Institute of Mathematics of the Czech Academy of Sciences

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The Institute of Mathematics of the Czech Academy of Sciences, located in the very centre of Prague with a small group of researchers working in Brno, is the leading research institution in mathematics in the Czech Republic. The mission of the Institute is to foster fundamental research in mathematics and its applications and to provide the necessary infrastructure. In cooperation with universities, the Institute carries out doctoral study programmes and provides training for young scientists.

The Institute was established in 1947 as the Institute for Mathematics of the Czech Academy of Sciences and Arts (Česká akademie věd a umění). The initiator and the first director was Eduard Čech. In 1953, the Institute was reorganized and incorporated into the newly established Czechoslovak Academy of Sciences. In 1993, when Czechoslovakia split, the Institute became a part of the newly established Czech Academy of Sciences. In 2007, the Institute, together with 53 other institutes of the Czech Academy of Sciences, was transformed into a public research institution. This status provides much broader autonomy, especially in research and personnel policy, but still under public control as the vast majority of funds come from public sources. Institutional funding forms about 60 % of the resources and is provided by the Czech Academy of Sciences based on regular evaluation of research quality. About 35 % of the revenue is earned through competitions for grant projects, and 5 % results from economic activities related mainly to the publication of research journals.

The Institute currently employs around 90 researchers. Almost half of them are foreigners, with more than 20 nationalities. Postdoctoral fellows and PhD students represent more than 25 % of the research staff. All researchers are hired in open competitions for 2–5 year contracts with the possibility of further extension based on a successful personal evaluation. Unfortunately, getting good PhD students is complicated by the legislation that allows the Institute to be involved in their education only in conjunction with a university. This means that a number of students are coming from abroad. The infrastructure is supported by 20 staff members who provide the services of the library, IT, project support, editorial office, administration and management.

Research

The research strategy of the Institute is based on bottom-up activities that are supported, encouraged and guided by the management in close cooperation with the Board of the Institute. The International Advisory Board is asked for advice on important decisions.

The fields of research include those connected with the best tradition of Czech mathematics as well as newly developed areas. The traditional fields are inherently connected with the founding members and strong personalities of the Institute such as Eduard Čech (Stone–Čech compactification in topology, Čech cohomology, Čech closure operator), Jaroslav Kurzweil (Henstock–Kurzweil integral, stability theory for ordinary differential equations), Ivo Babuška (theory of finite element method, Ladyzhenskaya–Babuška–Brezzi condition), Jindřich Nečas (regularity of generalized solutions of elliptic equation, theory of elasto-plastic bodies in continuum mechanics), Miroslav Fiedler (Fiedler algebraic connectivity in graph theory, Fiedler vector in linear algebra and matrix theory), and Vlastimil Pták (Pták topological vector spaces, Pták subtraction theorem and the notion of the critical exponent of iterative processes).

Following the high standards set by these distinguished personalities, research teams have been cultivating the traditional and strong mathematical disciplines while also opening new research directions. Current research focuses on mathematical analysis (differential equations, numerical analysis, functional analysis, theory of function spaces), mathematical logic and logical foundations of computer science, complexity theory, combinatorics, set theory, numerical linear algebra, general and algebraic topology, category theory, optimization and control theory, algebraic and differential geometry and mathematical physics.

The research at the Institute is organized in five departments that are described in the following paragraphs.

Abstract Analysis

Originally called Department of Topology and Functional Analysis, this department represents a continuation of one of the traditionally strong research directions in the Institute. Under the leadership of Wieslaw Kubiś, the team recently reassessed their research focus.

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The emphasis has shifted from the traditional topics of the theory of Banach spaces, operator theory, classical topology and functional analysis to those areas where mathematical logic plays a significant role, even though it is not the main object of study, namely descriptive set theory, algebraic topology, category theory, and the theory of C*-algebras. For this reason, the department has been renamed Abstract Analysis.

Several team members are currently involved in the prestigious EXPRO project of excellence funded in 2020–2024 by the Czech Science Foundation and lead by Wieslaw Kubiś. The project aims to explore and classify generic mathematical objects appearing in the above-mentioned areas of abstract analysis.

Algebra, Geometry and Mathematical Physics

Formed in 2014 on a bottom-up initiative of several members of other teams, this department steadily grows and continuously proves to be one of the most successful within the Institute. They investigate algebraic and differential geometry and closely related areas of mathematical physics. Their research focuses on mathematical aspects of modern theoretical physics, mathematical models aiming at understanding the nature of matter, fields, and spacetime. Research topics include representation theory and its applications to algebraic geometry, homological algebra, algebraic topology, applied category theory, tensor classification, mathematical aspects of string field theory, generalized theory of gravitation, and study of Einstein equations.

The team achieved excellent results in the theory of gravity, analytical solutions of Einstein equations, and modified theories of gravity. Using their conformal-to-Kundt method, Vojtěch Pravda and Alena Pravdová with their colleagues from the Charles University identified and studied several classes of new static spherically symmetric vacuum solutions of the field equations of modified gravity, including a new non-Schwarzschild black hole. This discovery attracted widespread attention and was even reported in the media. Martin Markl and his collaborators achieved the ultimate result on loop homotopy algebras in closed string field theory and constructed the disconnected rational homotopy theory. In 2018, he received the Praemium Academiae award of the Czech Academy of Sciences, connected with generous funding that allowed him to hire several talented postdocs and establish his own ambitious research group.

