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An energy estimate of an exterior problem and a Liouville theorem for harmonic maps

by

Dong ZHANG (*)

Department of Mathematics,
The Johns Hopkins University Baltimore,
MD 21218, U.S.A.

ABSTRACT. – We prove that the exterior harmonic maps, from $R^n \backslash \Omega$ ($n \geqq 3$) to a bounded strictly convex geodesic ball of some Riemannian manifolds, have finite conformal invariant energy. A consequence of this estimate is a Liouville theorem which states that harmonic maps between Euclidean space (R^n, g_0) and Riemannian manifolds are constant maps provided their image at infinity falls into a bounded strictly convex geodesic ball.

RÉSUMÉ. – Nous démontrons que des applications harmoniques extérieures, de $R^n \backslash \Omega$ ($n \geqq 3$) vers une boule géodésique donnée strictement convexe d'une variété riemannienne, ont une énergie invariante conforme finie. Une conséquence de ce résultat est un théorème de Liouville qui montre qu'une application harmonique, entre un espace euclidien (R^n, g_0) et des variétés riemanniennes, est constante dès que son image à l'infini est continue dans une boule géodésique bornée strictement convexe.

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1. INTRODUCTION

In this paper, we study the finiteness of the conformal invariant energy of an exterior boundary value problem of harmonic maps on $(R^n \backslash \Omega, g_0)$ to a bounded, strictly convex, domain of a Riemannian manifold (N, h) where $\Omega \subset R^n$ is a bounded domain. A domain of a Riemannian manifold is strictly convex if there exists a strictly convex function Φ defined on it such that its Hessian tensor has a positive lower bound. In particular, when the domain is a geodesic ball $B_r(p)$ centered at p of radius $r < \pi/(2\sqrt{\kappa})$ where κ is a upper bound for the sectional curvature of N, we choose $\Phi(q) = d^2(q)$ where $d(q)$ is the distance between $q \in K$ and the center p (*see* [H-J-W] and [Cho]). A general existence theorem of this kind of boundry value problem has been proved by Hildebrandt, Kaul and Widman [H-K-W]. Although their results are for compact manifolds with boundary, they are also true for complete manifolds with boundary without any alteration while the solution is of finite energy. As an application of this estimate, we prove a Liouville theorem which states that harmonic maps between Euclidiean space (R^n, g_0) and Riemannian manifolds are constant maps provided their image at infinity falls into a bounded, strictly convex, domain.

The classical theorem of Liouville ensures that a non-constant entire harmonic function on R^2 cannot be bounded. A well known generalization of this theorem states that a non-constant harmonic function on R^n must be unbounded. In 1975, Yau [Ya] proved that any bounded harmonic function on a complete Riemannian manifold of non-negative Ricci curvature is a constant function. On the other hand, there do exist many bounded harmonic functions on simply connected and negatively curved manifolds which were proved by Anderson [An] and Sullivan [Su]. In the study of Liouville type theorems of harmonic maps, mathematicians naturally chose to study problems with domain manifolds M of non-negative Ricci curvature. Schoen and Yau [S-Y] showed that a harmonic map of finite energy, from M to a manifold of non-positive sectional curvature, must be a constant map. Later, Cheng [Che] proved that Liouville's theorem is true for harmonic maps from M to a simply-connected complete manifold of sectional curvature $K \leqq 0$ provided its growth at infinity is slower than the linear rate. Choi [Cho] showed that a harmonic map U is a constant map if $U(M)$ falls into a strictly convex domain of the target manifold. Assuming finiteness of the energy, Hildebrandt [Hi] and Sealey [Se] studied harmonic maps from R^n with certain globally conformal flat metrics $f g_0$ to a Riemannian manifold.

Jin [Ji], following the works of Hildebrandt [Hi] and Sealey [Se], obtained a Liouville type of result for harmonic maps on R^n with the assumption of certain asymptotic behavior, namely, constant or almost boundary value at infinity. There are two kinds of assumptions in the Liouville type of theorems for harmonic maps, *i.e.*, either the finiteness of the energy of the smallnesss of the whole image and is interested in finding other kinds of conditions. For further information about this subject, we refer readers to a new report on harmonic maps by Eells and Lemaire [E-L].

