Mudumbai Seshachalu Narasimhan (1932–2021)

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On May 15, 2021, the eminent Indian mathematician Mudumbai Seshachalu Narasimhan passed away at his home in Bangalore. His work in the field of geometry is internationally recognised, having deep connections with different branches of mathematics and theoretical physics. Narasimhan spent much of his career at the Tata Institute of Fundamental Research (TIFR) in Mumbai, where he was a key figure in the creation and development of the internationally acclaimed modern Indian school of algebraic geometry. After retiring from TIFR, from 1993 to 1999, Narasimhan was Head of the Mathematics Section of the International Centre for Theoretical Physics (ICTP) in Trieste, an institution created in 1964 by the Pakistani 1979 Nobel Laureate in Physics Abdus Salam.

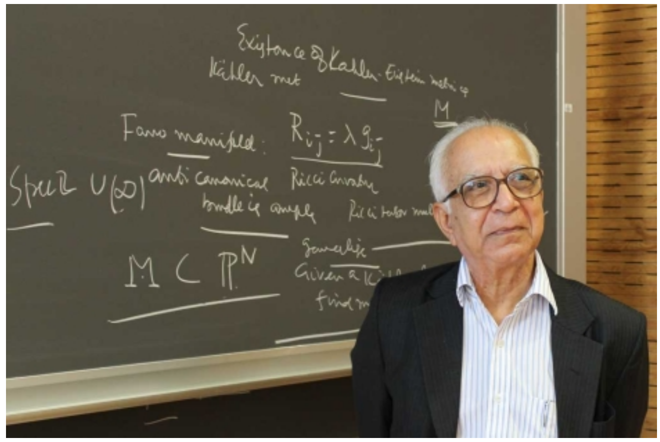

Figure 1. M. S. Narasimhan, ICMAT, Madrid, 2017

1 Life and career

Narasimhan was born on 7 June 1932 in Thandarai, a small town in Tamil Nadu (India), to a prosperous farming family. Although their circumstances were somewhat reduced after his father passed away when he was only thirteen, his family encouraged him to do what he wanted. From a young age he showed a great interest in mathematics and already in school he decided to become a researcher, even before really knowing what that meant. He completed his first university studies at Loyola College in Madras, in the heart of British India. There, he had as a teacher the French Jesuit Father Charles Racine, who was in contact with legendary figures of mathematics such as Elie Cartan, Jacques Hadamard, André Weil and Henri Cartan. Racine introduced him to modern mathematics, unknown in India, and, in particular, to the great French school. At Loyola College Narasimhan met C. S. Seshadri – also deceased in 2020 – who would later become one of his main collaborators.

Following his studies at Loyola College and on the advice of Father Racine, Narasimhan moved in 1953 to the newly created TIFR in Bombay to do his doctorate under the direction of K. Chandrasekharan, one of the founders of the centre's School of Mathematics. There he was able to interact with first-rate mathematicians who came as visitors to teach courses of two or three months. Among them was Laurent Schwartz – Fields medallist in 1950 – who would have a great influence on Narasimhan and would be his mentor during his three-year stay in Paris in the late 1950s, where he would also coincide with Seshadri. In the initial period of his stay in Paris he could not completely concentrate on mathematics as he was hospitalised due to a sickness. However, he used that time to read the paper of Kodaira and Spencer on deformations of complex structures which eventually played a great role in his future work. During his time in France he also collaborated with Japanese mathematician Takeshi Kotake, who was also in Paris to work with Schwartz.

When he returned to TIFR in 1960, Narasimhan and Seshadri started an intense collaboration that resulted in the famous Narasimhan–Seshadri theorem, published in 1965. A bit later, he began his long and fruitful collaboration with S. Ramanan. Along with Ramanan, who was his first student, Narasimhan's student roster includes other such illustrious names as N. Nitsure, R. Parthasarathy, V. K. Patodi, M. S. Raghunathan, T. R. Ramadas and R. R. Simha, who have made essential contributions to various areas of mathematics. Narasimhan's presence at TIFR was indeed a source of inspiration to several generations of young mathematicians.

