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An existence result for nonlinear elliptic problems involving critical Sobolev exponent

by

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ABSTRACT. — In this paper we consider the following problem:

$$(1) \qquad \begin{cases} -\Delta u - \lambda u = |u|^{2^*-2} \cdot u \\ u = 0 \quad \text{on} \quad \partial\Omega \end{cases} \qquad 2^* = 2n/(n-2)$$

where $\Omega \subset \mathbb{R}^n$ is a bounded domain and $\lambda \in \mathbb{R}$.

We prove the existence of a nontrivial solution of (1) for any $\lambda > 0$, if $n \geqslant 4$.

RÉSUMÉ. — Soient Ω un sous-ensemble ouvert borné de $\mathbb{R}^n$ et λ un nombre positif, le but de cette note c'est de montrer que le problème suivant :

$$\begin{cases} -\Delta u - \lambda u = |u|^{2^*-2} \cdot u \\ u|_{\partial\Omega} = 0 \end{cases} \qquad 2^* = 2n/(n-2)$$

admet, au moins, une solution non triviale, si $n \geqslant 4$.

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0. INTRODUCTION

Let $\Omega \subset \mathbb{R}^n$, $n \geq 3$, be an open bounded set with smooth boundary. Consider the problem

$$(0.1) \qquad \begin{cases} -\Delta u - \lambda u - u \cdot |u|^{2^*-2} = 0 \\ u \in H_0^1(\Omega) \end{cases}$$

where λ is a real parameter and $2^* = 2n/(n-2)$ is the critical Sobolev exponent for the Sobolev embedding $H_0^1(\Omega) \hookrightarrow L^p(\Omega)$.

The solutions of (0.1) are the critical points of the energy functional

$$(0.2) \qquad f_\lambda(u) = \frac{1}{2}\int_\Omega |\nabla u|^2 dx - \frac{\lambda}{2}\int_\Omega |u|^2 dx - \frac{1}{2^*}\int_\Omega |u|^{2^*} dx.$$

Since the embedding $H_0^1(\Omega) \hookrightarrow L^{2^*}(\Omega)$ is not compact the functional f_λ does not satisfy the Palais-Smale condition in the energy range $]-\infty, +\infty[$ (cfr. remark 2.3 of [*4*]).

Moreover if $\lambda \leq 0$ and Ω is starshaped (0.1) has only the trivial solution (cf. [*6*]).

Recently Brezis and Nirenberg in [*2*] have proved that if $n \geq 4$ and $0 < \lambda < \lambda_1$ (λ_1 is the first eigenvalue of $-\Delta$) then (0.1) has a positive solution. In [*4*] Cerami, Fortunato and Struwe have obtained multiplicity results for (0.1) in the case in which λ belongs to a suitable left neighbourhood of an arbitrary eigenvalue of $-\Delta$ (cf. also [*3*]).

In this paper we prove the following theorem:

THEOREM 0.1. — *If* $n \geq 4$ *the problem* (0.1) *possesses at least one non trivial solution for any* $\lambda > 0$.

A weaker result related to theorem 0.1 has been announced in [*5*].

We observe that if $n = 3$ and Ω is a ball, Brezis and Nirenberg [*2*] have proved that the problem (0.1) does not have nontrivial radial solutions if $0 < \lambda < \dfrac{\lambda_1}{4}$.

1. SOME PRELIMINARIES

Let $\|\cdot\|$, $|\cdot|_p$ denote respectively the norms in $H_0^1(\Omega)$ and $L^p(\Omega)$ $(1 \leq p \leq \infty)$, and let

$$S = \inf\{\, \|u\|^2/|u|_{2^*}^2 : u \in H_0^1(\Omega)\setminus\{0\}\,\}$$

denote the best constant for the embedding $H_0^1(\Omega) \hookrightarrow L^{2^*}(\Omega)$.

The following lemma shows that f_λ satisfies a local P. S. condition.

