

Gene Calabi at 100 – Memorable encounters with Eugenio Calabi

Jean-Pierre Bourguignon and Balázs Szendrői

It is rare indeed to be able to celebrate the centenary of a living legend. The last time this happened in mathematics may have been the 100th birthday of Leopold Vietoris in 1991; the spring of 2023 brings up the centenary of Eugenio Calabi. Born in Milan on 11 May 1923, he has lived in the United States since an early age. He completed a PhD at Princeton University in 1950. Following temporary positions, he was appointed to a professorship at the University of Minnesota, and finally settled at the University of Pennsylvania in 1964, where he held for several decades the Thomas A. Scott Professorship of Mathematics. He is Commander of the Order of Merit of the Italian Republic. We wish to celebrate the occasion by collecting personal reminiscences of encounters with Eugenio Calabi.

Jerry Kazdan

University of Pennsylvania

I first met Gene in Fall 1962, when I was a grad student at the Courant Institute of Mathematics at New York University. A group of us, mainly students of Lipman Bers, often met on Fridays for lunch with Bers: “Children’s Lunch.” Gene was visiting New York and had dropped in to visit Bers. He was directed to the restaurant. He overflowed with mathematics, a pleasure to see.

We next met at the January 1966 Annual Meeting of the AMS. He immediately began telling me about Kähler manifolds and his conjecture concerning what are now called Calabi–Yau manifolds. This was a bit technical. I confess that I did not follow everything and did not appreciate the depth of the ideas he revealed.

That Fall I moved to the University of Pennsylvania. Gene had moved there in 1964 from the University of Minnesota. I had the pleasure of frequent personal interaction. His amazing geometric insight was a gift. He had a deep intuitive sense of what was important and interesting. He always shared his ideas generously. Often he came to my office and began explaining some of his recent thoughts at the blackboard (this sometimes bewildered undergraduates who might have happened to be there for my office hours).

It was refreshing to see Gene’s original views on many things. When driving with him, those in the car were perplexed at the

sometimes circuitous routes he chose. When asked, Gene said that he was minimizing the number of traffic lights. Someone suggested calling these “Calabi geodesics.” The name fit.

Blaine Lawson

Stony Brook University

Eugenio Calabi has certainly been one of the most original geometers of the twentieth century. As a graduate student in the late 1960s, I was fortunate to witness him delivering an early address on one of his deep and beautiful results concerning minimal 2-spheres in the Euclidean n -sphere. That day is one I will never forget!

At that time, I thought I had a sufficient result for my thesis. I had mentioned it to several people on the faculty and had written it up. It said that a non-compact holomorphic curve of constant curvature in complex projective n -space $\mathbb{P}^n(\mathbb{C})$ (with its standard metric) had to have curvature $\frac{1}{k}$ for an integer $1 \leq k \leq n$, and would be locally congruent to a specific rational normal curve in the linear subspace $\mathbb{P}^k(\mathbb{C}) \subset \mathbb{P}^n(\mathbb{C})$. This generalized to spaces of all dimensions. Early on that day, my advisor, Bob Osserman, mentioned this to Calabi, who told him that, unfortunately, that result was part of his thesis. You can imagine my chagrin. Nevertheless, I went to his colloquium that afternoon, and for me it was one of the most beautiful and exciting lectures I had ever heard. I was so inspired, I threw away my old work and wrote my thesis on minimal surfaces in the sphere. Since that time, I have had many conversations with Gene. They have always been fascinating and actually wondrous (that is, the part from him to me). Over time, I realized how generous he has always been to young mathematicians.

Nigel Hitchin

University of Oxford

Thirty years ago, I spoke at a conference in Pisa for Eugenio Calabi’s 70th birthday. It is remarkable that today we can celebrate his achievements over a much greater span of time.

My first encounter with him was when I was a postdoc at the Institute for Advanced Study in 1972. One of the first invitations to give a seminar in nearby universities was at Penn. I spoke about

positive scalar curvature obstructions, and he was generous in his comments and advice. I was very much aware of bigger issues in differential geometry at the time as Shing-Tung Yau was also at the Institute and we frequently discussed whether the Calabi conjecture was true or false.