During his time at TIFR, Narasimhan had also important administrative activity. In particular, he was the first Chairman of the

National Board for Higher Mathematics, which was set up in 1983 by the Government of India, under the Department of Atomic Energy, to foster the development of higher mathematics in the country. Together with S. Ramanan, who acted as Secretary, Narasimhan undertook the task of setting it up in the initial years. He was also a member of the Executive Committee of the International Mathematical Union (IMU) during the period 1983–1986, as well as President of IMU's Commission on Development of Exchange.

After retiring from TIFR, Narasimhan was the Head of the ICTP Mathematics Section from 1993 to 1999. In this position, he carried out in particular important work in supporting young mathematicians from developing countries. When he retired from ICTP, he continued to be an adviser of ICTP and served as a member of its Scientific Council. In 2020, he was awarded the Spirit of Abdus Salam Award by the family of the ICTP founder at a ceremony where numerous mathematicians from around the world showed him their great admiration, respect and affection.

After his stay at the ICTP, Narasimhan spent three years at SISSA (Trieste), before returning to India, where he continued his mathematical activity at the Indian Institute of Science in Bangalore.

Narasimhan's work earned him many prestigious awards, including the Shanti Swarup Bhatnagar Prize (1975), Third World Academy of Sciences Prize for Mathematics (1987), the Srinivasa Ramanujan Medal (1989), the French Ordre National du Mérite (1990), the Padma Bhushan Award by the President of India (1990), the C. V. Raman Birth Centenary Award of the Indian Science Congress (1994), and the 2006 King Faisal International Prize in Science that he shared with Sir Simon Donaldson. He was also a Fellow of the Indian National Sciences Academy, Indian Academy of Sciences, the Royal Society of London and the Third World Academy of Sciences.

Narasimhan was a great fan of detective novels, and literature in general, in Tamil, English and French. He also liked Indian classical music, as well as Western classical music.

Narasimhan was married to Sakuntala Narasimhan, a renowned Indian classical music singer and journalist. The couple had a daughter, Shobhana Narasimhan, a physics researcher and professor at the Jawaharlal Nehru Center for Advanced Scientific Research, and a son, Mohan Narasimhan, who, after obtaining an MBA and having worked in the US for several years, returned to India, where he teaches martial arts.

2 Work

Narasimhan made important contributions in several areas of mathematics, including algebraic geometry, differential geometry, representation theory of Lie groups and analysis. Here, we will focus mostly on his work in algebraic geometry, and specially in the theory of moduli spaces of vector bundles on Riemann surfaces, with particular reference to works that are more familiar to the author. For details, one can consult the Collected Papers of M. S. Narasimhan [10].

The theorem of Narasimhan and Seshadri

Upon his return to TIFR in 1960, Narasimhan embarked on an intense collaboration with Seshadri that resulted in the famous Narasimhan–Seshadri theorem, published in 1965. This theorem captures the interconnection between various branches of geometry, topology and theoretical physics, and was the basis for later fundamental works by some of the greatest mathematicians of our time such as Michael Atiyah, Raoul Bott, Simon Donaldson, Karen Uhlenbeck, Shing-Tung Yau, Nigel Hitchin and Carlos Simpson, among others.

The problem of classifying holomorphic vector bundles over a compact Riemann surface X of genus g is a central one in algebraic geometry. The set of equivalence classes of holomorphic line bundles on X is given classically by the Picard group of X. For genus $g = 0$ higher rank holomorphic vector bundles were classified by Grothendieck (1957), and in a different fashion by earlier work of Birkhoff (1909). The case of elliptic curves ($g = 1$) was solved by Atiyah (1957).

For genus $g \geq 2$ the problem is much harder. Inspired by some remarks in the 1938 paper of A. Weil on "Généralisation des fonctions abéliennes", Narasimhan and Seshadri started looking in 1961–62 at unitary vector bundles. A unitary representation ρ of dimension n of the fundamental group of X defines a holomorphic vector bundle E_ρ of rank n and degree 0, which is referred to as a *unitary vector bundle*. This is called an *irreducible unitary vector bundle* if ρ is irreducible. They showed that the infinitesimal deformations of a unitary vector bundle E_ρ as a holomorphic bundle can be identified with the infinitesimal deformations of the representation ρ. From this, they deduced that the set of equivalence classes of unitary vector bundles had a natural structure of a complex manifold, and were able to compute the expected dimension.

