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A New Class of Knots with Property P

By

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§ 1. Introduction

It is known that several classes of knots have property P ([1], [3], [5], [9]).

In this paper, it will be shown that the special class of knots has property P.

To show the above, 3-dimensional homology spheres constructed by Dehn's method will be considered and it will be shown that they are not 3-dimensional homotopy spheres.

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A *singular disk* in the 3-sphere S^3 means a map f of an oriented disk $\bar{D}$ into S^3. For brevity, one may refer to the image $D=f(\bar{D})$ as the singular disk.

Among the singularities that a singular disk may have perhaps the simplest is a *clasping singularity* or just *clasp*. This consists of two mutually disjoint slits $\bar{S}$ and $\bar{\bar{S}}$ that are mapped by f topologically onto an arc S of D. The singular disks to be considered are those that have only simple clasps. Let us call such a disk an *elementary disk*. This is a natural class to consider, since it is known that any singular disk in general position can be deformed, without moving the boundary, into an elementary disk [10].

If D is an elementary disk then a regular neighborhood W of D in S^3 is a handlebody, and its boundary ∂W is an orientable surface of some

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genus g, where g is the same as the number of clasps. Let us call D *totally knotted if* ∂W is incompressible* in $S^3-\partial D$.

Let V be a tame solid torus in S^3, and α a simple closed curve contained in ∂V and not contractible on ∂V. Let φ be a homeomorphism of a torus $S^1\times\partial D^2$ onto the torus ∂V which maps $1\times\partial D^2$, $1\in S^1$, onto α. By this homeomorphism φ we will get a new 3-dimensional manifold $M\cong(S^3-\text{int}\,V)\underset{\varphi}{\cup}S^1\times D^2$, identifying $x(\in S^1\times\partial D^2)$ with $\varphi(x)$.

Let us give a canonical orientation to S^3 and M in such a way that M and S^3 induce the same orientation in $S^3-\text{int}\,V$.

Let m and l be a meridian and a longitude of ∂V respectively. We also denote by m and l the elements of $\pi_1(\partial V)$ or $\pi_1(S^3-\text{int}\,V)$ represented by these curves. Let α be a curve on ∂V which, when properly oriented, represents the element $m^\tau l^\nu$ (τ, γ: integers). Since the manifold M will be the homology sphere, τ must be $+1$ or -1. We may assume, changing the orientation of α if necessary, that α represents the element ml^ν ($\gamma\neq0$).

In this paper, let us choose only α that is not a meridian, i.e. α does not represent the element m of $\pi_1(\partial V)$.

Then the following main result will be proved.

Theorem I. *The knot type k which is equivalent to the boundary of a totally knotted disk with two clasping singularities has property P.*

Proving this theorem, it is equivalent to prove the following one.

Let M be a homology sphere constructed by the method in the above.

Theorem II. *A homology sphere M is not simply connected if the knot type k of the core of V is equivalent to the boundary of a totally knotted disk with two clasping singularities.*

§ 2. Lemma

A *normalized Alexander polynomial* means the Alexander polynomial [2] with the smallest positive but no negative powers of each

* ∂W is incompressible in $S^3-\partial D$ means the induced map of natural inclusion of $\pi_1(\partial W)$ into $\pi_1(S^3-\partial D)$ is a monomorphism.

generator.

By multiplying or diving by some powers of generators, any Alexander polynomial can be changed to a normalized Alexander polynomial.

To prove theorem II, we need the following lemmas.

Lemma 1. *Let G be a finitely presented group which is isomorphic to the non-trivial free product $G_1 * G_2$, and the abelianized groups of G_1 and G_2 be both infinite cyclic groups. If the ι-th normalized Alexander polynomial of G is not zero, then it is a product of two one variable polynomials.*

Proof. Let an $m_1 \times n_1$-matrix A_1 and an $m_2 \times n_2$-matrix A_2 be Alexander matrices of G_1 and G_2, respectively. Then an Alexander matrix A of the free product G of G_1 and G_2 is the $(m_1+m_2)\times(n_1+n_2)$-matrix $\begin{pmatrix} A_1 & 0 \\ 0 & A_2 \end{pmatrix}$. Let g_1 and g_2 be generators of the Abelianized group Z of G_1 and G_2, respectively. For any integer t, the t-th polynomials are defined for each G_1 and G_2. They are one variable polynomials including the case that the polynomial, is "1" or "0" after normalization, i. e. the t-th polynomial of G_1 is $\Delta^{(t)}(g_1)=\sum_{i\geqq 0} a_i g_1^i$, and the t-th polynomial of G_2 is $\Delta^{(t)}(g_2)=\sum_{j\geqq 0} b_j g_2^j$.