The most direct impact that Calabi had on my work was his paper [4], which gave constructions of complete Ricci-flat Kähler manifolds. I had learned of some 2-dimensional examples from Gary Gibbons and Stephen Hawking in Cambridge and also through the work of Penrose in Oxford, but this paper opened up a new world independent of any relativity connections. It is difficult to appreciate that only a few years earlier there were absolutely no examples or existence proofs for complete non-flat manifolds with zero Ricci tensor. The paper also introduced the word “hyperkähler” to describe the differential geometry which previously went under various names associated to the quaternions. It was an inspired choice emphasizing that one should view these manifolds as possessing many Kähler structures and not be led astray by trying to define quaternionic coordinates. It was a language which pointed the way to further discoveries and in particular our hyperkähler quotient construction.

It would be a long task to list the papers of Calabi which foresaw future developments, even those which have influenced my own work. I wish him well on this his 100th birthday.

Jean-Pierre Bourguignon

Institut des Hautes Études Scientifiques

The first time I met Eugenio Calabi was in 1972 in Oberwolfach during one of the biennial meetings *Differentialgeometrie im Großen* organised for many years by Martin Barner, Wilhelm Klingenberg and Chern Shiing Shen. Eugenio attended these sessions regularly; he had been there already in 1962 and 1969. That year, he gave Bures-sur-Yvette as address, so I may have met him already at IHÉS where I had started to attend the Thom seminar.

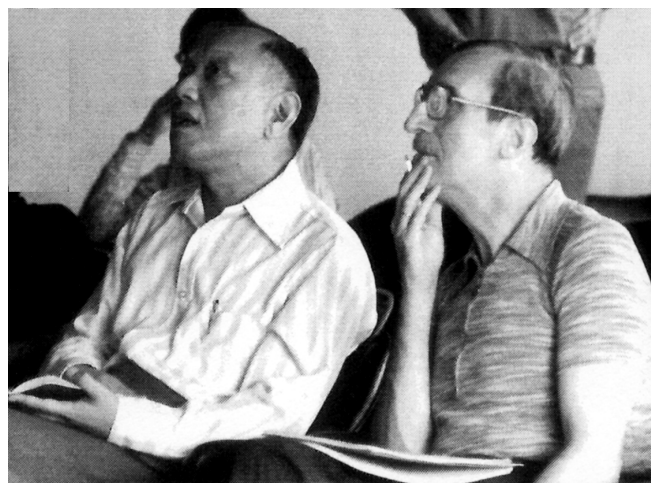
It is precisely in 1969 that, in a groundbreaking article [3], Eugenio discusses for the first time global properties of groups of diffeomorphisms preserving a symplectic structure. At that time, global symplectic geometry was still in its infancy. He introduces a very ingenious tool, often called the *Calabi morphism*. Very recently, the beautiful result establishing the non-simplicity of the group of homeomorphisms of the 2-sphere preserving the volume uses an extension of the Calabi morphism to the continuous context, showing the remarkable actuality and power of ideas Eugenio introduced more than 50 years ago.

Eugenio also visited Paris many times, lecturing at the Differential Geometry seminar led by Marcel Berger. And, early on in my life as a mathematician, Berger informed me of the importance of the Calabi conjecture, in particular in relation with the existence of metrics with vanishing Ricci curvature, an intriguing case connected to special holonomy groups. This is why, in 1972–1973, Yau Shing

Tung and I, both of us assistant professors in Stony Brook, tried hard to find obstructions to the existence of such metrics on K3 surfaces. There was a good reason for us to fail since, as is now well known, Yau proved the Calabi conjecture in 1976, opening the way to the epoch-making ‘Calabi–Yau’ metrics. This led me to a number of exchanges with Eugenio as the various follow-ups of the solution of the Calabi conjecture occupied me for quite some time. Through these contacts, I could appreciate his kindness as well as his exceptional capacity to use all kinds of tools to address geometric questions, often creating very innovative approaches.

Getting involved in the publication of Eugenio’s Complete Works [8] was therefore for me a natural gesture to pay back a little for all what I owe to him.

Dear Eugenio, happy 100th birthday and my best thanks for the inspiration and the support!