A breakthrough came with the work of Mumford on Geometric Invariant Theory. In the 1962 International Congress in Stockholm, he introduced the notion of stability of a vector bundle on a compact Riemann surface, and proved that the set of equivalence classes of stable bundles of fixed rank and degree has a natural structure of a non-singular quasi-projective algebraic variety, projective if the rank and degree are coprime. Let E be a holomorphic vector bundle over X. Define the *slope* of E as

$$\mu(E) = \frac{\deg(E)}{\operatorname{rank}(E)}.$$

The holomorphic vector bundle E is said to be *stable* if $\mu(F) < \mu(E)$ for every proper holomorphic subbundle $F \subset E$. One can similarly define *semistability* replacing the strict inequality by $\leq$ for every subbundle $F \subseteq E$.

After they became aware of Mumford's work, the relation with unitary bundles was clear to them. Narasimhan and Seshadri proved that an irreducible unitary bundle is stable. For arbitrary degree they showed that the stable vector bundles on X are precisely the vector bundles on X which arise from certain irreducible unitary representations of suitably defined Fuchsian groups acting on the unit disc and having X as quotient. The result that they proved in [8] can be easily reformulated as saying that a holomorphic vector bundle over X is stable if and only if it arises from an irreducible projective unitary representation of the fundamental group of X. From this, one deduces that a reducible projective unitary representation of the fundamental groups corresponds to a direct sum of stable holomorphic vector bundles of the same slope (what is nowadays referred as a *polystable* vector bundle). One can observe that the projective unitary representations lift to unitary representations of a certain *central extension* of the fundamental group of X.

The Narasimhan–Seshadri theorem has been a paradigm and an inspiration for almost 60 years now for many important developments. The theorem was generalised by Ramanathan (1975) to representations into any compact Lie group. The gauge-theoretic point of view of Atiyah and Bott (1982), using the differential geometry of connections on holomorphic bundles, and the new proof of the Narasimhan–Seshadri theorem given by Donaldson (1983) following this approach, brought new insight and new analytic tools into the problem. In this approach a projective unitary representation of the fundamental group is the holonomy representation of a unitary projectively flat connection.

The case of representations into a non-compact reductive Lie group G required the introduction of new holomorphic objects on the Riemann surface X called *G-Higgs bundles*. These were introduced by Hitchin (1987), who established a homeomorphism between the moduli space of reductive representation in $\mathrm{SL}_2(\mathbb{C})$ and polystable $\mathrm{SL}_2(\mathbb{C})$-Higgs bundles. This correspondence was generalised by Simpson (1988) to any complex reductive Lie group (and in fact, to higher dimensional Kähler manifolds). The correspondence in the case of non-compact G needed an extra ingredient – not present in the compact case – having to do with the existence of twisted harmonic maps into the symmetric space defined by G. This theorem was provided by Donaldson (1987) for $G = \mathrm{SL}_2(\mathbb{C})$ and by Corlette (1988) for arbitrary G. It is perhaps worth pointing out that this theorem is a twisted version of an existence theorem of harmonic maps of Riemannian manifolds proved by Eells–Sampson (1964) pretty much around the same time as the theorem of Narasimhan and Seshadri. Corlette's theorem, which holds for any reductive real Lie group, can be combined with an existence theorem for solutions to the Hitchin's equations for a G-Higgs bundle, given by the author in collaboration with Bradlow, Gothen and Mundet i Riera (2003, 2009) to prove the correspondence for any real reductive Lie group G. Earlier, Simpson (1992) gave an indirect proof of this by embedding G in its complexification.

