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Addendum to Closed orbits of fixed energy for a class of N-body problems

by

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In Lemma of 6 the paper *Closed Orbits of fixed energy for a class of N-body problems*, published on this Journal (Vol. 9, No. 2, 1992, pp. 187-200) it is claimed that critical points of f_ε are found by applying the Mountain-Pass Theorem. Actually, such a Theorem needs to be slightly changed, in order to make it applicable in Lemma 6. We indicate below the required modification.

Let $\Sigma_\rho=\{u\in\Lambda_0:\|u\|\geqq\rho\}$, $\Gamma_\rho=\{p\in C([0,1],\ \Sigma_\rho):\|p(0)\|=\rho,\ p(1)=u_1\}$, and

$$c=\inf_{p\in\Gamma_\rho}\max_{0\leqq\xi\leqq 1} f_\varepsilon(p(\xi)).$$

Hereafter, $0<\varepsilon\leqq\varepsilon_0$. By Lemma 2 (i), $a\geqq\beta>0$. Suppose c is not a critical level for f_ε. Since (PS^+) holds, then there exist $m>0$ and $k\in]0,\beta/2[$ such that $\|f'_\varepsilon\|\geqq m$, for all $u\in\Lambda_0\cap\{c-k\leqq f_\varepsilon(u)\leqq c+k\}$. Let $\eta(s,u)$, $\eta:[0,\tau_u[\times\Lambda_0\to E$ denote the steepest descent flow satisfying

$$\frac{d\eta}{ds}=-X(\eta),\qquad \eta(0,u)=u,$$

where X is, as usual, a pseudo-gradient vector field for f_ε, such that (i) $X(u)=0$ if $f_\varepsilon(u) \leqq c-2k$ or $f_\varepsilon(u) \geqq c+2k$, (ii) $X(u)=f'_\varepsilon$ if $c-k \leqq f_c(u)=c+k$, and (iii) $f_c(\eta(s, u)) \leqq f_\varepsilon(u)$, $\forall\, 0 \leqq s < \tau_u$ (*see* the Deformation lemma in [2]).

Since, as a consequence of (2.3), $f_\varepsilon(u) \to +\infty$ whenever $u \to v \in \partial\Lambda_0 - \{0\}$, then (iii) above readily implies that $\eta(s, u) \in \Lambda_0$ whenever $u \in \Lambda_0$ and $\eta(s, u) \neq 0$.

Let us show that $\tau_u \geqq \rho/2$, for all $u \in \Sigma_\rho$. According to the preceding remark, this follows in the usual way if $\|\eta(s, u)\| > \rho/2$, $\forall\, s < \tau_u$ (indeed, in such a case $\tau_u = +\infty$). Otherwise, let $S \in]0, \tau_u[$ be such that $\|\eta(S, u)\| = \rho/2$, for some $u \in \Sigma_\rho$. Then

$$\frac{\rho}{2} \leqq \|\eta(S, u) - u\| \leqq \int_0^S \|X(\eta)\| \leqq S,$$

proving the claim.

Note also that the same arguments used to prove Lemma 2 (i) yield that $(f'_\varepsilon(u) \mid u) > 0$, $\forall\, u \in \Lambda_0$, $\|u\| = \rho$. Therefore $\|\eta(s, u)\| < \rho$, whenever $s \geqq 0$ and $u \in \Lambda_0$, $\|u\| = \rho$.

After these preliminaries, let $k' < \min(k, \rho/4m)$ and let $p \in \Gamma_\rho$ be such that $\max_\xi f_\varepsilon(p(\xi)) < c + k'$. Consider the path $p_1(\xi) = \eta\left(\frac{\rho}{2}, p(\xi)\right)$. Using the properties of the vector field X, one shows in the standard way that $\max_\xi f_\varepsilon(p_1(\xi)) < c - k'$. Moreover, $p_1(1) = u_1$, whereas, as noted before, $\|p_1(0)\| < \rho$. Let $\xi_0 = \min\{\chi \in [0, 1] : \|p_1(\xi)\| > \rho,\ \forall\, \xi \in]\chi, 1]\}$. Setting $q(\xi) = p_1(\xi_0 + \xi(1 - \xi_0))$, it follows that $q \in \Gamma_p$ as well as $\max_\xi f_\varepsilon(q(\xi)) < c - k'$, a contradiction which shows that c is a critical level for f_ε.