$$A \sim \begin{array}{cc} \begin{array}{cc} n_1 & n_2 \end{array} & \\ \left(\begin{array}{c|c} A_1 & 0 \\ \hline \underbrace{0}_{\mu} & \underbrace{A_2}_{\iota-\mu} \end{array}\right) & \begin{array}{c} m_1 \\ m_2 \end{array} \end{array}$$

Let us consider ι-th polynomial of A. By the definition, this is the g.c.d. of determinants of all $(n_1+n_2-\iota)\times(n_1+n_2-\iota)$ submatrices of the matrix A. These submatrices are of the form $\left(\begin{array}{c|c} B_1 & 0 \\ \hline 0 & B_2 \end{array}\right)$, wehre B_1 is an $(n_1-\mu)\times(n_1-\mu)$ submatrix of A_1, and B_2 is an $(n_2-\iota+\mu)\times(n_2-\iota+\mu)$ submatrix of A_2. The determinant of this form is the product of det B_1 and det B_2. Fixing the number ι and μ, consider all the $(n_1-\mu)\times(n_1-\mu)$ submatrices of A_1, and the $(n_2-\iota+\mu)\times(n_2-\iota+\mu)$ submatrices of A_2. Let $p(g_1)$ be the g.c.d. of determinants of the above $(n_1-\mu)\times(n_1-\mu)$ submatrices of A_1. By the definition, $p(g_1)$ is the μ-th polynomial of A_1, and $q(g_2)$, similarly defined, is the $(\iota-\mu)$-th polynomial of A_2. The product $p(g_1)\cdot q(g_2)$ is the g.c.d. of all the determinants of the form $(\det B_1)\cdot(\det B_2)$, since no determinant of any submatrix of A_1 can have any factor in common with the determinant of any submatrix of A_2. Since μ ranges over $0, 1, \dots, \iota$, the ι-th polynomial of A is the g.c.d. of $\Delta^{(0)}(g_1)\times\Delta^{(\iota)}(g_2)$, $\Delta^{(1)}(g_1)\cdot\Delta^{(\iota-1)}(g_2)$, $\dots$, $\Delta^{(\mu)}(g_1)\cdot\Delta^{(\iota-\mu)}(g_2)$, $\dots$, $\Delta^{(\iota)}(g_1)\times\Delta^{(0)}(g_2)$. In general $\Delta^{(t)}$ is divisible by $\Delta^{(t+1)}$. So $\Delta^{(0)}, \dots, \Delta^{(\iota-1)}$ are divisible by $\Delta^{(\iota)}$. Since g_1 and g_2 are different variables, $\Delta^{(\mu)}(g_1)$ and $\Delta^{(\mu')}(g_2)$ can have no common factor other than a constant, and since G_1 and G_2 are abelianized to infinite cyclic groups, neither $\Delta^{(\mu)}(g_1)$ nor $\Delta^{(\mu')}(g_2)$ can have any constant factor other than 1. Thus the g.c.d. of the above products, i.e. the ι-th polynomial of A, is just the product $\Delta^{(\iota-\iota'')}(g_1)\cdot\Delta^{(\iota-\iota')}(g_2)$, where ι' and ι'' are the smallest number among that the $(\iota\text{-}\iota'')$-th and $(\iota\text{-}\iota')$-th Alexander polynomials of G_1 and G_2 are non-zero.

Corollary to Lemma 1. *Under the same conditions in Lemma* 1, *the ι-th normalized Alexander polynomial of G has non-zero constant term.*

Proof. It is almost trivial.

Lemma 2. *Let X be an orientable connected* 3*-manifold with boundary and the boundary ∂X is a closed surface with genus* 2. *If the homomorphism of $\pi_1(\partial X)$ into $\pi_1(X)$ induced by the natural inclusion map is not a monomorphism, then $\pi_1(X)$ is the free product of two non-trivial groups.*

Proof. Since the mapping of $\pi_1(\partial X)$ into $\pi_1(X)$ is not a monomorphism, there exists a disk D is X whose boundary J is a nontrivial closed curve on ∂X. By the Loop Theorem [11] and Dehn's Lemma [7], we can assume that D is nonsingular and J is a simple closed curve.