There is another direction in which the Narasimhan–Seshadri theorem has been generalised. This is by allowing punctures in the Riemann surface. Here one is interested in studying representations of the fundamental group of the punctured surface with fixed holonomy around the punctures. These representations now relate to the parabolic vector bundles introduced by Seshadri (1977). The correspondence in this case for $G = \mathrm{U}_n$ was carried out by Mehta and Seshadri (1980). A differential geometric proof modelled on that of Donaldson for the parabolic case was given by Biquard (1991). The case of a general compact Lie group has been studied by Bhosle–Ramanathan (1989), Teleman–Woodward (2003), Balaji–Seshadri (2015), Balaji–Biswas–Pandey (2017) and others, under suitable conditions on the holonomy around the punctures.

The non-compactness in the group and in the surface can be combined to study representations of the fundamental group of a punctured surface into a non-compact reductive Lie group G. Simpson (1990) considered this situation when $G = \mathrm{GL}_n\mathbb{C}$. Biquard and Mundet i Riera in collaboration with the author (2020) extended this correspondence to the case of an arbitrary real reductive Lie group G (including the case in which G is complex), establishing a one-to-one correspondence between reductive representations of the fundamental group of a punctured surface X with fixed arbitrary holonomy around the punctures and polystable parabolic G-Higgs bundles on X.

In 1972 Takemoto generalised Mumford's stability to holomorphic vector bundles on a higher dimensional complex projective variety. This was easily extended to any compact Kähler manifold and, in this setup, the projectively flat condition of the theorem of Narasimhan and Seshadri generalises to the Hermitian–Yang–Mills equation, whose existence of irreducible solutions is equivalent to Mumford–Takemoto stability of the bundle, as proved by Donaldson (1986, 1987) in the algebraic case, and by Uhlenbeck and Yau (1986) in the general Kähler situation.

In a very different direction, partial p-adic analogues of the Narasimhan–Seshadri theorem and the Hitchin–Simpson correspondence have been studied by Deninger–Werner (2005, 2010), Faltings (2005, 2011), Ogus–Vologodsky (2007), as well as Abbes–Gros (2016) and Xu (2017).

Collaboration with S. Ramanan

After his return to TIFR in 1960, Narasimhan also began his long and fruitful collaboration with S. Ramanan. Together, they developed over more than two decades the theory of moduli spaces of vector bundles on Riemann surfaces.

Their first collaboration, however, was in the area of differential geometry, proving the existence of universal connections. In a first paper (1961) they proved that for the unitary group, namely the Stiefel bundle over the Grassmannian, there was a natural homogeneous connection which could serve as a universal connection.

They later generalised this result to all compact Lie groups and in fact to all Lie groups (1963). This result has been extensively used by physicists and geometers, for instance in Chern–Simons theory and in the work of Quillen on superconnections.

After the work of Narasimhan and Seshadri, using Mumford's theory, Seshadri (1967) showed that on the set $M(n, d)$ of semistable vector bundles of rank n and degree d on X of genus $g \geq 2$, under a certain notion of equivalence introduced by Seshadri – what later was called S-equivalence –, there is a natural structure of a normal projective variety. In [6] Narasimhan and Ramanan showed that the smooth points of $M(n, d)$ correspond precisely to the stable vector bundles, except for the case $n = 2$, $g = 2$ in which case $M(2, 0)$ is smooth. They also gave an explicit description of $M(2, 0)$ and $M(2, 1)$ when $g = 2$. The explicit description of $M(2, 1)$ had also been given independently by Newstead (1968) using different methods, and was later extended by Desale–Ramanan (1976) to hyperelliptic curves. Later Narasimhan and Ramanan began studying the case of genus $g = 3$ for which an earlier purely geometric study by Coble was very helpful.

Their next joint endeavour was to study the geometry of the moduli spaces $M(n, d)$, in general, using the geometry of X. Narasimhan and Ramanan [7] proved an analogue for the moduli spaces of vector bundles of the Torelli theorem regarding the Jacobian of X. A significant difference is that, unlike the Jacobian, which can be deformed into abelian varieties which are not necessarily Jacobians, the deformations of the moduli spaces of fixed determinant are obtained only from deformations of the Riemann surface. In [7] they introduced and exploited the notion of *Hecke correspondence*. In particular, when the genus is 2, this is a correspondence between the moduli spaces $M(2, 0)$ and $M(2, 1)$ with fixed determinants that they had explicitly described. The Hecke correspondence has been extensively used in the study of moduli spaces and plays a central role in the Geometric Langlands Programme.

