^*/2} \| v_m \|^{2^*} + o(1) .$$

Or equivalently

$$\| v_m \|^2 (S^{2^*/2} - \| v_m \|^{2^*-2}) \leqslant o(1)\,.$$

Taking account of (2.14) this implies that $v_m \to 0$ strongly in $H_0^1(\Omega)$, concluding the proof. ■

REMARK 2.3. — Complementing the preceding lemma we have a non-compactness result for energies $\geqslant \frac{1}{n} S^{n/2}$. In fact we now show that for any $\lambda \in \mathbb{R}$ there exists a sequence $\{ u_m \} \subset H_0^1(\Omega)$ satisfying the P-S assumptions in $c = \frac{1}{n} S^{n/2}$, which is not relatively compact in $H_0^1(\Omega)$.

Let $x_0 \in \Omega$ and choose a function $\phi \in C_0^\infty(\Omega)$ such that $\phi \equiv 1$ in a neighbourhood $\mathcal{N}$ of x_0. The functions $u_\mu : \mathbb{R}^n \to \mathbb{R}$

$$u_\mu(x) = \frac{[n(n-2)\mu^2]^{\frac{n-2}{4}}}{[\mu^2 + |x - x_0|^2]^{\frac{n-2}{2}}}$$

solve the equation

$$- \Delta u_\mu = u_\mu |u_\mu|^{2^*-2} \quad \text{in } R^n\,. \tag{2.15}$$

Let

$$u_m = \phi u_{\mu_m} \qquad \mu_m = \frac{1}{m}.$$

Note that $u_m \in H_0^1(\Omega)$ and moreover

$$\{ u_m \} \text{ is uniformly bounded in } H_0^1(\Omega)\,. \tag{2.16}$$

Also we easily derive that as $m \to +\infty$

$$\nabla u_{\mu_m} \to 0 \quad \text{in } L^2(\mathbb{R}^n \backslash \mathcal{N}) \tag{2.17}$$

$$u_m \to 0 \quad \text{in } L^\infty_{\mathrm{loc}}(\Omega \backslash \{ x_0 \})\,. \tag{2.18}$$

Hence also

$$u_m \to 0 \text{ weakly in } H_0^1(\Omega) \qquad (m \to \infty)\,. \tag{2.19}$$

Using (2.17) and (2.18) we deduce that

$$\begin{aligned} f_\lambda(u_m) &= 1/2 \int_{R^n} |\nabla u_{\mu_m}|^2 dx - 1/2^* \int_{R^n} |u_{\mu_m}|^{2^*} dx + o(1) \\ &= \frac{1}{n} S^{n/2} + o(1) \end{aligned} \tag{2.20}$$

(cp. [1] [9]).

Also using (2.15)-(2.18) we obtain

$$\| df_\lambda(u_m) \|_{H^{-1}(\Omega)} = \sup_{\substack{v \in H_0^1(\Omega) \\ \|v\|_{H_0^1} = 1}} \int_{R^n} (\nabla u_{\mu_m} \nabla v - u_{\mu_m} | u_{\mu_m} |^{2^*-2} v) dx + o(1) = o(1)$$

Hence $\{ u_m \}$ satisfies the (P-S) assumptions with $c = \frac{1}{n} S^{n/2}$, however, by (2.19) and (2.20), $\{ u_m \}$ cannot be relatively compact in $H_0^1(\Omega)$.

LEMMA 2.4. — *For* $\lambda > 0$ *let* $\lambda_+ = \inf \{ \lambda_j \mid \lambda < \lambda_j \}$ *and set*

$$M_+ = \overline{\bigoplus_{\lambda_j \geq \lambda_+} M(\lambda_j)} \qquad \text{(the closure is taken in } H_0^1(\Omega))$$
$$M_- = \bigoplus_{\lambda_j \leq \lambda_+} M(\lambda_j)$$

where $M(\lambda_j)$ *denotes the eigenspace of* $-\Delta$ *corresponding to* λ_j. *Then*

$$\beta_\lambda := \sup_{u \in M_-} f_\lambda(u) \leq (\lambda_+ - \lambda)^{n/2} \frac{\text{meas}\,(\Omega)}{n},$$

moreover, there exist constants $\rho_\lambda > 0$, $\delta_\lambda \in]0, \beta_\lambda[$ *such that*

$$f_\lambda(u) \geq \delta_\lambda \qquad \text{for any } u \in M_+, \ \| u \| = \rho_\lambda .$$