LEMMA 1.1. — *For any* $\lambda \in \mathbb{R}$ *the functional* f_λ *(see* (0.2)) *satisfies the Palais-Smale condition in* $\left]-\infty, \frac{1}{n} S^{n/2}\right[$ *in the following sense:*

If $c < \frac{1}{n} S^{n/2}$ *and* $\{ u_m \}$ *is a sequence in* $H_0^1(\Omega)$ *such that*

(P. S.) $\left\{ \begin{array}{l} \textit{as } m \to \infty \; f_\lambda(u_m) \to c, f'_\lambda(u_m) \to 0 \textit{ strongly in } H^{-1}(\Omega), \textit{then } \{ u_m \} \\ \textit{contains a subsequence converging strongly in } H_0^1(\Omega). \end{array} \right.$

The proof of this lemma is in [*2*] and in [*4*]. We recall that a deeper compactness result has been proved in [*7*].

We recall a critical point Theorem (cf. [*1*, Theorem 2.4]) which is a variant of some results contained in [*0*].

THEOREM 1.2. — *Let* H *be a real Hilbert space and* $f \in C^1(H, \mathbb{R})$ *be a functional satisfying the following assumptions:*

(f_1) $f(u) = f(-u)$, $f(0) = 0$ *for any* $u \in H$

(f_2) *there exists* $\beta > 0$ *such that* f *satisfies (P. S.) in* $]0, \beta[$

(f_3) *there exist two closed subspaces* V, $w \hookrightarrow$ H *and positive constants* ρ, δ *such that*

(*i*) $f(u) < \beta$ *for any* $u \in W$

(*ii*) $f(u) \geq \delta$ *for any* $u \in V$, $\| u \| = \rho$

(*iii*) codim $V < +\infty$.

Then there exist at least m pairs of critical points, with

$$m = \dim (V \cap W) - \operatorname{codim} (V + W).$$

2. PROOF OF THEOREM 0.1

Our aim is to define two suitable closed subspaces V and W, with $V \cap W \neq \{ 0 \}$ and $V + W = H$, such that f_λ satisfies the assumptions f_2) and f_3) of Theorem 1.2 with $\beta = \frac{1}{n} S^{n/2}$.

In the sequel we denote by λ_j the eigenvalues of $-\Delta$ and by $M(\lambda_j)$ the corresponding eigenspaces.

Given $\lambda > 0$, we set

$$\lambda^+ = \min \{ \lambda_j \mid \lambda < \lambda_j \}$$

$$(2.1) \qquad H_1 = \overline{\bigoplus_{\lambda_j \geq \lambda^+} M(\lambda_j)} \qquad H_2 = \bigoplus_{\lambda_j < \lambda^+} M(\lambda_j)$$

where the closure is taken in $H_0^1(\Omega)$.

If $r > 0$ we set

$$N_r(0) = \{ x \in \mathbb{R}^n \mid \| x \| < r \}.$$

Without loss of generality we can suppose that $0 \in \Omega$ and that $N_1(0) \subset \Omega$. Given $\mu > 0$ we set (cf. [2] [7])

$$\psi_\mu(x) = \phi(x) \cdot u_\mu(x)$$

where $\phi \in C_0^\infty(N_1(0))$, $\phi(x) = 1$ on $N_{\frac{1}{2}}(0)$, and

$$u_\mu(x) = \frac{|\, n(n-2)\mu \,|^{(n-2)/4}}{|\, \mu + |x|^2 \,|^{(n-2)/2}}.$$

The following lemma holds:

LEMMA 2.1. — *If $\psi_\mu(x)$ is defined as in (2.1), then for any μ*

$$(2.2) \qquad \|\psi_\mu\|^2 = S^{n/2} + 0(\mu^{(n-2)/2}) \text{ (1)}$$

$$(2.3) \qquad |\psi_\mu|_{2^*}^{2^*} = S^{n/2} + 0(\mu^{n/2})$$

$$(2.4) \qquad |\psi_\mu|_2^2 = \begin{cases} K_1\mu + 0(\mu^{(n-2)/2}) & \text{if } n \geq 5 \\ K_1\mu\,|\log\mu| + 0(\mu) & \text{if } n = 4 \end{cases}$$

$$(2.5) \qquad |\psi_\mu|_1 \leq K_2\mu^{(n-2)/4}$$

$$(2.6) \qquad |\psi_\mu|_{2^*-1}^{2^*-1} \leq K_3\mu^{(n-2)/4}$$

where K_1, K_2, K_3 are suitable positive constants.

