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ScienceDirect
Ann. I. H. Poincaré – AN 34 (2017) 1633–1637
ANNALES DE L'INSTITUT HENRI POINCARÉ ANALYSE NON LINÉAIRE
www.elsevier.com/locate/anihpc

Corrigendum

Correction and addendum to "Boundary regularity of minimizers of $p(x)$-energy functionals" [Ann. Inst. Henri Poincaré, Anal. Non Linéaire 33 (2) (2016) 451–476]

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Received 5 October 2016; accepted 27 September 2017
Available online 6 October 2017

Abstract

In the paper [1] "Boundary regularity of minimizers of $p(x)$-energy functionals", some modifications are needed.

1. The exponent $p_2 = p_2(2R)$ in the statement of Theorem 2.6 should be $p_2(\rho)$. According to this correction, we should modify the proof of Theorem 3.2.
2. In Theorem 1.1, the domain Ω is assumed to have the Lipschitz boundary $\partial\Omega$. However, we need to assume that $\partial\Omega$ is in the class C^1.

MSC: 49N60; 35J50; 58E20

Keywords: $p(x)$-growth; Minimizer; Boundary regularity

In the study of regularity for minimizers of $p(x)$ energy functionals, in paper [1], we used the following well-known Lemma:

Lemma 0.1. *Let $f : [0,\infty) \to [0,\infty)$ be a non-negative and non-decreasing function satisfying*

$$f(\rho) \le A\left[\left(\frac{\rho}{R}\right)^{\alpha} + \varepsilon\right] f(R) + BR^{\beta}$$

for some A, B, ε, α, $\beta > 0$, with $\alpha > \beta$ and for all $0 < \rho \le R \le R_0$, where $R_0 > 0$ is given. Then there exist constants ε_0 and C such that if $\varepsilon < \varepsilon_0$, we have

DOI of original article: https://doi.org/10.1016/j.anihpc.2014.11.003.
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https://doi.org/10.1016/j.anihpc.2017.09.004
0294-1449/

$$f(\rho) \le C\Big[\Big(\frac{\rho}{R}\Big)^{\beta} f(R) + B\rho^{\beta}\Big],$$

for all $0 \le \rho \le R \le R_0$.

In the final step of the proof of Theorem 2.6, we obtain the estimate

$$\begin{aligned}\int_{B^+(\rho)} & |Dv|^{p_2} dx \\ & \le c\Big[\Big(\frac{\rho}{R}\Big)^{m} + k + R^{\sigma - m\delta}\Big] \int_{B^+(2R)} |Dv|^{p_2} dx \\ & \quad + c(k^{1-p_2}+1)R^{m-p_2 m/s}\Big(\int_{B^+(2R)} (1+|Dh|^2)^{s/2} dx\Big)^{p_2/s}. \end{aligned} \tag{1}$$

We applied Lemma 0.1 and got the assertion of the theorem. But, in the left hand side of (1) there is the exponent $p_2 = p_2(2R) = \sup_{B_{2R}} p(x)$: namely we observe that the quantity of the left hand side depends not only on ρ but also on R.

According to this, in the estimate (2.16) of [1, Theorem 2.6], the exponent $p_2(2R)$ on the left-hand side should be replaced by $p_2(\rho)$. More precisely, we state and prove Theorem 2.6 as follows:

Theorem 2.6. *Assume that $p(x)$ satisfies (1.9) Let $R > 0$ be sufficiently small so that*

$$\Big(1 + \frac{\delta}{2}\Big) p_2(2R) \le (1+\delta) p_1(2R). \tag{2.15}$$

Let $v \in W^{1,p(x)}(B^+(R), \mathbb{R}^n)$ a local minimizer of $\mathcal{D}_{p(x)}$ in the class

$$\{w \in W^{1,p(x)}\ ;\ w = h \ \text{ on } \Gamma(R)\},$$

where h is a given boundary data in the class $W^{1,s}(B^+(R), \mathbb{R}^n)$, $s > (1+\delta)p_2$. Assume that $\mathcal{D}_{p(x)}(v) \le K$ for some positive constant K. Then, for any $\varepsilon \in (0, mp_2(2R)/s)$, we have

$$\begin{aligned}\int_{B^+(\rho)} & |Dv|^{p_2(\rho)} dx \\ & \le c_0\Big[\Big(\frac{\rho}{R}\Big)^{m-\varepsilon} \int_{B^+(R)} |Dv|^{p_2(2R)} dx \\ & \quad + \rho^{m - mp_2(2R)/s}\Big(\int_{B^+(2R)} (1+|Dh|^2)^{s/2} dx\Big)^{p_2(2R)/s}\Big]. \end{aligned} \tag{2.16}$$