Figure 2. From left to right: M. S. Narasimhan, the author, C. S. Seshadri, S. Ramanan and M. S. Raghunathan, Indian Institute of Science, Bangalore, 2012

They later looked at direct images of line bundles on etale coverings of the Riemann surface, and described them as fixed-point subvarieties of the moduli space of vector bundles under a natural action given by tensoring by a line bundle of finite order. Using the fixed point theorems, they were able to compute some topological invariants of the moduli space. This provided a higher rank generalisation of the Prym construction that has been recently generalised to moduli spaces of principal bundles and Higgs bundles in joint work of the author with Ramanan (2019), and with Barajas (2021).

Jointly with A. Beauville, Narasimhan and Ramanan (1989) generalised the Hitchin integrable system, given by the moduli space of Higgs bundles, to the situation in which the Higgs field is twisted by an arbitrary line bundle. This was extensively used by Ngô (2010) in his proof of the fundamental lemma of the Langlands Programme. A generalisation of this system twisting by a higher rank vector bundle was given recently by Narasimhan in collaboration with G. Gallego and the author [2]. This generalisation was motivated by a problem in supersymmetric gauge theory, and made use of ideas of Chen and Ngô (2020) in their study of the Hitchin fibration for higher dimensional varieties. A generalisation of the results by Beauville–Narasimhan–Ramanan for higher dimensional varieties was given by Narasimhan and Hirshowitz (1994).

The Harder–Narasimhan filtration

Another seminal contribution of Narasimhan is his joint work with G. Harder [3] on the computation of the cohomology of the moduli space of vector bundles $M(n, d)$ with n and d coprime. Their number theoretical approach, counting points over finite fields, was based on the Weil conjectures that had just then been proved by Deligne (1974), and Siegel's formula. Earlier, Harder (1970), using the work by Newstead (1968) on the computation of the Betti numbers for $M(2, 1)$, had established a connection between the cohomology groups of the rank 2 moduli space and the Tamagawa number of $SL_2(\mathbb{C})$. This method, pursued by Desale–Ramanan (1975), led to an explicit inductive formula for the Betti numbers of the moduli spaces $M(n, d)$ in the coprime situation. Later, Atiyah and Bott (1983) used Yang-Mills theory to give an alternative computation of the Betti numbers.

An important concept introduced in [3] is that of *Harder–Narasimhan filtration*. Harder and Narasimhan proved that given any vector bundle E there is a canonical filtration

$$0 = E_0 \subset E_1 \subset \cdots \subset E_k = E$$

such that E_i/E_{i-1} is semistable for $i = 1, \ldots, k$, and

$$\mu(F_i/F_{i-1}) > \mu(F_{i+1}/F_i) \quad \text{for } i = 1, \ldots, k-1.$$

The Harder–Narasimhan filtration also played a central role in the approach of Atiyah and Bott, using the differential geometry of connections and holomorphic structures on vector bundles.

The notion of Harder–Narasimhan filtration has been extended to principal bundles, Higgs bundles and other similar objects with important applications. Analogues of this filtration have been used extensively in numerous other contexts in algebraic geometry and number theory.

Other contributions

There are many other important contributions of Narasimhan, some of which would deserve a section of their own, but for lack of space we will just briefly describe some of them here. For a more complete account we refer to [10].

It was in the mid 1950s that the first papers of Narasimhan appeared. They were devoted to the study of the Laplace operator on Riemannian manifolds (1956) and certain extensions of elliptic operators (1957). After these, he wrote a paper giving a new approach to the construction of Green's function of an open Riemann surface (1960), and another paper studying the local properties of variations of complex structures on a relatively compact subdomain of an open Riemann surface (1961).