Evolution Differential Equations

The research of this department focuses on theoretical analysis of complex multi-field evolution processes in physics, in particular continuum mechanics and thermodynamics. Special attention is paid to the description of interacting phenomena of different physical natures, such as biological systems, stratified or viscoelastic fluids, contact mechanics between fluids and solids or between rigid, elastic, or elastoplastic solids, fluid diffusion in deformable porous media, electric and magnetic effects in moving solids and fluids, magnetohydrodynamics, liquid crystals, hysteresis, thermal effects and radiation, or temperature-induced phase transitions in a large parameter range. The systems under consideration are based on physical laws of conservation of mass, momentum, energy, balance of entropy, including also energy exchange principles between mechanical, thermal, and electromagnetic energy in multifunctional materials.

Eduard Feireisl, the principal investigator of the ERC Advanced Grant MATHEF (Mathematical Thermodynamics of Fluids), 2013–2018.

An outstanding achievement was the ERC Advanced Grant MATHEF (Mathematical Thermodynamics of Fluids) awarded to Eduard Feireisl in 2013–2018. He and his collaborators built a complete mathematical theory describing the motion of compressible viscous heat-conducting fluids, including aspects of stochastic forcing and construction of convergent numerical schemes. The novel and original approach to the interpretation of the principles of continuum thermodynamics in modelling heat-conducting fluid flow turned out to be a rich source of results for the general theory, as for example, the concept of dissipative measure-valued solutions. Further essential results concerned well-posedness, regularity and stability of the Euler system and similar partial differential equations, including the construction of a stable finite volume scheme and proof of its convergence via dissipative measure-valued solutions.

The team members are involved in the Nečas Center for Mathematical Modeling, a research platform established by the Institute, the Charles University and the Institute of Computer Science of the Czech Academy of Sciences with the ambition of coordinating and supporting research and education activities in the theoretical and applied mathematics, particularly in the field of continuum mechanics. They are also active in the network for industrial mathematics EU-MATHS-IN.CZ (part of the European network EU-MATHS-IN).

Mathematical Logic and Theoretical Computer Science

The research programme of this department concerns mathematical problems arising from theoretical computer science, logic, set theory, finite combinatorics, and control theory. The main topics

Tomáš Vejchodský, Director of the Institute, in the promotional video presenting the cooperation with the company Doosan-Bobcat EMEA, youtube.com/watch?v=_I2KN-z_fo4.

studied by its members include proof and computational complexity, logical foundations of arithmetic, quantum information theory, graph theory, and set theory. The problems studied have foundational importance in themselves, and potentially also practical applications, for example in data security.

In the area of the logical foundations of mathematics, the team is one of the world's leading centres of research in bounded arithmetic and proof complexity. Computational complexity is a discipline with a short history that has only recently been recognized as an important field not only in computer science but also in mathematics. It is also due to the fact that fundamental questions in this domain (e.g. the famous "P versus NP" problem) belong to the set of mathematical problems which resist being solved for decades. Pavel Pudlák's group attacks these problems using methods of mathematical logic. He believes that the reason why we cannot answer these questions is fundamental in nature, and therefore their logical aspects should be studied. The research domain in which he and his colleagues work and have already reached important results is called proof complexity. While computational complexity deals with how difficult it is to compute something, proof complexity asks how difficult it is to prove it.

Pavel Pudlák, the principal investigator of the ERC Advanced Grant FEALORA (Feasibility, Logic and Randomness in Computational Complexity) in 2014–2018.

Numerical Analysis

Following a decades-long tradition, this department investigates both theoretical and practical aspects of computational science, mainly numerical methods for partial differential equations and numerical linear algebra, whereas classical and strong areas have been complemented with new research topics. Its members focus on questions of convergence, efficiency, and reliability of numerical methods for partial differential equations, including matrix computations and high-performance implementations on parallel computer architectures. Members of the team led by Michal Křížek are experts in the finite element method, saddle-point systems, preconditioning, domain decomposition methods, rounding error analysis, high-performance computing and computational fluid dynamics.

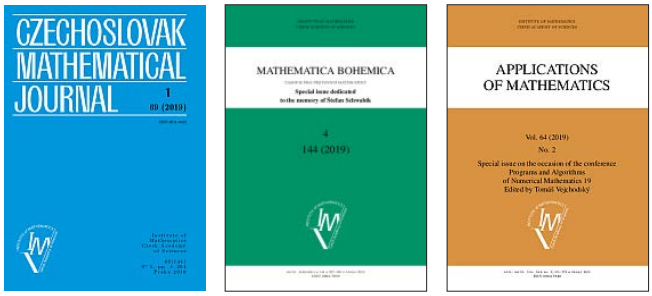
The team is involved in the Nečas Center for Mathematical Modeling and in the network for industrial mathematics EU-MATH-IN.CZ. It has succeeded in competitions for the CPU time at large European computers and cooperates with the IT4Innovations National Supercomputing Center of the Technical University in Ostrava.