Proof. We can proceed as in the proof in [1] and get (1). From (1), it is easy to see that

$$\begin{aligned}\int_{B^+(\rho)} & (1+|Dv|^2)^{p_2(\rho)/2} dx \\ & \le c\Big[\Big(\frac{\rho}{R}\Big)^{m} + k + R^{\sigma - m\delta}\Big] \int_{B^+(2R)} (1+|Dv|^2)^{p_2(2R)/2} dx \\ & \quad + c(k^{1-p_2(2R)}+1)R^{m-p_2(2R)m/s}\Big(\int_{B^+(2R)} (1+|Dh|^2)^{s/2} dx\Big)^{p_2(2R)/s} \end{aligned}$$

Now we can apply Lemma 0.1, and get the assertion. □

According to the above modification of Theorem 2.6, we must modify the proof of Theorem 3.2 as follows.

Proof of Theorem 3.2. Until (3.13), no change is necessary. (3.13) must be modified as

$$\int_{B_\rho^+} (1+|Dv|^2)^{p_2(\rho)/2}dx \le c\Big[\Big(\frac{\rho}{R}\Big)^{m-\beta}\int_{B_R^+}(1+|Dv|^2)^{p_2(2R)/2}dx + \rho^{m-mp_2(2R)/s}\Big(\int_{B_{2R}^+}(1+|Dh|^2)^{s/2}dx\Big)^{p_2(2R)/s}\Big]. \tag{3.13}$$

We can estimate the integral of the first term of the right-hand side of (3.13) exactly as in [1] to get

$$\begin{aligned}\int_{B_R^+}&(1+|Dv|^2)^{p_2(2R)/2}dx\\ &\le cR^{-m\omega_1(2R)}\int_{B_{2R}^+}(1+|Du|^2)^{p_2(2R)/2}dx\\ &\quad + cR^{m(1-(1+\delta)p_2(2R)/s)}\Big(\int_{B_{2R}^+}(1+|Dh|^2)^{s/2}dx\Big)^{(1+\delta)p_2(2R)/s}.\end{aligned}$$

So, we can proceed as in [1] and obtain the following modified (3.16)

$$\begin{aligned}\int_{B_\rho^+}&(1+|Dv|^2)^{p_2(\rho)/2}dx\\ &\le K_1\Big(\frac{\rho}{R}\Big)^{m-\beta}\int_{B_{2R}^+}(1+|Du|^2)^{p_2(2R)/2}dx + K_2\rho^{m-m\overline{p}_2(2R)/s}\hat{K}(h),\end{aligned}\tag{3.16}$$

for some positive constants K_1 and K_2, where $\overline{p}_2=(1+\delta)p_2(2R)$.

For the estimates on $\int_{B_R^+}|Du-Dv|^{p_2(2R)}dx$, no modification is necessary. Proceeding as [1], we arrive at the following modified (3.29)

$$\begin{aligned}\int_{B_\rho^+}&(1+|Du|^2)^{p_2(\rho)/2}dx\\ &\le K_3\Big[\Big(\frac{\rho}{r}\Big)^{m-\beta}+r^{\sigma-m\delta}+\hat{\omega}_G+\hat{\omega}_g\Big]\int_{B_r^+}(1+|Du|^2)^{p_2(r)/2}dx\\ &\quad + K_4[1+r^{\sigma-m\delta}+\hat{\omega}_G+\hat{\omega}_g]r^{m(1-\overline{p}_2(r)/s)}\hat{K}(h),\end{aligned}\tag{3.29}$$

for some constants K_3 and K_4.

In the following step (iteration procedure), we must proceed carefully, since some of p_2's must be $p_2(\tau r)$ and the others $p_2(r)$. For $\tau\in(0,1)$ which will be specified later, put $\rho=\tau r$ in the above estimate and multiply both sides by $(\tau r)^{p_2(r)-m}$, then we have

$$\begin{aligned}(\tau r)^{p_2(r)-m}&\int_{B_{\tau r}^+}(1+|Du|^2)^{p_2(\tau r)/2}dx\\ &\le K_3[\tau^{p_2(r)-\beta}+\tau^{p_2(r)-m}r^{\sigma-m\delta}+\tau^{p_2(r)-m}\hat{\omega}_G+\tau^{p_2(r)-m}\hat{\omega}_g]\\ &\quad\times r^{p_2(r)-m}\int_{B_r^+}(1+|Du|^2)^{p_2(r)/2}dx\end{aligned}$$

$$+ K_4[\tau^{p_2(r)-m} + \tau^{p_2(r)-m} r^{\sigma-m\delta} + \tau^{p_2(r)-m}\hat{\omega}_G + \tau^{p_2(r)-m}\hat{\omega}_g] \times r^{p_2(r)-m\overline{p}_2/s}\hat{K}(h). \tag{3.30}$$