Proof. — For any $u \in M_-$ we have

$$f_\lambda(u) = 1/2 \int_\Omega (|\nabla u|^2 - \lambda |u|^2) dx - 1/2^* \int_\Omega |u|^{2^*} dx$$
$$\leq 1/2(\lambda_+ - \lambda) \int_\Omega |u|^2 dx - 1/2^* \int_\Omega |u|^{2^*} dx$$
$$\leq 1/2(\lambda_+ - \lambda)\, \text{meas}\,(\Omega)^{2/n} \left\{ \int_\Omega |u|^{2^*} dx \right\}^{2/2^*} - 1/2^* \int_\Omega |u|^{2^*} dx .$$

Let

$$g(\rho) = 1/2(\lambda_+ - \lambda)\, \text{meas}\,(\Omega)^{2/n} \rho^2 - 1/2^* \rho^{2^*} .$$

Then

$$\sup_{u \in M_-} f_\lambda(u) \leq \sup_{\rho \geq 0} g(\rho) = \frac{1}{n} (\lambda_+ - \lambda)^{n/2}\, \text{meas}\,(\Omega)$$

proving the first part of the lemma.
Since for $u \in M_+$ we obtain

$$\int_\Omega (|\nabla u|^2 - \lambda |u|^2) dx \geq \left(1 - \frac{\lambda}{\lambda_+}\right) \| u \|^2$$

while

$$|u|_{2^*}^{2^*} \leq \text{const}\, \| u \|^{2^*} .$$

The second part of the claim is immediate. ■

By lemmata 2.1, 2.4, Theorem 1.1 can be deduced by the following result of Bartolo, Benci, Fortunato (cp. Theorem 2.4 of [3]), which is a variant of some results contained in [0].

THEOREM 2.5. — *Let* H *be a real Hilbert space with norm* $\| \cdot \|$ *and suppose* $I \in C^1(H, \mathbb{R})$ *is a functional on* H *satisfying the following conditions:*

I_1) $I(u) = I(-u)$, $I(0) = 0$;

I_2) *There exists a constant* $\beta > 0$ *such that the Palais-Smale condition* (P-S) *holds in* $]0, \beta[$;

I_3) *There exist two closed subspaces* $V, W \subset H$ *and positive constants* ρ, δ, β', *with* $\delta < \beta' < \beta$ *such that*

i) $I(u) \leqslant \beta'$ *for any* $u \in W$

ii) $I(u) \geqslant \delta$ *for any* $u \in V, \| u \| = \rho$

iii) codim $V < +\infty$ and dim $W \geqslant$ codim V.

Then there exists at least

$$\dim W - \operatorname{codim} V$$

pairs of critical points of I *with critical values belonging to the interval* $[\delta, \beta']$.

We are now ready to prove Theorem 1.1.

Proof of Theorem 1.1. — Let $H = H_0^1(\Omega)$, $I = f_\lambda$, $V = M_+$, $W = M_-$, $\beta = \frac{1}{n} S^{n/2}$, $\beta' = \beta_\lambda$, $\delta = \delta_\lambda$, $\rho = \rho_\lambda$ and apply Theorem 2.5 together with lemmata 2.1, 2.4. ■

For the proof of Theorem 1.3 the following result from [2] is needed.