Let us assume first that J is homologous to zero on ∂X, i.e. that J is separates ∂X into two surfaces T_1 and T_2 with common boundary J; T_1 and T_2 are both surfaces of genus 1.

Let D_1 and D_2 be copies of D in X that have J as common boundary. Along J sew D_1 to T_1, and D_2 to T_2. Then we get two tori. Let M_i be the manifold in X bounded by $D_i \cup T_i$, thus M_i is a 3-dimensional manifold whose boundary is a torus in $X (i=1, 2)$. Since the intersection of M_1 and M_2 consists of a nonsingular disk D, by the Van Kampen Theorem, $\pi_1(X)$ is isomorphic to the nontrivial free product $\pi_1(M_1)*\pi_1(M_2)$.

To complete the proof, it will be shown that J must be homologous to zero on ∂X. Suppose, to the contrary, that J, suitably oriented, represents an element α of $\pi_1(\partial X)$ that does not lie in the commutator subgroup $[\pi_1(\partial X), \pi_1(\partial X)]$. Then there exists a simple closed curve J' on ∂X that intersects J at just one point. Let β denote the element of $\pi_1(\partial X)$ represented by J' (suitably oriented).

Since $\alpha\neq 1$ and $\beta\neq 1$ it follows from a result of Greendlinger [4] that either $\alpha\beta\bar{\alpha}\bar{\beta}\neq 1$ or $\alpha=\gamma^m$ and $\beta=\gamma^n$ for some element γ of $\pi_1(\partial X)$ and integers m and n.

If, in fact, $\alpha=\gamma^m$, $\beta=\gamma^n$ then, since the intersection number $S(\alpha, \beta)= S(J, J')$ is equal to ± 1, and $S(\alpha, \beta)=S(\gamma^m, \gamma^n)=mnS(\gamma, \gamma)$, it must be that $m=\pm 1$ and $n=\pm 1$. But this means that $\alpha=\beta^{\pm 1}$, and hence that J' can be deformed into J in the complement of J. This contradiction shows that $\alpha\beta\bar{\alpha}\bar{\beta}\neq 1$.

Consequently if N is a regular neighborhood of $J\cup J'$ on ∂X, then its boundary ∂N is not contractible on ∂X. Hence ∂N separates ∂X into two surfaces N and $\partial X-N$, each of genus 1. As shown in the first part of this proof it follows that $\pi_1(X)$ must be the free product of two nontrivisl groups.

Lemma 3. *Let X be an orientable connected 3-manifold with boundary, $i: \partial X\rightarrow X$ an inclusion map and i_*: $H_1(\partial X)\rightarrow H_1(X)$ an induced map. Then the rank of the kernel of i_* is exactly half of the rank of $H_1(\partial X)$.*

Proof. Let us consider the following exact sequences:

$$\begin{aligned} 0 \longrightarrow H_3(X, \partial X) &\longrightarrow H_2(\partial X) \longrightarrow H_2(X) \\ \longrightarrow H_2(X, \partial X) &\longrightarrow H_1(\partial X) \xrightarrow{i_*} H_1(X) \\ \longrightarrow H_1(X, \partial X) &\longrightarrow H_0(\partial X) \longrightarrow H_0(X) \longrightarrow 0. \end{aligned}$$

Let $2g$ be the rank of $H_1(\partial X)$, a of $H_0(\partial X)$, b of $H_1(X, \partial X)$ and c of $H_1(X)$.

By Poincaré Duality $H_2(\partial X)$ has rank a, $H_2(X)$ has rank b and $H_2(X, \partial X)$ has rank c. Also $H_3(X, \partial X)$ has the same rank 1 as $H_0(X)$. Among the numbers a, b, c, 1 and g, by the exactness, there is an equation

$2g=(c-b+a-1)+(c-b+a-1)$. Moreover from the exactness, the rank of ker i_* is $c-b+a-1$; this is just half of the rank of $H_1(\partial X)$, i.e. $g=c-b+a-1$.

