ERME column

regularly presented by Jason Cooper and Frode Rønning

In this issue, with a contribution by
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CERME12

The 12th Congress of the European Society for Research in Mathematics Education, CERME12, was held as an online conference from 2 to 6 February 2022, organized by the Free University of Bolzano, Italy. The congress was due to be held in 2021, but was postponed because of the COVID-19 pandemic. with the hope that it would be possible to run it as a physical conference. Unfortunately, this did not work out. However, the organizers in Bolzano did a great job turning the conference into an online event, including also social events in addition to the sessions in the Thematic Working Groups (TWGs).

A total of more than 900 participants attended CERME12, distributed over 27 TWGs. Around 540 papers and 130 posters were presented. Two plenary lectures were delivered: one titled *Enhancing language for developing conceptual understanding – a research journey connecting different research approaches*, by Susanne Prediger from TU Dortmund, Germany, and another, titled *Conceptualizing individual-context relationships in teaching: Developments in research on teachers' knowledge, beliefs and identity*, by Jeppe Skott from Linneaus University, Sweden, and University of Agder, Norway. A plenary panel around the topic of *Big Questions in Mathematics Education* was held on the final day of the conference.

Two new Thematic Working Groups began their activity at the 12th conference: TWG11 – Algorithmics (taking the number 11 from the discontinued *Comparative studies in mathematics education*), and TWG27 – *The Professional Practices, Preparation and Support of Mathematics Teacher Educators*.

Planning for CERME13 is well underway. CERME13 will be held at the Eötvös Loránd University, Budapest, Hungary from 10 to 14 July 2023. See http://erme.site/cerme-conferences/. Scheduling the conference in the summer is hoped to improve the chances of holding it face-to-face, and to bring us back to schedule with CERME14, to be held in February 2025.

CERME Thematic Working Groups

We continue the initiative of introducing the CERME working groups, which we began in the September 2017 issue, focusing on ways in which European research in the field of mathematics education may be interesting or relevant for people working in pure and applied mathematics. Our aim is to enrich the ERME community with new participants, who may benefit from hearing about research methods and findings and contribute to future CERMEs.

Introducing CERME Thematic Working Group 4 – Geometry Teaching and Learning

Michela Maschietto and Alik Palatnik

CERME's Thematic Working Group on geometry was created at CERME3 in 2003 [5]. Throughout the history of CERME, this TWG has had different names, so as to take into account different aspects of teaching and learning geometry and/or to emphasize particular ones: from *Geometrical thinking*, through *Geometry*, and currently *Geometry teaching and learning*. Typically, around 25 participants from all over Europe, the Americas, Asia, Africa, and Australia attend the working group, and 15–20 papers/posters are presented and discussed per conference.

Research on geometry teaching and learning has several components that have been addressed in the conferences. Emphasis and interest have varied from one conference to another, depending on the papers presented in the working group. Four main topics can be identified: specific aspects of mathematical activity in geometry, including what it means to be "doing geometry" and how to characterize geometrical thinking; learning geometry, in terms of students' processes in solving geometrical tasks, with attention to visualization, language, argumentation, transition between different representations and use of tools; teaching geometry, from the point of view of curriculum, methodologies, tools, tasks and competencies; teacher education in geometry, referring to contexts, practices, content and perspectives. On the one hand, this richness of components shows the complexity of the geometry thematic; on

the other hand, participants critically comment that some components seem to be less present in the range of papers for discussion because those components are within the scope of other working groups. In general, the papers discussed in TWG4 concern research carried out from kindergarten to secondary school and, at the university level, mainly prospective primary school teachers; there are no papers concerning other university students or the transition to tertiary geometry (which might be addressed in TWG14 on university mathematics).

As said above, a recurring topic of discussions has been to characterize what is meant by "doing geometry", i.e., geometrical thinking and its development. Since the first meeting of the TWG, the development of shared theoretical frameworks has been crucial to ground collaboration among the participants. Besides the always-mentioned van Hiele levels [6], other approaches and frameworks, such as the geometrical paradigms, the geometrical working space, the formulation of geometrical thinking in terms of (four) competencies have been discussed [5]. Other approaches have been added over the years, referring to spatial skills. The metaphor of space has also been used to articulate three facets of doing geometry – in the realm of physical space, geometrical space, and graphical space [4]. During CERME10 [3] and CERME11 [4], the discussions were about the characterization of these spaces and their mutual relationships, from the psychological and mathematics education points of view. The psychological point of view refers to the exploration of relationships between physical and graphical space, while from the mathematics education point of view, the focus is traditionally on the links between graphical and geometrical space. In this perspective, geometry consists of establishing relationships between these spaces, and to solve geometry problems one needs to "grasp space" and make the information usable in another kind of space [1]. In the papers presented in CERME12, some of these frameworks have become less prominent or even missing in their theoretical references, although they do emerge in the discussions. For instance, research on spatial skills and on spaces was not present, though questions on relationships between geometrical knowledge and spatial knowledge arose. Other frameworks, such as van Hiele levels, were critically discussed, especially in relation to the tests based on them and to the components of geometrical thinking/competencies that they do or do not help to grasp. Finally, two other theoretical elements have become relevant in recently discussed research: the embodied approach in experiments carried out with students at different school levels, especially in relation to the use of tools, and emotional and motivational aspects emerging-in and accompanying activities in geometry. In general, these new theoretical elements allow us to reexamine the questions "What does it mean to learn geometry?", "What geometry should our students know when they move from primary to secondary to tertiary education?" and "What skills (visual, reasoning, operational and figural) should students/pupils acquire/develop by the end of a given school level?".