Eugenio Calabi with Shiing-Shen Chern in Oberwolfach in 1976
(© Dirk Ferus)

Simon Salamon

King’s College London

If the exact dates of most of my encounters with Eugenio Calabi are vague, I can pinpoint with precision one day that I spent in his company. This was Monday 30 March 1981, when he had invited me to give a seminar at the University of Pennsylvania. I must have left Union Station in DC early in the morning, and President Ronald Reagan was to joke some hours later (quoting W. C. Fields) about rather being in Philadelphia. My talk was around lunchtime, and I recall being shocked by the setup: two blackboards on opposite walls with a long table at right angles stretching between them. The audience sat on both sides of the table, possibly eating sandwich lunches, whilst any speaker (without advanced practice) was forced to move to and fro between the boards. My talk went well

enough, though I am not sure I succeeded in rousing Calabi, who had assumed his characteristic pose with eyes shut in the middle of the table (though Jim Eells had warned me not to underestimate his perceptive powers in the lecture room.) About the time I was speaking, Reagan was addressing union representatives (no doubt with eyes wide open) in the Washington Hilton. What happened next is well documented, also in movie form. As his team emerged from a side entrance of the hotel, John Hinckley Jr fired six .22 calibre pistol shots. The first was to disable press secretary James Brady for life, and others were injured. The last bullet ricocheted and (as was soon to become apparent) punctured the President's lung. News of the shooting probably reached us around 3pm and immediately put a stop to any mathematics. I recall a frantic search for a transistor radio, which was installed in Calabi's office and became a focal point for colleagues, many of whom will have recalled events in Dallas almost two decades before. But within a couple of hours, it was clear that Reagan was out of danger, though subsequent details revealed how lucky he had been. The rest of the day is a blur, but I do remember Calabi walking me to the station, probably anxious to resume serious scientific discussion. We waited together on the concourse, where I picked up a late edition of the Philadelphia Daily News with a stark 300 point headline "REAGAN SHOT; IS ALERT, STABLE" – it remains in my office as a reminder of Calabi, not Reagan. Despite the drama of that ultimately tragic day, my overriding recollection is of my host's kindness, which was to provide a model of how to treat a visiting speaker.

I am sure that the mathematics I learnt from Eugenio in Philadelphia and on other precious occasions (whether it related to special holonomy, minimal surfaces, twistor spaces, non-Kähler geometry, or Cayley numbers) had a significant influence on my subsequent work. I was especially proud to be present with Calabi at the first joint international meeting of the AMS and UMI in Pisa in 2002, at which Vestislav Apostolov and I conceived of our paper that helped develop closer links between Calabi–Yau spaces and metrics with holonomy G_2 .

Claude LeBrun

Stony Brook University

I am delighted to have been recently assured by close mutual friends that Gene Calabi remains intellectually active, in good health, and in excellent spirits, even as he embarks on the second century of his remarkable life. Calabi's work has had an overwhelming impact on my own career, but a large contingent of the geometers of my generation would no doubt say much the same thing. However, I have also had the good fortune to have gotten to know Gene personally, to have had many delightful and informative conversations with him, and to have been able to become a member of his large circle of friends.

Although I first met Gene in Italy in 1980, when I was straight out of graduate school, it was not until half-a-dozen years later

that I had a truly life-altering encounter with him. This happened during a brief visit to the University of California at San Diego, where I'd gone to run a few questions past Rick Schoen. Suddenly, Gene materialized in Rick's office, and immediately launched into an extensive lecture on extremal Kähler metrics. I soon realized that I would need to carefully read Gene's groundbreaking papers on the subject, because my own work on anti-self-dual 4-manifolds had recently led me into the realm of scalar-flat Kähler metrics, which represented one tiny piece of Gene's grand vision. Over the following decades, I had the pleasure of discussing related areas of differential geometry with Gene many times, sometimes at University of Pennsylvania, sometimes at Stony Brook, and sometimes at conferences held at other universities. His beneficent influence on my life has continued to exert itself in many ways – directly through ideas, of course, but also indirectly, through mathematicians whose research directions have been molded by Gene's mathematical taste.

I last saw Gene in person in 2019, a few months before the pandemic. The director of the Simons Center for Geometry and Physics proposed that a filmed interview with Calabi would be of great interest to both mathematicians and theoretical physicists, and I had the good fortune to then be asked to go conduct the interview in Philadelphia. The resulting documentary [7], entitled *Quintessentially Science Fiction*, aims to capture Gene's charm and intelligence. The title, incidentally, comes from a comment that Gene made during the interview regarding the nature of mathematics: we invent imaginary worlds, and scientists only decide long afterwards whether any of these places might actually make good homes for genuine scientific theories. That's vintage Gene. Simple, terse, and perhaps a little cryptic; but definitely worth pondering at length!