Together with T. Kotake, Narasimhan proved a theorem characterising real analytic functions by Cauchy-type inequalities satisfied with respect to powers of a linear elliptic operator with analytic coefficients (1962). This result was used in the original proof of the Atiyah–Bott fixed point theorem, and has been generalised in several directions by many authors, including Lions–Magenes, Bouendi–Goulaouic, Bouendi–Metvier and Bolly–Camus–Mattera.

Narasimhan and R. R. Simha [9] proved, using differential geometric methods, that the set of isomorphism classes of complex structures with ample canonical line bundle on a compact connected real analytic manifold has a natural structure of a Hausdorff complex space.

Jointly with K. Okamoto [4], Narasimhan made an important contribution to the theory of representation of Lie groups. It had been suggested by Langlands (1966) that, in analogy to the Borel–Weil–Bott theorem for compact groups, the Harish-Chandra discrete series of a real semisimple non-compact Lie group defining a symmetric space of Hermitian type could be realised as square-integrable harmonic forms in certain holomorphic vector bundles. The work of Narasimhan and Okamoto was the first breakthrough in the proof of this conjecture. Although Narasimhan did not pursue this any further, his student Parthasarathy has contributed in an important way to this field.

Narasimhan wrote two joint papers with H. Lange: the first one (1983) on the study of maximal subbundles of rank two vector bundles on curves, and a second one (1989) on squares of ample line bundles on abelian varieties.

J.-M. Drezet and Narasimhan [1] proved that the moduli space of vector bundles on a curve is locally factorial and determined the Picard group, showing that this is isomorphic to the integers. Their results enable one to define a generalisation of the Riemann theta divisor of the Jacobian. The famous Verlinde formula gives the dimension of the space of sections of powers of the theta line bundle (generalised theta functions) on the moduli space. Tsuchiya–Ueno–Yamada (1989) had proved factorisation theorem and the Verlinde formula in the context of Conformal Field Theory. Narasimhan and Ramadas [5] gave an algebro-geometric proof of this in the rank 2 case, which they extended also to parabolic bundles. In a previous collaboration, Narasimhan and Ramadas (1979) studied Yang–Mills theory on the product of the 3-sphere with the real line, using topological and differential geometric techniques to identify the configuration space as the base space of a principal bundle with the gauge group as structure group.

In joint work with S. Kumar (1997), Narasimhan extended his result with Drezet to the moduli space of principal bundles over a compact Riemann surface with a simple, simply-connected connected complex affine algebraic structure group. And with S. Kumar and A. Ramanathan (1997), using the relation between principal bundles and infinite Grassmanians, they elucidated the relation between conformal blocks and generalised theta functions. This enables one to compute the dimension of the space of generalised theta functions using the Verlinde formula. This was also proved by Beauville–Lazlo (1994) in the vector bundle case.

Narasimhan and M. Nori (1981) proved that there are only finitely many smooth curves having a given abelian variety as the Jacobian. I. Biswas and Narasimhan (1997) studied Hodge classes of moduli spaces of parabolic bundles on general curves. With Y. I. Holla (2001), Narasimhan proved a generalisation of a theorem of Nagata on a ruled surface to the case of a bundle of flag varieties associated to a principal bundle.

Narasimhan also worked on vector bundles on higher dimensional varieties. He studied the moduli space M of stable vector bundles of rank 2, vanishing first Chern class and second Chern class $c_2 = 2$ on complex projective 3-space. With A. Hirschowitz (1982) he proved that M is rational. A compactification of M was given by Narasimhan and G. Trautmann (1990) as the closure in the moduli space of sheaves constructed by Maruyama (1978). Later, Narasimhan and Trautmann (1991) computed the Picard group of the compactification. With W. Decker and F.-O. Schreyer (1990), he studied rank 2 vector bundles on projective 4-space, developing a construction by Barth (1980) of irreducible rank 2 bundles with first Chern class $c_1 = -1$. G. Elencwajg and Narasimhan (1983) wrote a paper on projective bundles on complex tori.

Jiayu Li and Narasimhan (1999) proved a correspondence relating the existence of a Hermitian–Einstein metric on a rank 2 parabolic bundle over a Kähler surface to the stability of the parabolic bundle. This was related to work by Munari (1993) and Biquard (1997).