Besides psychology, another area of shared interest between mathematics education and other disciplines concerns language, both in terms of the emergence of the geometrical lexicon and the construction of the meanings of words used in geometry (considering that it is a long and complex process that cannot be reduced to the matter of "vocabulary"). Many contributions in TWG04 pertain to argumentation, justification, reasoning or demonstration, topics in which geometry has a privileged status, because it is often the only context in which school students engage in proving.

In all CERMEs, research on the use of tools (both material and digital) in geometry teaching and learning have been discussed. Many contributions have focused on dynamic geometry software for 2D geometry, mainly at the secondary level, but with some contributions concerning teacher education. With respect to the previous CERMEs, in CERME12 we discussed a paper on the 3D environment of GeoGebra that demonstrated and discussed how in solving a geometrical construction, students intertwined the 2D procedure of construction, visualization of 3D objects, and relationships between procedures and representations. 3D geometry learning environment is an interesting topic yet to be developed in future CERMEs, also including Augmented and Virtual Reality for 3D geometry. In addition, the situation of online teaching, compelled by the current pandemic, has raised some new questions that need to be studied pertaining to teaching geometry online, its consequences for learning, and its influence on students' conceptualization.

In addition to digital tools, research on material tools has been presented and discussed at several CERMEs. For instance, in CERME10, a paper on the Pythagorean theorem proposed the analysis of an experiment on the use of material artifacts (called "mathematical machines" and related to one of the proofs of the theorem), intending to discuss the mediation of these artifacts in the construction of meanings in geometry. Regarding 3D geometry, at CERME12 there were two contributions on the use of construction kits and 3D pens, which allow the creation and exploration of solids in microspace and mesospace [2]. Research on 3D geometry with different types of tools could be a fruitful topic for further development, with implications for teacher education in geometry.

In this working group, the variety of nationalities of the participants (from different countries of the world) has always allowed a comparison of teacher training programs, mathematics curricula (in particular, in geometry) and teaching practices that have developed in the different countries, even when this was not the main object of investigation. In the perspective of the vertical development of geometric thinking, the link between primary and secondary school has emerged as a fundamental question.

At CERME12, the works on teacher education accounted for half of the accepted contributions. While there is a group at CERME dedicated to teacher education, the discussion in TWG4 allowed us to focus on the specificities of geometry and its unique mathematical processes. The papers presented in CERME12, on the one

hand, have led to the question of which tasks are emblematic for prospective teachers; on the other hand, they have paid attention to teachers' beliefs relating to geometry itself and the "ideas" that prospective teachers have about the fundamental objects of geometry. This aspect is closely linked to the study of the motivational and emotional aspects in doing geometry, as mentioned above, from the students' perspective. Classification tasks in the plane and in 3D space have emerged as particularly interesting in terms of enabling geometric processes for primary-school prospective teachers; these are tasks in which, after analyzing the content knowledge, prospective teachers are asked to anticipate or analyze pupils' solving processes. With these tasks, it is possible to work simultaneously on content knowledge, which is often lacking in prospective teachers according to research reports, and on the specific competence for teaching geometry. Further studies are needed to characterize design principles and applications of emblematic tasks for teacher education in geometry.

Finally, during CERME12 some researchers proposed to explore new topics, such as non-Euclidean geometries, topology, and analytical geometry at various levels of education.

To summarize, TWG4 is an active group whose work reflects both long-standing and emerging trends in the field of geometry education and stimulates further research in this area. On all the topics of this TWG, contributions are welcome not only from researchers in mathematics education, but also from research mathematicians, to enrich the discussion on geometry teaching and learning even more. We leave this suggestion for the next CERME. The diversity and inclusiveness of our group embodies the spirit of communication and collaboration of CERME and contributes to our understanding of international perspectives on geometry education, to advance the teaching and learning of geometry.

References

[1] H. Freudenthal, Major problems of mathematics education. *Educational Studies in Mathematics*, **12**, 133–150 (1981)

[2] P. Herbst, T. Fujita, S. Halverscheid and M. Weiss, *The learning and teaching of geometry in secondary schools: A modeling perspective*. Routledge (2017)

[3] K. Jones, M. Maschietto, J. Mithalal-Le Doze, Introduction to the papers of TWG04: Geometry education. In *Proceedings of CERME 10*, edited by T. Dooley and G. Gueudet, DCU Institute of Education and ERME, Dublin, https://hal.archives-ouvertes.fr/hal-01925528/document (2017)

[4] K. Jones, M. Maschietto, J. Mithalal-Le Doze, C. Papadaki, Introduction to the papers of TWG04: Geometry teaching and learning. In *Proceedings of CERME 11*, edited by U. T. Jankvist, M. Van den Heuvel-Panhuizen and M. Veldhuis, Freudenthal Group & Freudenthal Institute, Utrecht University and ERME, Utrecht, https://hal.archives-ouvertes.fr/hal-02402091/document (2019)

[5] A. Kusniak, P. R. Richard, P. Michael-Chrysanthou, From geometrical thinking to geometrical working competencies. In *Developing Research in Mathematics Education*, edited by T. Dreyfus, M. Artigue, D. Potari, S. Prediger and K. Ruthven, Routledge, 8–22 (2018)

[6] P. M. Van Hiele, *Structure and insight: A theory of mathematics education*. Academic Press, Orlando (1986)

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