Gene, I've said it before, but, on this very happy occasion, let me say it again: Your visionary ideas have made our world a richer and more interesting place. Thank you for your ideas, thank you for your kindness, and, above all, thank you for your friendship!

Fabrizio Catanese

University of Bayreuth

I had a few friendly encounters with Eugenio Calabi, first when he was visiting the Scuola Normale Superiore of Pisa as a guest of the late Edoardo Vesentini, and, later on, as I had the honour to be invited as a speaker at the conferences held in Italy in honour of some of his birthdays. On these occasions, I had the opportunity of talking to him, and to get to know about his real and mathematical life through Italy and then in the USA.

He looked to me like an old-fashioned gentleman, a species of mathematician in danger of extinction. Yet, his eyes and quiet speech were sparkling of a deep intellectual life. I did not dare to ask him many mathematical questions, even if at a certain point I had been quite involved with some of his constructions. For me,

Calabi was like a grandfather, since I regarded him as a teacher of Edoardo Vesentini, one of my teachers who introduced me to differential geometry back in 1970–71.

Making difficult things simple and finding elegant solutions has been the great talent of Eugenio Calabi. In the words of Vesentini: “Amidst intimidating theories and theorems which were tormenting me, came the simple explanations by Calabi: everything seemed just straightforward linear algebra, and easy calculations with matrices were yielding the desired curvature results.” I later read their paper [10] myself, I loved it, and I fully agree with Vesentini’s statement: explicit calculations are easily understood, and concrete mathematics will live longer than awe-generating abstract theories.

Wolfgang Ziller

University of Pennsylvania

Some of my fondest and most important experiences at Penn were my mathematical interactions with Gene Calabi. He would often come by at my office when arriving by train and would explain to me what he was thinking about in the shower that morning. It was always fascinating and I was able to ask him questions about what I was working on. His insight into what the core of a problem was, and his ability of coming up with relevant examples always amazed me.

Let me tell one of my favourite stories. In 1982, I was teaching a course on closed geodesics and asked Gene over tea about a conjecture of Poincaré, which states that for a metric on S^2 with positive curvature any shortest closed geodesic is simple. He thought about it for 5–10 minutes and told me the problem was very subtle. On a bi-equilateral triangle (which can be blown up to a convex surface) there are two closed geodesics of the same length, one simple and one with a self intersection. Thus the answer is no in the non-smooth limit. He was intrigued by the question. A year later he told me how to prove the positive answer in the smooth case by modifying the Birkhoff curve shortening process to 1-cycles, not just simple curves, a technique that foreshadowed developments that are much more recent. He only published the result 10 years later [9] together with a graduate student at the time, Jianguo Cao.

A second example is his discovery, more or less at the same time, of the grim reaper for the curve shortening flow of closed curves on the plane, again over tea at Penn (it is not well known that he discovered, and named, this example). It is the unique simple ancient solution of the flow, foreshadowing that this is a crucial property for the flow, only understood much later. His influence on me and many others was through mathematical conversations, with observations that often became crucial in later developments. His love for mathematics (but not for writing papers) was obvious to everyone. Happy Birthday Gene!



Eugenio Calabi with Xiuxiong Chen at IHÉS, 2007 (© Jean-François Dars)

Xiuxiong Chen

Stony Brook University

Shortly after I arrived at the University of Pennsylvania to pursue my PhD studies, I ran into Prof. Eugenio Calabi. Following a brief introduction, he started to explain something that he believed or hoped I would find interesting. Little did I know that this would become our routine for the ensuing years.

Prof. Calabi would pen his explanations or thoughts on whatever was available in hand or at hand, be it an envelope or a napkin, or a blackboard in a nearby classroom. We would talk hours and end often in his office, but also in the mail room and in the hallways. I would take home those envelopes and napkins (regrettably many of them got lost during our many moves), but most of the time I would jot down on my notepad what he wrote or said, or occasionally my own musings.

Prof. Calabi would ask me to repeat what he said or what I heard the next time we met without consulting any notes. As he explained, “it wouldn’t become yours until it’s imprinted in your memory”. Though not an immediate embracer, it didn’t take too long for me to appreciate that advice. Now I am a fervent adherent of the doctrine, and I have been passing it on to my own students.