The monotonicity formula implies (*see* [Hi] and [Se]) that a non-constant harmonic map from $(R^n, g = f g_0)$ must have infinite energy and provides a lower bound for the energy growth when the energy is infinite. We shall call the following integral

$$\int_{R^n \backslash \Omega} \frac{|\nabla U|_g^2(x)}{[\mathrm{dist}_g(O, x)]^{n-2}} \, dvol_g$$

the conformal invariant energy of U. Instead of estimating the energy growth directly as in [Ji], we first study the finiteness of the conformal invariant energy of the exterior problem for harmonic maps equation. Under the assumption that their image at infinity falls into a bounded strictly convex domain, we prove that the solution has finite conformal invariant energy. Aplying the above estimate to the harmonic maps on R^n whose image at infinity falls into a bounded strictly convex domain, we are able to show that, in fact, they have finite energy, *i.e.*,

$$\int_{R^n} |\nabla U|^2 \, dx < \infty$$

where $O \in R^n$ is the origin. Then, the uniqueness and a Liouville type of result follows.

THEOREM 1. – *Suppose* $U : (R^n, g_0) \to (N, h)$ *is a weak* $W^{1,2}_{\mathrm{loc}}(R^n, N)$ *harmonic map* $(n \geqq 3)$ *and let* $K \subset N$ *be a bounded strictly convex geodesic ball. If*

$$U(\{x \in R^n \mid |x| > r > 0\}) \subset K$$

for some large r, *then* U *has finite energy and is a constant map.*

We will show that the above result is true on R^n with certain globally conformal flat metrics, *i.e.*, $(R^n, f g_0)$ as in [Hi] and [Se].

2. THE EXTERIOR PROBLEM

A map $U : M^n \to N^m$ between two Riemannian manifolds (M^n, g) and (N^m, h) is a harmonic map if it is a critical point of the energy functional

$$E(U) = \frac{1}{2} \int_{M^n} e(U)\, dV_g$$

where $e(U) = \mathrm{Tr}(g)(U^* h)$ is the energy density of the map U. $e(U)$ can be written in terms of a local coordinate as follows

$$e(U)(x) = h_{ij}(u(x))\, g^{\alpha\beta}(x)\, \partial_\alpha u^i(x)\, \partial_\beta u^j(x)$$

where $(g^{\alpha\beta}) = (g_{\alpha\beta})^{-1}$, x and u are local coordinates in M^n and N^m respectively,

$$g = g_{\alpha\beta}(x)\, dx^\alpha\, dx^\beta \qquad \text{and} \qquad h = h_{ij}(u)\, du^i\, du^j$$

and $u(x)$ is a local representation of U. Or, equivalently, a map $U(x) \in W^{1,2}_{\mathrm{loc}}(M, N)$ is harmonic if it satisfies the corresponding system of Euler equations

$$\Delta_{M^n} u^i = \Gamma^i_{jk}(u)\, g^{\alpha\beta}\, \partial_\alpha u^j\, \partial_\beta u^k, \quad 1 \leqq i \leqq m$$

where $\Delta_{M^n} = [\det(g_{\alpha\beta})]^{-1/2}\, \partial_\alpha \{[\det(g_{\alpha\beta})]^{1/2}\, g^{\alpha\beta}\, \partial_\beta\}$ is the Laplace-Beltrami operator on (M^n, g) and Γ^i_{jk} denote the Christoffel symbols of (N^m, h)

$$\Gamma^i_{jk} = \frac{1}{2} h^{li} \left(\frac{\partial h_{lk}}{\partial u^j} - \frac{\partial h_{jk}}{\partial u^l} + \frac{\partial h_{il}}{\partial u^k} \right).$$

In this section, we study the solutions of the exterior Dirichlet problem for harmonic maps from $(R^n \backslash \Omega, f g_0)$ to a bounded, strictly convex domain K of a Riemannian manifold (N, h) where $\Omega \subset R^n$ is a bounded domain. We will prove that these solutions have finite conformal invariant energy. As in [Hi], [Ji] and [Se], we only consider the case of $M = R^n$ and $g = f g_0$ where f will be satisfying some of the following conditions