On the other hand, by virtue of the minimality of u and the reverse Hölder inequality ([1, Proposition 2.2]), $\int_{B^+(\tau r)} |Du|^{p_2(\tau r)} dx$ is bounded from above by a constant M depending only on the functional and the boundary condition. So, we can see that

$$\begin{aligned}
\Psi(x_1,\tau r)^{p_2(r)} &= \Big\{\tau r\Big((\tau r)^{-m}\int_{B_{\tau r}}(1+|Du|^2)^{p_2(\tau r)/2}dx\Big)^{1/p_2(\tau r)}\Big\}^{p_2(r)}\\
&= \Big\{\tau r\Big((\tau r)^{-m}\int_{B_{\tau r}}(1+|Du|^2)^{p_2(\tau r)/2}dx\Big)^{1/p_2(r)}\\
&\quad\times\Big((\tau r)^{-m}\int_{B_{\tau r}}(1+|Du|^2)^{p_2(\tau r)/2}\Big)^{\frac{1}{p_2(\tau r)}-\frac{1}{p_2(r)}}dx\Big\}^{p_2(r)}\\
&\le c(M)(\tau r)^{-m\omega_1(r)p_2(r)}(\tau r)^{p_2(r)-m}\int_{B_{\tau r}}(1+|Du|^2)^{p_2(\tau r)/2}dx,
\end{aligned}$$

where $c(M)$ is a positive constant depending on M. Here, we used the assumptions that $p(x) > 1$ and that $\tau, r < 1$. Using the boundedness of $r^{-\omega_1(r)}$, we obtain

$$\Psi(x_1,\tau r)^{p_2(r)} \le C^*\tau^{-m\omega_1(r)p_2(r)}(\tau r)^{p_2(r)-m}\int_{B_{\tau r}}(1+|Du|^2)^{p_2(\tau r)/2}dx$$

for some positive constant C^*.

Now, combining the above inequality with (3.30), we obtain the following estimate instead of [1, (3.31)].

$$\begin{aligned}
&\Psi(x_1,\tau r)^{p_2(r)}\\
&\quad\le K_3C^*\tau^{-m\omega_1(r)p_2(r)}\big[\tau^{p_2(r)-\beta}+\tau^{p_2(r)-m}\{r^{\sigma-m\delta}\\
&\qquad+\hat{\omega}_G\big(\Psi(x_1,r)^2+r^{2(1-q/s)}K(h)^{2q}\big)+\hat{\omega}_g(r)\}\big]\times\Psi(x_1,r)^{p_2(r)}\\
&\qquad+\tau^{p_2(r)-m}r^{p_2(r)-m\overline{p}_2/s}C(g,G,p,h)\\
&\quad= K_3C^*\tau^{p_2(r)-\beta-m\omega_1(r)p_2(r)}\big[1+\tau^{\beta-m}\{r^{\sigma-m\delta}\\
&\qquad+\hat{\omega}_G\big(\Psi(x_1,r)^2+r^{2(1-q/s)}K(h)^{2q}\big)+\hat{\omega}_g(r)\}\big]\times\Psi(x_1,r)^{p_2(r)}\\
&\qquad+\tau^{p_2(r)-m}r^{p_2(r)-m\overline{p}_2/s}C(g,G,p,h),
\end{aligned}\tag{3.31}$$

where $C(g,G,p,h)$ is a positive constant depending only on $g^{\alpha\beta}(x)$, $G_{ij}(u)$, $p(x)$ and $h(x)$. The main difference between [1, (3.31)] and the above (3.31) is the appearance of $-m\omega_1(r)p_2(r)$ at the exponent of τ. However, by choosing $r > 0$ sufficiently small, we can assume that $\omega_1(r)$ is as small as we like. So, we can proceed slightly modifying the corresponding part of [1].

