Addenda: Complete Boolean algebras of type I factors

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Lemmas 4.4, 4.6 and a slightly strengthened version of 4.7 can be proved in one step as follows.

Lemma. Let $\{R_\alpha\}_{\alpha\in A}$ be a type I factorization. If there exists a vector ψ such that

$$\inf d(\psi;\ R_{\alpha_1}, \cdots R_{\alpha_n}, R(\{\alpha_1, \cdots \alpha_n\}^c)) = \varepsilon > 0$$

where inf is taken over all n, $\alpha_1, \cdots \alpha_n$. then R_α is a TPF.

Proof. For each finite $J\subset A$ choose a minimal projection $P_j^{(J)}\in R_j$ for all $j\in J$ such that

$$(\psi, \prod_{j\in J} P_j^{(J)}\psi)\geq\varepsilon.$$

Let

$$\psi(J)=\prod_{j\in J} P_j^{(J)}\psi,$$

$$S(J)=\bigcup_{K\supset J}\psi(K),$$

$$S=\bigcap_J S(J)^{(W)}.$$

Let $\Phi\in S$. By lemma 4.1 we have $(\psi, \Phi)\geq\varepsilon$. For any $j\in J$, all vectors in $S(J)$ are product vectors in

$$H=H_j\otimes H'_j,$$

where

$$R_j=B(H_j)\otimes\mathbf{1}.$$

By lemma 6.2 any vector in $S(J)^{(W)}$ is a limit of a sequence of ele-

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ments in $S(J)$. By lemma 3.3 $\Phi \in S$ is thus a product vector. Hence there exists a minimal projection $P_j \in R_j$ such that $P_j \Phi = \Phi$ Hence Φ is factorizable and R_α is a TPF by lemma 4.3. Q. E. D