Proof. — The proof of (2.2), (2.3), (2.4) is contained in [2], moreover (2.5) and (2.6) can be straightforward verified.

Now we shall prove some technical lemmas. We set

$$\overline{W}(\mu) = \{\, u \in H_0^1 \mid u = u^- + t\psi_\mu,\ u^- \in H_2,\ t \in \mathbb{R} \,\}.$$

The following lemma holds:

LEMMA 2.2. — *If $u \in \overline{W}(\mu)$, then for any $\mu > 0$*

$$(2.7) \qquad |u|_{2^*}^{2^*} \geq |t\psi_\mu|_{2^*}^{2^*} + \frac{1}{2}|u^-|_{2^*}^{2^*} - K_4 t^{2^*}\mu^{n(n-2)/(2n+4)} \quad \textit{for any} \quad t \in \mathbb{R}.$$

Proof. — By the identity

$$(2.8) \qquad |u|_{2^*}^{2^*} = 2^* \int_\Omega dx \int_0^u |s|^{2^*-2} s\, ds$$

(1) In the sequel we denote by $0(\mu^\alpha)$, $\alpha > 0$ a function $|f(\mu)| \leq \text{const}\ \mu^\alpha$ near $\mu = 0$, and by $0(\mu)$, a function such that $f(\mu)/\mu \to 0$ as $\mu \to 0$.

it follows that

(2.9) $| u^- + t\psi_\mu |_{2^*}^{2^*} - | t\psi_\mu |_{2^*}^{2^*} - | u^- |_{2^*}^{2^*} =$

$$= 2^* \int_0^1 d\tau \int_\Omega [| t\psi_\mu + \tau u^- |^{2^*-2} \cdot (t\psi_\mu + \tau u^-) - | \tau u^- |^{2^*-2} \cdot \tau u^-] u^- dx =$$

$$= 2^*(2^* - 1) \int_0^1 d\tau \int_\Omega | \tau u^- + t\psi_\mu \theta |^{2^*-2} \cdot t\psi_\mu \cdot u^- dx$$

where $\theta = \theta(x)$ is a measurable function such that $0 < \theta(x) < 1$.

By (2.9) and by (2.5), (2.6) we have that

(2.10) $\left| | u |_{2^*}^{2^*} - | t\psi_\mu |_{2^*}^{2^*} - | u^- |_{2^*}^{2^*} \right|$

$$\leq c_1 \int_0^1 d\tau \int_\Omega \{ | u^- | \cdot | t\psi_\mu |^{2^*-1} + \tau^{2^*-2} \cdot | t\psi_\mu | \cdot | u^- |^{2^*-1} \} dx \leq$$

$$\leq c_2 \{ | t\psi_\mu |_{2^*-1}^{2^*-1} \cdot | u^- |_\infty + | t\psi_\mu |_1 \cdot | u^- |_\infty^{2^*-1} \} \leq$$

$$\leq c_3 \{ | t\psi_\mu |_{2^*-1}^{2^*-1} \cdot | u^- |_2 + | t\psi_\mu |_1 \cdot | u^- |_{2^*}^{2^*-1} \} \leq$$

(2.10)a $\leq e_3 . t^{2^*-1} . \mu^{(n-2)/4} | u^- |_2 + \frac{1}{4} | u^- |_{2^*}^{2^*} + c_4 . t^{2^*} . \mu^{n/2} \leq$

$$\leq \frac{1}{2} | u^- |_{2^*}^{2^*} + k_4 t^{2^*} . \mu^{n(n-2)/2n+4}$$

and the lemma is proved.