From (3.31), we obtain

$$\begin{aligned}
&\Psi(x_1,\tau r)\\
&\quad= K_5\tau^{1-(\beta/p_2(r))-m\omega_1(r)}\big[1+\tau^{(\beta-m)/p_2(r)}\{r^{(\sigma-m\delta)/p_2(r)}\\
&\qquad+\hat{\omega}_G^{1/p_2(r)}\big(\Psi(x_1,r)^2+r^{2(1-q/s)}K(h)^{2q}\big)+\hat{\omega}_g^{1/p_2(r)}(r)\}\big]\times\Psi(x_1,r)\\
&\qquad+\tau^{1-m/p_2}r^{1-mq/s}C_0(g,G,p,h),
\end{aligned}\tag{3.32}$$

where K_5 depends only on K_3, C^*, $p_2(r)$, and we are putting $q = 1+\delta$, $C_0(g,G,p,h) = C(g,G,p,h)^{1/p_2(r)}$.

Since $0 < \beta < 1$, $m > 2$, $\gamma_1 \le p_2(r) \le \gamma_2$, and $\tau < 1$, we have

$$\tau^{(\beta-m)/p_2(r)} \le \tau^{(\beta-m)/\gamma_1}. \tag{3.33}$$

Without loss of generality we can assume that $0 < r < 1$, so we see that

$$r^{(\sigma-m\delta)/p_2(r)} \le r^{(\sigma-m\delta)/\gamma_2}, \quad (\tau r)^{1-(\beta/p_2(r))} \le (\tau r)^{1-(\beta/\gamma_1)}. \tag{3.34}$$

In the following, since we consider the case that ω_G and ω_g are sufficiently small, we can assume that $\omega_G,\ \omega_g < 1$. So, we have

$$\hat{\omega}_G^{1/p_2(r)} \le \hat{\omega}_G^{1/\gamma_2}, \quad \hat{\omega}_g^{1/p_2(r)} \le \hat{\omega}_g{}^{1/\gamma_2}. \tag{3.35}$$

Now, let us take $r > 0$ so small that $2m\omega_1(r) < 1 - (\beta/\gamma_1)$, then we have

$$\tau^{1-(\beta/p_2(r))-m\omega_1(r)} \le \tau^{(1-\beta/\gamma_1)/2}.$$

For the sake of simplicity, let us put

$$\mu_1 := \frac{1}{2}\Big(1 - \frac{\beta}{\gamma_1}\Big), \quad \tilde{\omega}_G := \hat{\omega}_G^{1/\gamma_2}, \quad \tilde{\omega}_g := \hat{\omega}_g^{1/\gamma_2},$$

and let μ_2 be a positive constant such that

$$\mu_2 < \min\Big\{\mu_1, 1 - \frac{mq}{s}\Big\}.$$

Then, from (3.32), assuming $\Psi(r) < 1$, we get

$$\begin{aligned}\Psi(x_1, \tau r) \le K_5 \tau^{\mu_1}\big[1 + \tau^{(\beta-m)/\gamma_1}\big\{r^{(\sigma-m\delta)/\gamma_2} \\ + \tilde{\omega}_G\big(\Psi(x_1, r) + r^{2\mu_2} K(h)^{2q}\big) + \tilde{\omega}_g\big\}\big]\Psi(x_1, r) \\ + \tau^{1-m/p_2} r^{\mu_2} C_0(g, G, p, h).\end{aligned} \tag{3.36}$$

Now, we can proceed exactly as in [1] to get the assertion, mentioning that in (3.38) $\tau^{(\beta-m)\gamma_1} r_0^{(\sigma-m\delta)/\gamma_2}$ is a typo and should be $\tau^{(\beta-m)/\gamma_1} r_0^{(\sigma-m\delta)/\gamma_2}$. □

Finally, we mention also that in the proof of [1, Theorem 1.1, p. 475] the transformation that straighten partly the boundary $\partial\Omega$ should be C^1-map. Since the coefficients of the integrand must be continuous, the Jacobian of the transformation must be continuous, and therefore the transformation should be of class C^1. Consequently, in the statement of [1, Theorem 1.1] the boundary $\partial\Omega$ should be assumed to be of class C^1.

Conflict of interest statement

This article has no conflict of interest.

References

[1] M.A. Ragusa, A. Tachikawa, Boundary regularity of minimizers of $p(x)$-energy functionals, Ann. Inst. Henri Poincaré, Anal. Non Linéaire 33 (2) (2016) 451–476, http://dx.doi.org/10.1016/j.anihpc.2014.11.003.