Ludmil Katzarkov

University of Miami

I met Gene Calabi for the first time in September 1990. His class immediately impressed me: he introduced me to special metrics

and multiplier ideal sheaves; later I wrote papers on these subjects. Gene also introduced me to Shing-Tung Yau.

I have had many discussions with Gene on European history – his knowledge of the subject was spectacular.

Gene was also my ride to Princeton. He regularly drove me from University of Pennsylvania to IAS to attend a course on harmonic maps to buildings by Richard Schoen. Later, I used the knowledge acquired in the proof of the Shafarevich conjecture. With the exception of his adventurous Italian driving style (the Honda Civic felt more like a Ferrari), these were memorable drives – really unforgettable scientific, intellectual and cultural experiences.

Happy 100th birthday Gene! Thank you very much for teaching me so many things, in particular that mathematics can be a subject for gentlemen.

Antonella Grassi

University of Pennsylvania and Università di Bologna

When I started on a tenure-track position at the University of Pennsylvania, Eugenio Calabi had just retired. More precisely, he had to retire, having reached the age of 70, the mandatory retirement age at the time. Colleagues were saying that he was the last person to whom the mandate applied, and commented on the irony of it, as Gene did not show any sign of slowing down. He never said a word about this, and he continued his activities as usual. This was typical of Eugenio: quiet, understated, reserved, dignified, and at the same time determined.

I was quite intimidated when he asked me to tell him about the interest of the physics community in certain mathematical objects, whose properties he described in impeccable formal Italian; with a shy smirk he eventually used the words *Calabi–Yau*. He told me how Yau and he, and others, met on a Christmas day at New York University, to discuss Yau’s proof of his conjecture (Giuliana, Eugenio’s wife, later commented that Eugenio’s profession is to create problems for others to solve). Years later, when Yau came to Penn to deliver the Rademacher Lecture in 1999, I spent time with both of them together.

Over the years I grew very fond of Eugenio. He likes mathematics, but above all, from our conversations his passion emerged for justice, art, music and dedication to his family. He told me very proudly of the social accomplishments of his sister the journalist, who had moved back to Italy. He was careful never to mention her name, as in a riddle, and he was delighted when I eventually figured out which important public figure she was (Tullia went by her husband’s last name). He would tell me about the current math question he had come up with, then he would give me practical advice for an Italian in the United States. He also shared his very useful method to walk safely on a narrow busy road in the Italian Alps. The last time we spoke in person, after he gave a seminar on the occasion of his 95th birthday, shortly before I moved to the Università di Bologna, he shared humorous, but as always



Eugenio Calabi with Shing-Tung Yau at École Polytechnique in 2007
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humble, reflections on his career. He then went on to talk about his “nipotini,” with an affection that warms the heart.

One winter, years before, I had returned to the Department with short hair. The difference must have been so drastic that several colleagues did not recognize me. Eugenio, after the ever polite and warm greetings, commented that Giuliana had also cut her hair short after the birth of their first child. Owio!

Claudio Arezzo

The Abdus Salam International Centre for Theoretical Physics

Calabi’s work has had a huge impact on my education and research for the depth and beauty of his results, and for the elegance and simplicity in which he wrote, and spoke about, them. It is well known that his work on the existence of Kähler metrics with prescribed Ricci curvature has changed algebraic and differential geometry, as well as mathematical physics, forever. Three other themes of his work that I find as important are holomorphic isometric immersions of Kähler manifolds [1], minimal surfaces in spheres [2] and extremal Kähler metrics [5, 6], representing Calabi’s proposal for a “best metric” on the largest possible space of Kähler manifolds. While these papers in particular have attracted a large amount of attention, the most fundamental existence question remains unanswered.

I want to stress especially the beauty of the presentation Calabi uses in his papers. It is a pure joy to read his works; I still remember with great nostalgia the many nights in the library studying the paper [4], which became famous as “Calabi’s Ansatz” ... He manages to teach the reader not just about the specific topic, but also how to choose a good problem and how a good idea is born, without tricks or intimidation. I cannot think of a better example of what Plato meant when stating in the Republic that “... the object of education is to teach us to love beauty”.

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