Survey on Research in University Mathematics Education at ICME 14

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At the 14th International Congress on Mathematics Education, which took place in a hybrid mode in Shanghai from July, 11 to 18, 2021, the survey team on Research in University Mathematics Education (RUME) presented an overview of their work. As noted in the presentation, it is an exciting time for RUME. There are now several major conferences every year across the globe, as well as the fairly new *International Journal of Research in Undergraduate Mathematics Education*, now in its seventh year. The significant growth in the number of researchers focused on university mathematics education has led to the development of research groups and the consolidation of a diverse academic community; RUME is coming to age as a field of research that is beginning to coalesce and develop an identity.

To explore this identity, we surveyed 218 RUME scholars across the world, both well-established scholars and rising stars. We invited these scholars to respond to the following prompt:

> What do you see as the most significant advances, changes, and/or gaps in the field of research in university mathematics education? These advances, changes, or gaps might relate to theory, methodology, classroom practices, curricular changes, digital environments, purposes and roles of universities, social policies, preparation of university teachers, etc. Please elaborate on just *one or two advances, changes, or gaps* most relevant to your experience and expertise.

We received 119 responses. Our next step was to conduct a thematic analysis[1], which led to the identification of five areas in which there has been considerable progress (Theoretical Perspectives, Instructional Practices, Professional Development of University Teachers, Digital Technology, and Service-Courses in University Mathematics Education) and seven, non-disjoint areas in need of further research (Theories and Methods, Linking Research and Practice, Professional Development of University Teachers, Digital Technology, Curriculum, Higher Years, and Interdisciplinarity). We then conducted a literature review, guided by the identified themes. We hope that this brief report offers those less familiar with RUME an overview of the progress to date and spurs interest in areas in which the reader might want to contribute to the knowledge base.

One of the field's major advances is that we now have a plethora of theoretical perspectives, and hence tensions among them can sharpen their constructs and methodologies and open the possibility of finding commonalities. This diversification has contributed to the development of new methods, research topics, and the development and research on theory-based teaching experiences. Recent years have seen the emergence of an interdisciplinary group of scholars interested in using a variety of approaches (logical, cognitive, historical, philosophical, etc.) to address questions which have always been of interest to RUME. Another theoretical advance that is of growing interest is the use of theories that enable insights into the interrelatedness of knowledge, identity, power, and social discourses [1]. While there is still much research that is needed here, we see this new direction as an important advance for the field of university mathematics education research.

The research of instructional practices at university level is another rapidly developing area of research. Much of the research on this topic relates to active or inquiry-based mathematics education [2, 6]. Given the myriad calls for instructional reform in university mathematics classrooms, researchers and educators have challenged conventional lecture-based instruction by conducting studies that have provided evidence for the positive effects of innovative student-centered instructions on students' cognitive and affective development. Active learning, broadly defined as classroom practices that engage students in activities such as reading, writing, discussing, or problem solving, that promote higher-order thinking, has repeatedly been shown to improve student success and to reduce the equity gap for women and underrepresented students [3, 7]. For example, a meta-analysis of 225 studies that compared student success in traditional lecture versus active learning in postsecondary science, engineering, and mathematics courses and found that average examination scores improved by about 6 % in active learning sections, and that students in classes with traditional lecturing were 1.5 times more likely to fail than were students in classes with active learning; further, the effectiveness of active learning was found across all

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class sizes [3]. On the other hand, RUME has only just begun to deeply explore the culture, experiences, and gendered/racialized interactions in these classes, and how those social factors may be obstructing the students' opportunities to learn [4].

Another area in need of research is on the learning and teaching of advanced mathematics. Historically, the work of Felix Klein is most relevant here. Core parts of his "Elementary Mathematics from a Higher Standpoint" [5] actually refer to mathematics that many of today's future teachers do not even get to know in the course of their academic studies. This applies, for example, to knowledge of Fourier analysis that goes beyond the basics, but especially also to knowledge of function theory, e.g. Riemann surfaces and value assignment theorems. Even when students hear about function theory, for example, they usually do not get as far as understanding what Felix Klein considered, more than a century ago, appropriate knowledge for prospective teachers. Klein considered this knowledge appropriate because it explains why, for example, certain elementary operations have to be restricted in certain ways for mathematical reasons (and not just for didactic reasons of reduction!), and related curricular decisions.

Also ripe for further investigation is the cooperation with mathematicians, engineers, economists, psychologists, etc. For many years, there have been many different kinds of cooperation, for example, agreements between faculties with regard to teaching. What does not seem to exist so far is, among other things, systematic research on these cooperations. What are the benefits of these? How do they take shape? How do they function? Possibilities, limits, etc.? Related to these cooperations is the relationship of mathematics to other sciences or the use of mathematics in other sciences. There are several places, such as philosophy or the history of science, in which such connections are examined and the question of what distinguishes mathematics itself and its respective role in other sciences is explored. Research on this is dependent on the respective ideological assumptions, and accordingly there are no unambiguous and generally accepted answers here. From the point of view of didactics, however, clarifications in this regard could certainly be regarded as desirable, since they would be of great help in answering the question of which goals, and how mathematicians and even more engineers, economists, psychologists, etc., are to be taught.

Last but not least are questions concerning mathematics itself. Mathematics, too, changes its inherent orientation, and to some extent its character, over time. New fields, such as Big Data and Data Science, are continually emerging. Correspondingly, there are new fields of application in other sciences, such as discrete mathematics in electrical engineering, numerical methods in psychology, etc. This leads directly to questions of what should be taught in service courses.

The video of the oral ICME presentation on the survey can be found here: https://drive.google.com/file/d/1LTBDl_KNZ371SL5ahvN2x_TA09PQQRlB/view?usp=sharing.

As we noted at the start of this brief overview, there is now much research-based wisdom, while at the same time there are exciting opportunities for new research. In particular, research mathematicians are welcome to join the systematic reflection and empirical investigation of university mathematics teaching.

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