(i) $f \in C^1(R^n)$ is positive and

$$r \frac{\partial f}{\partial r}(x) \geqq -(n-2) f(x), \qquad x \in R^n;$$

(ii) there exist constant $\sigma > 0$ and $r_0 > 1$ such that

$$r \frac{\partial f}{\partial r}(x) \geqq -\left(\frac{2\sigma}{n-2} - 2 \right) f(x), \qquad |x| \geqq r_0;$$

(iii) for some $c > 0$

$$f(x) \leqq c\, |x|^{\frac{2\sigma}{n-2} - 2}, \qquad |x| \geqq r_0.$$

(ii) implies a lower bound for f, *i.e.*, for some $c' > 0$

$$c' \,|\, x \,|^{\frac{2\sigma}{n-2} - 2} \leqq f(x), \qquad |\, x \,| \geqq r_0.$$

In the case of Euclidean space, we have $f \equiv 1$ and $\sigma = n - 2$.

Our objective is to study the solutions of the following exterior Dirichlet problem,

$$(\star) \quad \begin{cases} \Delta_g u^i = \Gamma^i_{jk}(u)\, g^{\alpha\beta}\, \partial_\alpha u^j\, \partial_\beta u^k, & x \in R^n \backslash \Omega, \quad 1 \leqq i \leqq m; \\ U \in W^{1,2}_{\text{loc}}(R^n \backslash \Omega,\, K); \\ U \,|_{\partial\Omega} = U_0; \end{cases}$$

where $K \subset N$ is a bounded strictly convex domain. By our assumption, K possesses a strictly convex function $\Phi \in C^2(\bar{K})$. We will show that the solutions of the exterior Dirichlet problem $(\star)$ have finite conformal invariant energy. From (ii)$'$ and (iii), we have

$$[\text{dist}_g(O,\, x)]^{n-2} \sim |\, x \,|^{\sigma} \tag{1}$$

at the infinity where $O \in R^n$ is the origin. Therefore, the conformal invariant energy can be written as the following,

$$\int_{R^n \backslash \Omega} |\, \nabla U \,|_g^2(x) \cdot |\, x \,|^{-\sigma}\, dVol_g < \infty.$$

THEOREM 2. – *Suppose that f satisfies* (ii) *and* (iii), *U is a solution of the exterior Dirichlet problem $(\star)$. Then, U has finite conformal invariant energy, i.e.,*

$$\int_{R^n \backslash \Omega} |\, \nabla U \,|_g^2(x) \cdot |\, x \,|^{-\sigma}\, dVol_g < \infty.$$

Proof. – We shall prove our theorem by contradiction. Let $U \in W^{1,2}_{\text{loc}}(R^n \backslash \Omega)$ be a solution of $(\star)$ for some U_0. From now on, we will embed K into a Euclidean space isometrically. $e(U)$ can be written as $|\, \nabla U |_g^2$.

Suppose the above integral is not finite. By the assumption $K \subset N$ is bounded and strictly convex, there exists a positive, strictly convex, function $\Phi \in C^2(\bar{K})$. Let $\lambda > 0$ denote a lower bound of the eigenvalues of Φ's Hessian tensor (Φ_{ij}) on $\bar{K}$, then, we have

$$\Delta_g [\Phi \circ U(x)] \geqq \lambda \,|\, \nabla U \,|_g^2(x), \qquad x \in R^n \backslash \Omega. \tag{3}$$

The above inequality appeared in both [Cho] and [GH]. Choi used it to study Liouville theorems for harmonic maps. Giaquinta and Hildebrandt used it to study the regularity of harmonic maps. Let $B_r \subset R^n$ denote the ball centered at the origin of radius r. Without losing generality, we may assume $\Omega \subset B_{r_0}$. We will apply Green's formula to $\Phi \circ U(x)$ over the annulus $B_r \backslash B_{r_0}$. Since f satisfies (ii), it easy to verify that function $|x|^{-\sigma}$ is super harmonic, *i.e.*,

$$\Delta_g (|x|^{-\sigma}) \leqq 0. \tag{4}$$

Multiply the above inequality by $|x|^{-\sigma}$ and integrate it over the annulus $B_r \backslash B_{r_0}$ for some $r > r_0$ and we then have

$$\begin{aligned} &\int_{B_r \backslash B_{r_0}} (\Delta_g [\Phi \circ U(x))]) \cdot |x|^{-\sigma} \, dvol_g \\ &\geqq \lambda \int_{B_r \backslash B_{r_0}} |\nabla U|_g^2 (x) \cdot |x|^{-\sigma} \, dvol_g. \end{aligned} \tag{5}$$