LEMMA 2.3. — *If μ is sufficiently small, then*

(2.11) $$\frac{\| \psi_\mu \|^2 - \lambda | \psi_\mu |_2^2}{| \psi_\mu |_{2^*}^2} = \begin{cases} S - K_5 \mu + 0(\mu^{\frac{n-2}{2}}) & \text{if } n \geq 5 \quad (2.11)a \\ S + K_5 \mu \log \mu + 0(\mu) & \text{if } n = 4 \quad (2.11)b \end{cases}$$

Proof. — The evaluation (2.11) follows immediately by (2.2), (2.3) and (2.4).

REMARK 2.4. — Suppose that $\lambda = \lambda_j$, with $\lambda_j \in \sigma(-\Delta)$ and denote by P_j the projector on the eigenspace M_j corresponding to λ_j.

We set

(2.12) $$\tilde{\psi}_\mu = \psi_\mu - P_j \psi_\mu .$$

Let $\{ v_k \}$ an orthonormal family spanning M_j, then by (2.5) we have

(2.13) $$| P_j \psi_\mu |_2^2 = \sum_k \left(\int_\Omega \psi_\mu v_k dx \right)^2 \leq \text{const } | \psi_\mu |_1^2 \leq K_6 \mu^{\frac{n-2}{2}}$$

then

(2.14) $$| P_j \psi_\mu |_\infty \leq K_7 \mu^{\frac{n-2}{4}} .$$

Moreover we have

$$\left|\int_\Omega \{|\tilde{\psi}_\mu|^{2^*} - |\psi_\mu|^{2^*}\}\,dx\right| = 2^*\left|\int_0^1 d\tau \int_\Omega |\psi_\mu - \tau P_j\psi_\mu|^{2^*-2}(\psi_\mu - \tau P_j\psi_\mu)P_j\psi_\mu dx\right| \leq$$

$$\leq 2^*\cdot 2^{2^*-1}\int_0^1 d\tau \int_\Omega \{|\psi_\mu|^{2^*-1} + \tau^{2^*-1}|P_j\psi_\mu|^{2^*-1}\}|P_j\psi_\mu|\,dx$$

$$\leq \text{const}\,\{|\psi_\mu|_{2^*-1}^{2^*-1}\cdot|P_j\psi_\mu|_\infty + |P_j\psi_\mu|_2^{2^*}\}.$$

Then by (2.14) and (2.6) it follows that

$$(2.15)\qquad \big|\,|\tilde{\psi}_\mu|_{2^*}^{2^*} - |\psi_\mu|_{2^*}^{2^*}\big| \leq c_1\mu^{\frac{n-2}{2}}.$$

Moreover by (2.14) and (2.6) we have

$$(2.16)\qquad |\tilde{\psi}_\mu|_{2^*-1}^{2^*-1} = |\psi_\mu - P_j\psi_\mu|_{2^*-1}^{2^*-1} \leq \text{const}\,\{|\psi_\mu|_{2^*-1}^{2^*-1} + |P_j\psi_\mu|_{2^*-1}^{2^*-1}\} \leq \text{const}\,\mu^{\frac{n-2}{4}}.$$

Analogously by (2.14) and (2.5) we have

$$(2.17)\qquad |\tilde{\psi}_\mu|_1 \leq \text{const}\,\mu^{\frac{n-2}{4}}.$$

By (2.15), (2.16), (2.17) it easily follows that (2.11) holds if we replace ψ_μ with $\tilde{\psi}_\mu$.

Moreover, by (2.15), (2.16), (2.17), also (2.7) holds (for μ small) if we replace ψ_μ with $\tilde{\psi}_\mu$ and $\overline{W}(\mu)$ with

$$\overline{\overline{W}}(\mu) = \{u \in H_0^1 \mid u = u^- + t\tilde{\psi}_\mu,\ u^- \in H_2,\ t \in \mathbb{R}\}.$$

Now we can prove a crucial lemma:

LEMMA 2.5. — *For μ sufficiently small*

$$(2.18)\qquad \sup_W f(u) < \frac{1}{n}S^{n/2}$$

where $W = \overline{W}(\mu)$ (resp. $\overline{\overline{W}}(\mu)$) *if* $\lambda \notin \sigma(-\Delta)$ (resp. $\lambda \in \sigma(-\Delta)$).