LEMMA 2.6. — *If* $\{ v_m \}$ *is a sequence in* $H^1(M)$ *such that* $v_m \rightharpoonup 0$ *weakly in* $H^1(M)$ *as* $m \to \infty$, *then*

$$\left(\int_M | v_m |^{2^*} dM \right)^{2/2^*} \leqslant S^{-1} \| v_m \|_M^2 + o(1).$$

Proof. — By [2, Theorem 2.21] for all $\phi \in H^1(M)$, $\varepsilon > 0$

$$\left(\int_M | \phi |^{2^*} dM \right)^{2/2^*} \leqslant (S^{-1} + \varepsilon) \int_M | \nabla \phi |^2 dM + A(\varepsilon) \int_M | \phi |^2 dM$$

with a constant $A(\varepsilon)$ independent of ϕ. Applying this inequality with $\phi = v_m$, and noting that by weak convergence $v_m \to 0$ $(m \to +\infty)$ we have

$$\int_M | v_m |^2 dM \to 0 \qquad m \to +\infty$$

we deduce that for any $\varepsilon > 0$

$$\left(\int_M | v_m |^{2^*} dM \right)^{2/2^*} \leqslant (S^{-1} + \varepsilon) \| v_m \|_M^2 + o(1).$$

The lemma follows on letting $\varepsilon \to 0$. ■

Proof of Theorem 1.3. — Going through the proof of Lemma 2.1 — keeping in mind Lemma 2.6 and the fact that, for any sequence $\{ v_m \}$

in $H^1(M)$ tending to 0 weakly in this space, $\| v_m \|_2 = o(1)$ — it is now immediate that also for the functional on $H^1(M)$

$$f_\lambda(u) = 1/2\int_M (\,|\nabla u|^2 - \lambda\,|u|^2)dM - 1/2^*\int |u|^{2^*}dM$$

corresponding to problem (1.1) the Palais-Smale condition is satisfied in the interval $\left]-\infty, \frac{1}{n}S^{n/2}\right[$.

Moreover it is easy to see that the same estimates of lemma 2.4 continue to hold (obviously $\lambda_j, \lambda_+, H_0^1(\Omega)$, meas Ω are replaced respectively by $\mu_j \cdot \mu_+ \cdot H^1(M), \int_M dM$). Then Theorem 1.3 can be proved by using again the abstract critical point Theorem 2.5. ∎

REFERENCES

[0] A. AMBROSETTI, P. H. RABINOWITZ, Dual variational methods in critical point theory and applications, *J. funct. Anal.*, t. **14**, 1973, p. 349-381.

[1] Th. AUBIN, Problèmes isopérimétriques et espaces de Sobolev, *J. Diff. Geom.*, t. **11**, 1976, p. 573-598.

[2] Th. AUBIN, *Nonlinear analysis on manifolds, Monge-Ampere equations*. Springer Grundelehren 252, 1982.

[3] P. BARTOLO, V. BENCI, D. FORTUNATO, Abstract critical point theorems and applications to some nonlinear problems with « strong resonance » at infinity, *Journal of nonlinear Anal. T. M. A.*, t. **7**, 1983, p. 981-1012.

[4] V. BENCI, D. FORTUNATO, The dual method in critical point Theory. Multiplicity results for indefinite functionals, *Ann. Mat. Pura Appl.*, t. **32**, 1982, p. 215-242.

[5] H. BREZIS, L. NIRENBERG, Positive solutions of nonlinear elliptic equations involving critical Sobolev exponents, *Comm. Pure Appl. Math.*, t. **XXXVI**, 1983.

[6] H. BREZIS, T. KATO, Remarks on the Schrödinger operator with singular complex potential, *J. Math. Pures et Appl.*, t. **58**, 1979, p. 137-151.

[7] S. LUCKHAUS, Existence and regularity of weak solutions to the Dirichlet problem for semilinear elliptic systems of high order, *J. Reine und Angew. Math.*, t. **306**, 1979, p. 192-207.

[8] S. J. POHOZAEV, Eigenfunctions of the equation $\Delta u + \lambda f(u) = 0$, *Soviet Math. Doklady*, t. **6**, 1965, p. 1408-1411 (Translated from the Russian *Dokl. Akad. Nauk SSSR*, t. **165**, 1965, p. 33-36.)

[9] G. TALENTI, Best constants in Sobolev inequality, *Ann. Mat. Pure Appl.*, t. **110**, 1976, p. 353-372.

[10] N. TRUDINGER, Remarks concerning the conformal deformation of Riemannian structure on compact manifolds, *Ann. Sc. Norm. Sup. Pisa*, t. **22**, 1968, p. 265-274.

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