3 Some personal reminiscences

I first met Narasimhan quite soon after having completed my doctoral thesis in 1991. From the very beginning, he was very kind to me, and extremely generous in the exchange of ideas. In those years, we mostly met in conferences in Europe, but thanks to my collaboration with S. Ramanan, whom I had met soon after Narasimhan, I started travelling regularly to India, where we also met.

We were very lucky to have Narasimhan in Madrid on several memorable occasions. In 2006 he participated in a panel, jointly with Sir Michael Atiyah, Jean-Pierre Bourguignon, Philip Candelas, José Manuel Fernández de Labastida, and Shing-Tung Yau on "New Interactions between Geometry and Physics", organised in the context of a conference in honour of Nigel Hitchin for his 60th birthday, that took place in Madrid soon after the International Congress. In 2012, the Instituto de Ciencias Matemáticas (ICMAT) in Madrid organised a conference in his honour for his 80th birthday, and later in 2017 he was invited as a special guest for a conference that ICMAT organised celebrating Ramanan's 80th birthday. On that occasion he participated in a special panel jointly with Antonio Córdoba, Nigel Hitchin and S. Ramanan on "Mathematics in India and Europe". A photographic exhibition on "Kolam, an Ephemeral Women's Art of South India" by photographer and anthropologist Claudia Silva was opened after the panel.

Over the years, we had many discussions on the possibility of establishing a scheme for mathematical collaboration between India and Europe in our research field. There had been some bilateral programmes between France and India, and we were contemplating the idea of bringing that to a larger context. It took a long time, but eventually we established a collaboration programme involving four nodes in Europe (Aarhus, Madrid, Oxford and Paris) and four in India (Bangalore, Mumbai and two in Chennai). This was the Indo-European Project on Moduli Spaces that was operating during the period 2013–2017, involving more than eighty mathematicians, funded under the Marie Curie Programme by the European Commission, and coordinated by ICMAT in Madrid. Narasimhan played an important role in the gestation of this project.

Figure 3. From left to right: M. S. Narasimhan, S. Ramanan, N. Hitchin and A. Córdoba, ICMAT, Madrid, 2017

Figure 4. From left to right: Guillermo Barajas, Guillermo Gallego, Gadadhar Misra and M. S. Narasimhan, Bangalore, 2020

In addition to discussing mathematics and scientific collaboration, Narasimhan and I very much liked to enjoy a glass (or two!) of good red wine, very often in company of our common friend and collaborator Ramanan, and other good friends. My wife and I were very fortunate to enjoy his great hospitality and that of his wife Sakuntala and daughter Shobhana, at his home during our many visits to Bangalore over the last few years.

I last saw Narasimhan in person in Bangalore in February 2020, during an activity on Moduli Spaces organised at the International Centre for Theoretical Sciences (ICTS). On that occasion we also had the opportunity to have a very nice dinner, accompanied as usual by good red wine, with our friend Gadadhar Misra and other friends. After the ICTS meeting, I went to Chennai for a few days, for a visit to the Chennai Mathematical Institute (CMI), where as a matter of fact I also saw C. S. Seshadri for the last time. I had actually met Seshadri in the late 1980s when I was still a graduate student at Oxford, where he gave a talk on parabolic bundles, a subject of great interest at the time in relation to Jones–Witten theory and the Atiyah–Segal approach to topological quantum field theory.

My students Guillermo Barajas and Guillermo Gallego also came to the workshop at ICTS in February 2020, and after that, while I was visiting the CMI, they went to the Indian Institute of Science to discuss with Narasimhan for a week. As always Narasimhan was extremely generous, spending a lot of time talking with them and, together with Gadadhar Misra, entertaining them (Figure 4). The last paper of Narasimhan, written jointly with Gallego and the author [2] appeared just a few days before his passing. The discussions with Barajas were very useful in connection with a joint paper of Barajas and the author (2021), which generalises to principal bundles and Higgs bundles the Prym-type construction given by Narasimhan and Ramanan (1975).

In addition to being a great mathematician, Narasimhan was a wonderful human being. He was kind, generous and sympathetic, and is very much missed by many people who loved him.

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