§ 3. Proof of the Theorem

Proof of the TheoremII. Let k be a knot which is equivalent to the boundary of a totally knotted disk E with two clasping singularities, V a regular neighborhood of k and W a regular neighborhood of E in S^3. The homology sphere $(S^3\text{-int } V) \underset{\varphi}{\cup} S^1 \times D^2 \cong (S^3\text{-int } W) \cup ((W\text{-int } V) \underset{\varphi}{\cup} S^1 \times D^2)$ will be denoted by M, where φ is a homeomorphism of $S^1 \times \partial D^2$ onto ∂V which maps $1 \times \partial D^2$, $1 \in S^1$, onto α, where α is a simple closed curve on ∂V and not a meridian curve of ∂V.

Since ∂W is incompressible in S^3-k and $S^3-\text{int} W \subset S^3-k$, the map from $\pi_1(\partial W)$ into $G_1=\pi_1(S^3-\text{int} W)$ is a monomorphism. To show that $\pi_1(M)$ is not trivial, by the Van Kampen Theorem, it is enough to show that a map from $\pi_1(\partial W) \cong \pi_1(\partial((W-\text{int} V) \underset{\varphi}{\cup} S^1 \times D^2))$ into $G_2=\pi_1((W-\text{int} V) \underset{\varphi}{\cup} S^1 \times D^2)$ is a monomorphism. If both maps from $\pi_1(\partial W)$ into G_1 and G_2 are monomorphisms, then $\pi_1(M)$ is isomorphic to the free product with amalgamation $G_1 \underset{\pi_1(\partial W)}{*} G_2$.

Let us consider $G_2=\pi_1((W-\text{int} V) \underset{\varphi}{\cup} S^1 \times D^2)$. Let W be a handlebody with two handles, which may be knotted or linked in S^3. To calculate G_2, it is enough to consider a handlebody W' in standard position in S^3; thus there is an autohomeomorphism of S^3 which maps W onto W', whose two handles are neither knotted nor linked with each other. By this mapping, V is mapped onto $V' \subset W'$ and α is mapped onto $\alpha' \subset \partial V'$. The simple closed curve α' is not a meridian of $\partial V'$.

Let us construct another homeomorphism of W' onto a handlebody W'' as follows: take two meridian cells in W', m_1 and m_2. Cut W' along these meridian cells and turn the exposed faces a suitable number of times, to untwist V', and sew back together again. Then we get a new solid torus V'' in W''. By this homeomorphism, α' is mapped onto α'' on $\partial V''$. The curve α'' is not a meridian curve of $\partial V''$. With the appropriate orientation α'' represents an element of $\pi_1(\partial V'')$ of the form ml^ν, where m

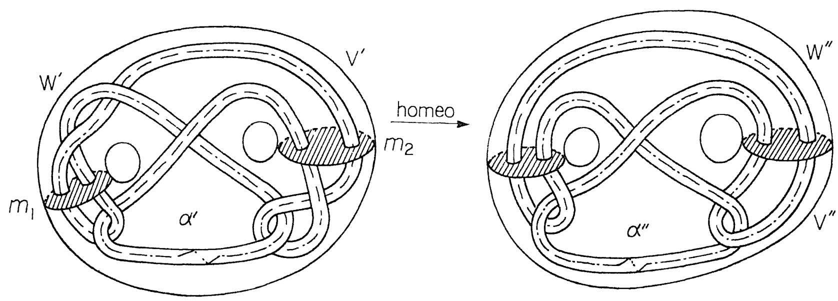


Fig. 1

is represented by a meridian, and l by a longitude of $\partial V''$. Since α'' is not a meridian, ν is a non-zero integer. Then the manifold $(W'-\mathrm{int}\,V') \cup_{\varphi'} S^1\times D^2$ is homeomorphic to the manifold $(W''-\mathrm{int}\,V'') \cup_{\varphi''} S^1\times D^2$, where φ'' is a homeomorphism of $S^1\times\partial D^2$ onto $\partial V''$ that maps $1\times\partial D^2$, $1\in S^1$, onto α''.

Thus the manifold $(W-\mathrm{int}\,V) \cup_{\varphi} S^1\times D^2$ is seen to be homeomorphic, by the composition of above maps, to the manifold $(W''-\mathrm{int}\,V'') \cup_{\varphi''} S^1\times D^2$.

