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Corrigendum

Corrigendum to "Singularity formation of the Yang-Mills flow" [Ann. I. H. Poincaré – AN 35 (6) (2018) 1655–1686]

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It was brought to our attention by A. Waldron, whom we thank for the correction, that Theorems 1.1 and 1.3 (cf. Theorems 4.1 and 6.1) misstate what is shown by our methods. The errors concern the manner of convergence and the space in which the limit lies. Theorem 1.1 was an informal statement, with the precise statement appearing in Theorem 4.1. In lieu of these separate statements, we simply give a corrected version of Theorem 4.1:

Theorem 4.1. *Given a vector bundle $E \to M$, suppose $\{\nabla_t^i\}$ is a sequence of smooth solutions to Yang-Mills flow defined on $M \times [-1, 0]$ with $\mathcal{YM}(\nabla_t^i) \le \mathcal{YM}(\nabla_{-1}^i) < C$. There exists a closed set $\Sigma \subset M \times [-1, 0]$ of locally finite $(n-2)$-dimensional parabolic Hausdorff measure, a limit vector bundle $E_\infty \to M \times (-1, 0] \setminus \Sigma$, a smooth solution ∇_t to Yang-Mills flow on $M \times (-1, 0] \setminus \Sigma$, and a subsequence, still denoted $\{\nabla_t^i\}$, such that $\nabla_t^i \to \nabla_t$ in $C^\infty_{loc}(M \times (-1, 0] \setminus \Sigma)$.*

Proof. The definition of Σ is as written of page 11. The proof of Lemma 4.2 is unchanged, showing Σ is closed. The proof of Lemma 4.3 gives the existence of the limiting connection, limiting bundle, and the subsequence converging outside Σ as claimed. The limit bundle E_∞ is constructed over the spacetime region $M \times (-1, 0] \setminus \Sigma$ by a standard argument which we sketch here. In particular, covering $M \times (-1, 0] \setminus \Sigma$ by parabolic balls as described in Lemma 4.3, we can choose gauges at the final time slices of each ball so that the connection has C^∞ estimates on that slice. A bootstrapping argument gives C^∞ estimates for the connection in these gauges on a parabolic sub-ball of half the given radius. Examining the transformation law for the connection on overlaps, another bootstrapping argument gives C^∞ estimates for the transition maps themselves (cf. [1] in the elliptic case), allowing us to take a further subsequence for which these transition maps converge in C^∞, yielding the limiting bundle E_∞. Note here a mild generalization of the definition of Yang-Mills flow, here defined as a single connection on a bundle defined over a dense open subset of a cylinder, satisfying the Yang-Mills flow in the usual sense. The proofs of Lemmas 4.4 and 4.5 are unchanged, yielding that Σ has locally finite $(n-2)$-dimensional parabolic Hausdorff measure. □

Section 4.2 analyzes the difference between curvature density of the limit obtained in Theorem 4.1 and the limit of the curvature densities. These results are unaffected by the changes above. However Lemma 4.10 has the first of

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a few instances where convergence/nonconvergence of curvature densities is erroneously referred to as strong/weak convergence in H^2_1. We corrected the statement and proof:

Lemma 4.10. *One has that* $\left\{\frac{1}{2}|F_{\nabla^i_t}|dV\right\}$ *does not converge to* $\frac{1}{2}\left|F_{\nabla_t}\right|dV$ *if and only if* $\mathcal{P}^{n-2}(\Sigma) > 0$ *and* $\nu(M \times [-1,0]) > 0$.

Proof. It follows from Lemma 4.7 that if $\mathcal{P}^{n-2}(\Sigma) > 0$ then for $\mathcal{P}^{n-2}$ almost everywhere $z \in \Sigma$ one has

$$\Theta(\nu, z) = \Theta(\mu, z) \geq \epsilon_0,$$

hence $\nu(M \times [-1,0]) = \nu(\Sigma) > 0$, and $\frac{1}{2}|F_{\nabla^i_t}|^2 dV dt$ does not converge to $\frac{1}{2}\left|F_{\nabla_t}\right|^2 dV dt$. The converse direction is clear from the definition of ν. □

The results of Section 5, including the proof of Theorem 1.2, are unaffected by the changes above. Section 6 gives the proof of Theorem 1.3, again an informal statement made precise in Theorem 6.1. We correct Theorem 6.1:

Theorem 6.1. *Suppose* $\{\nabla^i_t\}$ *is a sequence of smooth solutions to Yang-Mills flow on* $E \to M \times [-1,0]$ *with*

$$\sup_i \mathcal{YM}(\nabla^i_{-1}) < \infty, \qquad \sup_i \int_{M\times[-1,0]} \left|\frac{\partial \nabla^i_t}{\partial t}\right| dV dt < \infty,$$

and suppose ∇_t *is a limit as guaranteed by Theorem 4.1. Then exactly one of the following holds:*

- *There exists a blowup sequence converging to a Yang-Mills connection on* S^4.
- *One has*

  $$\left|F_{\nabla^i_t}\right|^2 dV dt \to \left|F_{\nabla_t}\right|^2 dV dt$$

  as convergence of Radon measures, and $\mathcal{P}^{n-2}(\Sigma) = 0$.

Proof. The proofs of the preliminary results of Section 6 are unaffected by the changes above. In the proof of Lemma 6.9, which contains the final argument, the sentence beginning, “There is a local H^2_1 estimate...” should be replaced by: “Using (6.13), we can apply Theorem 3.8 to obtain a subsequence so that $\widetilde{\nabla^i_t} \to \widetilde{\nabla}^\infty_t$ smoothly on compact subsets of $\mathbb{R}^n \times (-\infty, 0]$.” □

The statement of Corollary 1.4 is unchanged, while the final two sentences of the proof should be amended as follows: “If the curvature densities do not converge, Theorem 6.1 yields the further blowup sequence which converges to a Yang-Mills connection on S^4. If the curvature densities do converge, as the Ψ functional is becoming constant along the blowup sequence, the second term of the entropy monotonicity formula of (3.2) converges to zero, which implies that the blowup limit is a soliton.”

References

[1] H. Nakajima, Compactness of the moduli space of Yang-Mills connections in higher dimensions, J. Math. Soc. Jpn. 40 (3) (1988).