The right hand side of the above inequality if of the form of our conformal invariant energy. We shall control the right hand side of the above inequality. Integrate the left side by part and we get

$$\begin{aligned} &\int_{B_r \backslash B_{r_0}} \Delta_g (|x|^{-\sigma} \cdot [\Phi \circ U(x)] \, dvol_g \\ &+ \left(\int_{S_r} - \int_{S_{r_0}} \right) f^{\frac{n-2}{2}} (x) \cdot |x|^{-\sigma} \cdot \frac{\partial}{\partial |x|} [\Phi \circ U(x)] \cdot |x|^{n-1} \, d\omega \\ &+ \left(\int_{S_r} - \int_{S_{r_0}} \right) f^{\frac{n-2}{2}} \frac{\partial}{\partial |x|} (|x|^{-\sigma}) \cdot [\Phi \circ U(x)] \cdot |x|^{n-1} \, d\omega \end{aligned}$$

where $S_r = \partial B_r$ and $d\omega$ is the volume form of the unit sphere S_1. From (4), we know that the first term is not greater than zero. We may dismiss the first term. Using the assumption that f satisfies (iii) and K is bounded, it is easy to see that the third term is bounded by a constant wich depends only on σ, c, n and K, in fact, the constant can be written as the following

$$c^{(n-2)/2} \sigma \int_{S_1} [\Phi \circ U(r_0 \omega) + \Phi \circ U(r_0 \omega)] \, d\omega.$$

The second term is dominated by

$$\begin{aligned} &\left(\int_{S_r} + \int_{S_{r_0}} \right) f^{\frac{n-2}{2}} (x) \cdot |x|^{-\sigma} \cdot |\nabla \Phi (U(x))| \\ &\times f^{\frac{1}{2}} (x) \cdot [|\nabla U|_g^2 (x)]^{\frac{1}{2}} (x) \cdot |x|^{n-1} \, d\omega \end{aligned}$$

The above integral over S_{r_0} may be controled by

$$c^{\frac{n-2}{2}} r_0 \int_{S_1} |\nabla \Phi (U (r_0 \omega))| \cdot |\nabla U (r_0 \omega)| \, d\omega$$

which is a constant. To estimate the above integral over S_r, we first write it as a integal over S_1. Apply Schwarz inequality, we have that the integral over S_r is dominated by

$$\left(\int_{S_1} |\nabla \Phi (U (r \omega))|_h^2 \, d\omega\right)^{\frac{1}{2}} \times \left(\int_{S_1} |\nabla U|_g^2 (r \omega) \cdot r^{-2\sigma} \cdot f^{n-1} \cdot r^{2(n-1)} \, d\omega\right)^{\frac{1}{2}} \tag{6}$$

where the first integral

$$\left(\int_{S_1} |\nabla \Phi (U (r \omega))|_h^2 \, d\omega\right)$$

is bounded. Since f satisfies (iii),

$$r^{-2\sigma} . f^{n-1} . r^{2(n-1)} = (r^{-\sigma} . f^{\frac{n-2}{2}} . r^{n-2}) . [r^{-\sigma} . f^{\frac{n}{2}} . r^{n-1}] \leqq cr \, [r^{-\sigma} . f^{\frac{n}{2}} . r^{n-1}].$$

Therefore, the second integral is bounded by

$$\left(r \int_{S_1} |U|_g^2 (r \omega) \cdot r^{-\sigma} \cdot f^{\frac{n}{2}} \cdot r^{n-1} \, d\omega\right)^{\frac{1}{2}} \tag{7}$$

Let E_r denote

$$\int_{B_r \backslash B_{r_0}} |\nabla U|_g^2 (x) \cdot |x|^{-\sigma} \, dvol_g.$$

Then, E_r' is the second integral of (6), *i.e.*,

$$\frac{d}{dr} E_r = \int_{S_1} |\nabla U|_g^2 (r \omega) \cdot r^{-\sigma} . f^{\frac{n}{2}} \cdot r^{n-1} \, d\omega.$$