Let us assume that the map from $\pi_1(\partial W)$ into $\pi_1((W-\mathrm{int}\,V) \cup_{\varphi} S^1\times D^2)$ is not a monomorphism. Since $(W-\mathrm{int}\,V) \cup_{\varphi} S^1\times D^2$ is homeomorphic to $(W''-\mathrm{int}\,V'') \cup_{\varphi''} S^1\times D^2$ it is equivalent to assume that the map of $\pi_1(\partial(W''-\mathrm{int}\,V'') \cup_{\varphi''} S^1\times D^2)) \cong \pi_1(\partial W'')$ into $\pi_1((W''-\mathrm{int}\,V'') \cup_{\varphi''} S^1\times D^2)$ is not a monomorphism.

By lemma 2, there exist manifolds M_1 and M_2 such that $M_1\cup M_2 = (W''-\mathrm{int}\,V'') \cup_{\varphi''} S^1\times D^2$, $M_1\cap M_2$=nonsingular disk, and both the boundaries ∂M_1 and ∂M_2 are tori; then $\pi_1(M_1\cup M_2)$ is isomorphic to the nontrivial free product $\pi_1(M_1)*\pi_1(M_2)$. By lemma 3, since M_i is a 3-manifold whose boundary is a torus, $H_1(M_1)$ and $H_1(M_2)$ are not tivial. Since $\pi_1(M_1\cup M_2)/[\pi_1(M_1\cup M_2), \pi_1(M_1\cup M_2)]$ is $Z\times Z$, both $H_1(M_1)$ and $H_1(M_2)$ are infinite cyclic groups. So by Cor. to lemma 1, the ι-th polynomial of the free product $\pi_1(M_1)*\pi_1(M_2)$ must have non-zero constant term.

Let us calculate the polynomial of the group $\pi_1((W''-\mathrm{int}\,V'') \cup_{\varphi''} S^1\times D^2)$. To complete the proof it is enough to consider four different cases; that are depending on the order of the inverse images of slits along knot and the intersection number $S(k, E)$ at the end of slit.

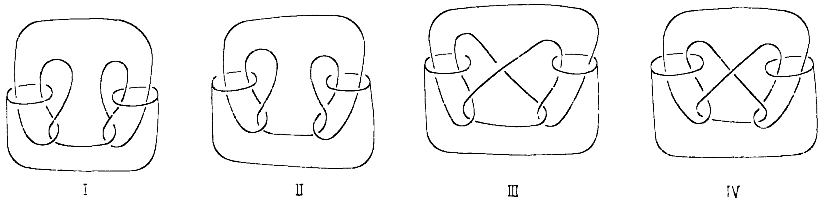


Fig. 2

Take the first case.

To consider the group $\pi_1((W''-\operatorname{int} V'')\underset{\varphi''}{\cup} S^1\times D^2)$ for this case is equivalent considering the fundamental group of the complement of the graph in Fig. 2 with more relations corresponding to a homeomorphism φ'' of $S^1\times\partial D^2$ onto $\partial V''$ that maps $1\times\partial D^2$, $1\in S^1$, onto α'' representing an element of $\pi_1(\partial V'')$ of the form $ml^{\nu}(\nu\neq 0)$, where m is represented by a meridian, and l by a longitude of $\partial V''$.

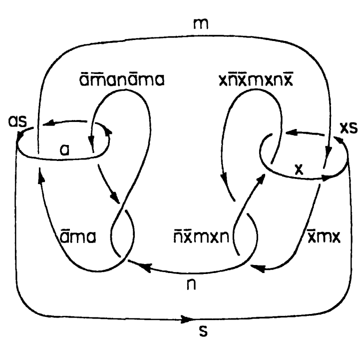


Fig. 3

Let a, x, m, n and s be the generators. From Fig. 3 we will get relations:

1) $\bar m\ a\ s\ m=\bar a\ \bar m\ a\ \bar n\ \bar a\ m\ a\ \bar m\ a\ n\ \bar a\ m\ a$

2) $\bar a\ \bar m\ a\ n\ \bar a\ m\ a=\bar m\ a\ n\ \bar a\ m\ a\ \bar n\ \bar a\ m$

3) $\bar m\ x\ s\ m=x\ \bar n\ \bar x\ \bar m\ x\ n\ x\ \bar n\ \bar x\ m\ x\ n\ \bar x$

4) $x\ \bar n\ \bar x\ m\ x\ n\ \bar x=\bar n\ \bar x\ m\ x\ n\ \bar x\ \bar m\ x\ n$