Proof. — Observe that if we fix $u \in H_0^1(\Omega)$, $u \neq 0$, then

$$(2.19)\qquad \max_t f_\lambda(tu) = \frac{1}{n}\left(\frac{\|u\|^2 - \lambda|u|_2^2}{|u|_{2^*}^2}\right)^{n/2}.$$

Then in order to prove (2.18) we need to evaluate

$$(2.20)\qquad \sup_{\substack{u\in W(\mu)\\ |u|_{2^*}=1}} \{\|u\|^2 - \lambda|u|_2^2\}.$$

We distinguish two cases:

Case i) $\lambda \notin \sigma(-\Delta)$.

Let $u = u^- + t\psi_\mu \in \overline{W}(\mu)$ with $|u|_{2^*} = 1$.

Observe that t is bounded if μ is small, in fact by (2.7) and (2.3) we get

$$1=|u|_{2^*}^{2^*}\geq |t\psi_\mu|_{2^*}^{2^*}-K_4t^{2^*}\mu^{n/2}+\frac{1}{2}|u^-|_{2^*}^{2^*}=t^{2^*}[S^{n/2}+0(\mu^{n/2})]+\frac{1}{2}|u^-|_{2^*}^{2^*}.$$

Then by (2.5) we have that

$$(2.21)\quad \|u\|^2-\lambda|u|_2^2=|\nabla u^-|_2^2-\lambda|u^-|_2^2+|\nabla t\psi_\mu|_2^2-\lambda|t\psi_\mu|_2^2-$$
$$-2\int_\Omega\{|t\psi_\mu||\Delta u^-|+\lambda|u^-||t\psi_\mu|\}dx\leq$$
$$\leq|\nabla u^-|_2^2-\lambda|u^-|_2^2+|\nabla t\psi_\mu|_2^2-\lambda|t\psi_\mu|_2^2+c_1\{|\Delta u^-|_\infty|t\psi_\mu|_1+|u^-|_\infty|t\psi_\mu|_1\}\leq$$
$$\leq|\nabla u^-|_2^2-\lambda|u^-|_2^2+|\nabla t\psi_\mu|_2^2-\lambda|t\psi_\mu|_2^2+c_2|u^-|_2\cdot\mu^{\frac{n-2}{4}}\leq$$
$$\leq(\bar\lambda-\lambda)|u^-|_2^2+\frac{|\nabla t\psi_\mu|_2^2-\lambda|t\psi_\mu|_2^2}{|t\psi_\mu|_{2^*}^2}\cdot|t\psi_\mu|_{2^*}^2+c_2|u^-|_2\cdot\mu^{\frac{n-2}{4}}$$

where $\bar\lambda=\max\{\lambda_j\,|\,\lambda_j<\lambda\}$.

We set $A(u^-,\mu,c)=(\bar\lambda-\lambda)|u^-|_2^2+C|u^-|_2\mu^{\frac{n-2}{4}}$ and observe that

$$(2.22)\qquad A(u^-,\mu,c)\leq 0\qquad\text{or}\qquad A(u^-,\mu,c)\leq c^2/(\lambda-\bar\lambda)\mu^{(n-2)/2}$$

If $|u^-|_{2^*}^{2^*}\leq 2K_4t^{2^*}\mu^{\frac{n(n-2)}{2n+4}}$, by (2.10)$a$ and the boundness of t,

$$|t\psi_\mu|_{2^*}^{2^*}\leq\left(1-\frac{3}{4}|u^-|_{2^*}^{2^*}+c_3\mu^{\frac{n-2}{4}}|u^-|_2+c_4\mu^{\frac{n}{2}}\right)^{\frac{2}{2^*}}$$
$$\leq 1+\frac{2}{2^*}(c_3\mu^{\frac{n-2}{4}}|u^-|_2+c_4\mu^{\frac{n}{2}})^{\frac{2}{2^*}},$$

then, if $n\geq 5$, by (2.11)a, (2.21)

$$(2.23)\quad \|u\|^2-\lambda|u|_2^2\leq(S-K_5\mu+0(\mu^{\frac{n-2}{2}}))(1+c_5\mu^{n/2})+A(u^-,\mu,c_6).$$

If $|u^-|_{2^*}^{2^*}>2K_4t^{2^*}\mu^{\frac{n(n-2)}{2n+4}}$, by (2.7), $|t\psi_\mu|_{2^*}<1$, then, by (2.21)

$$(2.24)\quad \|u\|^2-\lambda|u|_2^2\leq(S-K_5\mu+0(\mu^{\frac{n-2}{2}}))+A(\mu^-,\mu,c_2),$$

then, by (2.22), the conclusion follows in the case $n\geq 5$.