Combining the above estimates with (5) and (7), for some constant C, we have the following

$$C + C \, [r \, E_r']^{\frac{1}{2}} \geqq \lambda \, E_r.$$

Since we assume that lim E_r as $r \to \infty$ is infinite, for some large $r_1 > 0$, we have

$$2 \, C \, [r \, E_r']^{\frac{1}{2}} \geqq \lambda \, E_r$$

when $r \geqq r_1$. This inequality implies, for $r \geqq r_1$,

$$\frac{\lambda^2}{4\,C^2}\,(\ln r - \ln r_1) \leqq \frac{1}{E_{r_1}} - \frac{1}{E_r}\,.$$

This contradiction finishes our proof. □

Remark 1. – Technically, we only need a Φ and control of

$$\int_{S_1} \Phi \circ U\,(r\,\omega)\,d\omega \qquad \text{and} \qquad \int_{S_1} |\,\nabla\,\Phi\,(U\,(r\,\omega))\,|_h^2\,d\omega$$

when r is large.

A consequence of the above theorem is the following energy growth estimate,

COROLLARY 1. – *Suppose that f satisfies* (ii) *and* (iii), *U is a solution of the exterior Dirichlet problem* ($\star$). *Then,*

$$\lim_{r\to\infty}\left[r^{-\sigma}\int_{B_r\setminus\Omega} |\,\nabla\,U\,|_g^2\,(x)\,dvol_g\right] = 0.$$

Proof. – We write

$$r^{-\sigma}\int_{B_r\setminus\Omega} |\,\nabla\,U\,|_g^2\,(x)\,dvol_g$$

as follows

$$r^{-\sigma}\int_{B_{r_2}\setminus\Omega} |\,\nabla\,U\,|_g^2\,(x)\,dvol_g + r^{-\sigma}\int_{B_r\setminus B_{r_2}} |\,\nabla\,U\,|_g^2\,(x)\,dvol_g$$

for some big $r_2 > r_1$. The second integral is less than

$$\int_{B_r\setminus B_{r_2}} |\,\nabla\,U\,|_g^2\,(x)\cdot|\,x\,|^{-\sigma}\,dvol_g$$

which is small when r_2 is large by Theorem 2. We then fix r_2 and let r goes to infinity. □

3. A LIOUVILLE THEOREM FOR HARMONIC MAPS

In this section, we will apply our energy estimates obtained in the last section for the exterior problems to the harmonic maps on $(R^n,\ f g_0)$ and prove the following Liouville type theorem.

THEOREM 1. – *Suppose* $U : (R^n, f g_0) \to (N, h)$ *is a weak* $W^{1,2}_{\mathrm{loc}}(R^n, N)$ *harmonic map* $(n \geqq 3)$ *where* f *satisfies* (i), (ii) *and* (iii). *Let* $K \subset N$ *be a bounded, stricly convex, domain. If*

$$U(\{x \in R^n \mid 0 < r_0 < |x|\}) \subset K$$

for some large r_0, *then* U *is a constant map.*

Proof. – We will prove this theorem by showing that $E(U)$ is finite. Because f satisfies (i), Corollary 1 of [Se] says that U is a constant map.

Let's assume $E(U) = \infty$. We have (*see* [Hi] and [Se]) the monotonicity formula, *i.e.*,

$$r^{-\sigma} \int_{B_r} |\nabla U|_g^2 \, dvol_g$$

is a non-decreasing function of r. Therefore, we have

$$r^{-\sigma} \int_{B_r \setminus B_{r_0}} |\nabla U|_g^2 \, dvol_g \geqq c > 0$$

for some positive constant c which depends on σ, r_0 and U when r is large. The above inequality contradicts the conclusion of our Corollary. Therefore, U has finite Dirichlet energy. □

Remark 2. – The following examples indicate the complexity of this kind of problem.

(1) Let $S^n \subset R^{n+1}$ be the unit sphere, the following map

$$R^n \ni (x) \mapsto \left(\frac{x}{|x|}, 0\right) \in S^n \subset R^{n+1}$$

is harmonic and has infinite conformal invariant energy. Its image is the boundary of the convex set, *i.e.*, semisphere.