Members of the five above-mentioned departments organize a dozen regular seminars and about the same number of international workshops and conferences. Around 150 foreign researchers visit the Institute every year. In 2016, the Institute established Eduard Čech Distinguished Visitor Programme with the aim of significantly enhancing its creative environment by attracting highly distinguished mathematicians for a longer period of time. One visitor is selected every year to deliver a series of lectures and

to essentially develop scientific collaboration with researchers in the Institute. The visitor is also expected to deliver the prestigious Eduard Čech Lecture for the general mathematical community.

Other activities and service to the community

Although the emphasis is on fundamental research, attention is also paid to connections with applications. The Institute is involved in the Strategy AV21 programme "Hopes and Risks of the Digital Era" run by the Czech Academy of Sciences. The role of the Institute is to develop mathematical models for engineering applications. The Institute cooperates on a long-term basis with the Innovation Centre of the company Doosan Bobcat EMEA, the renowned producer of compact loaders and excavators.

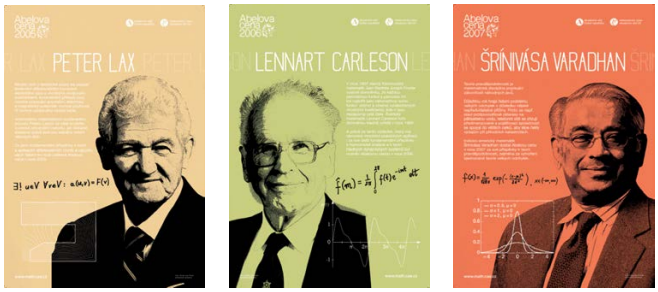


The Institute publishes three mathematical journals. The Czechoslovak Mathematical Journal and Mathematica Bohemica are continuations of Časopis pro pěstování mathematiky a fysiky (Journal for Cultivation of Mathematics and Physics) established in 1872. The aim of these two journals is to publish original research papers of high scientific quality in all fields of mathematics. The third journal, Applications of Mathematics, specializes in mathematical papers directed at applications in various branches of science.

The Institute also provides several services for the wide mathematical community and public. Its library, with almost 100,000 volumes including 35,000 monographs and 1,300 journal titles, is the largest public mathematical library in the country. Since 1996, the Prague editorial Group cooperates with zbMATH to produce metadata and reviews of mathematical publications. Since 2009, the Institute has been developing the Czech Digital Mathematics Library (DML-CZ, https://dml.cz) with the aim of digitizing, organizing and archiving the relevant mathematical literature published throughout history in the Czech lands, and providing free access to metadata and full texts. DML-CZ currently includes 17 journal titles, proceedings of 8 conference series, and about 300 books. The Institute is a member of the international consortium that has developed the European Digital Mathematics Library (EuDML, https://eudml.org).

Students during the Open House Days and the exhibition of Imaginary posters demonstrating the beauty of mathematical surfaces.

Close attention is paid to the popularization of mathematics. Public lectures in the annual Open House Days used to be attended by more than a thousand visitors, mostly high-school students. The restrictions connected with the Covid-19 pandemic inspired us to create a webpage for students and the general public presenting various mathematical problems, popular lectures and other interesting materials like posters celebrating the laureates of the Abel Prize.



Posters presenting the winners of the Abel Prize.

The well-being of the Institute employees and their work-life balance is supported in various ways. There is a tradition of cultivating a friendly atmosphere and an effort to approach and comply with individual needs of employees. The main objective of the currently running project "Institute of Mathematics CAS goes for HR Award – implementation of the professional HR management" is to improve the stimulating and attractive work environment in the Institute and to apply for the HR Excellence in Research Award (known as the HR Award) granted by the European Commission.

To learn more about the Institute, please visit the webpage www.math.cas.cz.

A group photo of members of the Institute at the annual bike trip, July 30, 2020.

Jiří Rákosník obtained his CSc (PhD equivalent) in 1980 at the Charles University in Prague. Since then, he has been working in the Institute of Mathematics of the Czech Academy of Sciences, in the position of the director in 2014–2019. His research interests focus on the theory of function spaces. For 25 years, he has been active in the digitization of mathematical literature, in close cooperation with zbMATH, and in building the Czech Digital Mathematics Library and the European Digital Mathematics Library. He serves as the current Secretary of the EMS.

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Miroslav Rozložník graduated in Mathematics in 1992 and obtained his PhD in Applied Mathematics in 1997 at the Faculty of Nuclear Sciences and Physical Engineering of the Czech Technical University in Prague. In 1998–2000, he worked on a postdoctoral fellow position at the Swiss Federal Institute of Technology (ETH) in Zürich. He is the author or co-author of one book and more than 40 journal publications. His research interests include numerical linear algebra, saddle point problems, parallel computing and rounding error analysis. From 2001 he was a research fellow at the Institute of Computer Science of the Czech Academy of Sciences until 2017, when he moved to the Institute of Mathematics, where he currently serves as the deputy director.

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