If $n=4$ the proof is the same. In this case (2.11)b replaces (2.11)a in (2.22).

Case ii) $\lambda=\lambda_{\bar j}\in\sigma(-\Delta)$.

Let $u=u^-+t\tilde\psi_\mu\in\overline{\overline{W}}(\mu)$ with $|u|_{2^*}=1$. We set $u=u^-+t\tilde\psi_\mu=\tilde u+P_ju^-+t\tilde\psi_\mu$, then

$$\|u\|^2-\lambda_{\bar j}|u|_2^2$$
$$=|t\nabla\tilde\psi_\mu|_2^2-\lambda_{\bar j}|t\tilde\psi_\mu|_2^2+|\nabla\tilde u_-|_2^2-\lambda_{\bar j}|\tilde u_-|_2^2-2\int_\Omega(t\tilde\psi_\mu\Delta u^-+\lambda_{\bar j}\tilde\psi_\mu u_-)dx\,.$$

Observe that

$$\int_\Omega (t\tilde{\psi}_\mu \Delta u^- + \lambda_{\bar{j}} t\tilde{\psi}_\mu u^-)dx = \int_\Omega (t\tilde{\psi}_\mu \Delta \tilde{u}_- + \lambda_{\bar{j}} t\tilde{\psi}_\mu \tilde{u}_-)dx \leq$$
$$\leq |\Delta \tilde{u}_-|_\infty |t\tilde{\psi}_\mu|_1 + \lambda_{\bar{j}} |\tilde{u}_-|_\infty |t\tilde{\psi}_\mu|_1 \leq c_3 |\tilde{u}_-|_2 \mu^{\frac{n-2}{4}}.$$

Now the proof follows by using the previous arguments.

Proof of theorem 0.1. — If $\lambda \notin \sigma(-\Delta)$ $(\lambda > 0)$ we set $V = H_1$ and $W = \overline{W}(\mu)$ with μ suitably small in order that (2.18) is verified. We see that the assumptions of Theorem 1.2 are satisfied. Obviously (f_1) and $(f_3.iii)$ are verified. Moreover (f_2) is verified with $\beta = \frac{1}{n} S^{n/2}$ by lemma 1.1 and $(f_3.i)$ $\left(\text{with } \beta = \frac{1}{n} S^{n/2}\right)$ is verified by lemma 2.5.

Finally observe that if $u \in H_1$, then

$$(2.25) \quad f_\lambda(u) = \frac{1}{2}\int_\Omega |\nabla u|^2 dx - \frac{\lambda}{2}\int_\Omega |u|^2 dx - \frac{1}{2^*}\int_\Omega |u|^{2^*} dx \geq$$
$$\geq \frac{1}{2}\left(1 - \frac{\lambda}{\lambda^+}\right)\|u\|^2 \frac{1}{2^*}|u|_{2^*}^{2^*} \geq \frac{1}{2}\left(\lambda - \frac{\lambda}{\lambda^+}\right)\|u\|^2 - \text{const}\,\|u\|^{2^*} \geq \delta > 0$$

if $\|u\| = \rho$ with ρ suitably small.

Hence by (2.27) also $(f_3.ii)$ is verified. Since $\dim V \cap W = 1$ and $V + W = H_0^1(\Omega)$, then by Theorem 1.2, we deduce that problem (0.1) has at least one non trivial solution.

If $\lambda \in \sigma(-\Delta)$ we set $W = \overline{\overline{W}}(\mu)$ with μ suitably small in order that (2.18) is verified and, by repeating the above arguments, the conclusion follows.

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