(2) Let p be the stereographic projection from R^2 to S^2. The map

$$R^2 \times S^n \ni (x, y) \mapsto p(x) \in S^2$$

is harmonic.

The following is a special case of our Remark 1 after theorem 2 of $N = R^1$ and is probability known, but since we do not know of any literature with this result, we state it here as a corollary.

COROLLARY 2. – *Suppose* $u \in W^{1,2}_{\mathrm{loc}}(R^n)$ *is a harmonic function and for some large* $c > 0$

$$\left\{ \int_{S_1} u^2(r\,\omega)\, d\omega \mid c < r \right\} \subset R^1$$

is a bounded set. Then, u *is a constant function.*

Proof. – For the case of $n \geqq 3$, we take $\Phi(u) = u^2$ and apply Remark 1. When $n = 2$ and 1, the proof is trivial. □

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REFERENCES

[An] M. T. ANDERSON, The Dirichlet Problem at Infinity for Manifolds of Negative Curvature, *J. Diff. Geo.*, Vol. **18**, 1983, pp. 701-721.

[Che] S. Y. CHENG, Liouville Theorem for Harmonic Maps, *Proc. Sympos. Pure Math.*, Vol. **36**, AMS, Providence, R.I., 1980, pp. 147-151.

[Cho] H. I. CHOI, On the Liouville Theorem for Harmonic Maps, *Proc. of AMS*, Vol. **85**, No. 1, 1980, pp. 91-94.

[E-L] J. EELLS Jr. and L. LEMAIRE, Another Report on Harmonic Maps, *Bull. London Math. Soc.*, Vol. **20**, 1988, pp. 385-524.

[E-S] J. EELLS Jr. and J. H. SAMPSON, Harmonic Mappings of Riemannian Manifolds, *Amer. J. Math.*, Vol. **86**, 1964, pp. 109-160.

[GH] M. GIAQUINTA and S. HILDEBRANDT, *A priori* Estimates for Harmonic Mappings. *J. Reine Angew. Math.*, Vol. **336**, 1982, pp. 124-164.

[Hi] S. HILDEBRANDT, Liouville Theorems for Harmonic Maps and an Approach to Beinstein Theorems, *Ann. of Math. Studies*, Vol. **102**, 1982, S. T. Yau ed., pp. 107-131.

[H-J-W] S. HILDEBRANDT, J. JOST and K. O. WIDMAN, Harmonic Mappings and Minimal Submanifolds. *Inventiones Math.*, Vol. **62**, 1980, pp. 269-298.

[H-K-W] S. HILDEBRANDT, H. KAUL and K. O. WIDMAN, An Existence Theory for Harmonic Mappings of Riemannian Manifolds. *Acta Math.*, Vol. **138**, 1977, pp. 1-16.

[Ji] Z. JIN, Liouville Theorems for Harmonic Maps, *Inventiones Math.*, to appear.

[Sc] R. SCHOEN, Analytic Aspects of the Harmonic Map Problem, Seminar on Non-linear PDE, S. S. Cherned. ed, pp. 321-358.

[Se] H. C. J. SEALEY, Some Conditions Ensuring the Vanishing of Harmonic Differential Forms with Applications to Harmonic Maps and Yang-Mills Theory, *Math. Proc. Camb. Phil. Soc.* Vol. **91**, 1982, pp. 441-452.

[S-U] R. SCHOEN and K. UHLENBECK, A Regularity Theory for Harmonic Maps, *J. Diff. Geom.*, Vol. **60**, 1984, pp. 307-335.

[Su] D. SULLIVAN, The Dirichlet Problem at Infinity for a Negatively Curved Manifold, *J. Diff. Geo.*, Vol. **18**, 1983, pp. 723-732.

[S-Y] R. SCHOEN and S. T. YAU, Harmonic Maps and the Topology of Stable Hypersurfaces and Manifolds with non-negative Ricci Curvature, *Comm. Math. Helv.*, Vol. **51**, 1976, pp. 333-341.

[Ya] S. T. YAU, Harmonic Functions on Complete Riemannian Manifolds, *Comm. Pure Appl. Math.*, Vol. **28**, 1975, pp. 201-228.

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