Let l be a longitude of $\partial V''$; then l is denoted by

$$l=\cancel{a}\cdot\cancel{\bar a}\ \bar m\ a\ \bar n\ \bar a\ m\ \cancel{a}\cdot\cancel{\bar a}\cdot\bar a\ \bar m\ a$$
$$\times\bar n\ \bar x\ \bar m\ x\ n\cdot x\cdot\bar n\ \bar x\ m^4.$$

Corresponding to the map φ'', we need one more relation: $ml^{\nu}=1$, i.e. $\pi_1((W''-\operatorname{int} V'')\underset{\varphi''}{\cup} S^1\times D^2)\cong\{a,\ x,\ m,\ n,\ s,\ l$:

1) $\bar m\ a\ s\ m=\bar a\ \bar m\ a\ \bar n\ \bar a\ m\ a\ \bar m\ a\ n\ \bar a\ m\ a$

2) $\bar a\ \bar m\ a\ n\ \bar a\ m\ a=\bar m\ a\ n\ \bar a\ m\ a\ \bar n\ \bar a\ m$

3) $\bar m\ x\ s\ m=x\ \bar n\ \bar x\ \bar m\ x\ n\ x\ \bar n\ \bar x\ m\ x\ n\ \bar x$

4) $x\ \bar n\ \bar x\ m\ x\ n\ \bar x=\bar n\ \bar x\ m\ x\ n\ \bar x\ \bar m\ x\ n$

5) $ml^{\nu}=1$

6) $l=\bar m\ a\ \bar n\ \bar a\ m\ \bar a\ \bar m\ a\ \bar n\ \bar x\ \bar m\ x\ n\ x\ \bar n\ \bar x\ m^4\}$.

Since abelianized group of $\pi_1((W''-\operatorname{int} V'')\underset{\varphi''}{\cup} S^1\times D^2)$ is $Z\times Z$, whose generators are represented by a and x, the each entry of the Alexander matrix is a polynomial with two variables at most. By the free calculus, the Alexander matrix A is equivalent to the following one:

$$A \sim \begin{pmatrix} 0 & 0 & -1+a & 1-a & a & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & x & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & \nu \\ 0 & 0 & \bar{a}+\bar{x}-4 & a+x & 0 & 1 \end{pmatrix}$$

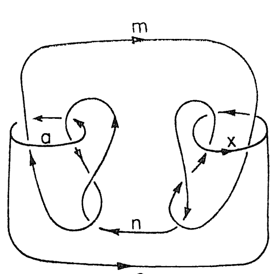
$$\sim \begin{pmatrix} 0 & 0 & 0 & a & 0 \\ 0 & 0 & 0 & x & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \nu \\ 0 & 0 & \bar{a}+\bar{x}-4+a+x & 0 & 1 \end{pmatrix}$$

$$\sim \begin{pmatrix} 0 & 0 & 1 & \nu \\ 0 & 0 & \bar{a}+\bar{x}-4+a+x & 1 \end{pmatrix}.$$

Then we get the second polynomial,

$$\Delta^{(2)}(a, x) = (4\nu+1)ax - \nu(x+a) - \nu(ax^2+a^2x).$$

For the other three cases, we get similarly:



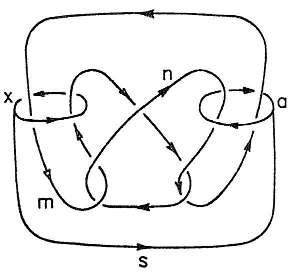
$$\pi_1((W''-\operatorname{int} V'') \underset{\varphi''}{\cup} S^1 \times D^2) =$$
$$\{a, x, m, n, s, l:$$
$$\bar{m}\ a\ s\ m = \bar{a}\ \bar{m}\ a\ \bar{n}\ \bar{a}\ m\ a\ \bar{m}\ a\ n\ \bar{a}\ m\ a$$
$$\bar{a}\ \bar{m}\ a\ n\ \bar{a}\ m\ a = \bar{m}\ a\ n\ \bar{a}\ m\ a\ \bar{n}\ \bar{a}\ m$$
$$\bar{m}\ x\ s\ m = \bar{x}\ \bar{m}\ x\ \bar{n}\ \bar{x}\ m\ x\ \bar{m}\ x\ n\ \bar{x}\ m\ x$$
$$\bar{x}\ \bar{m}\ x\ n\ \bar{x}\ m\ x = \bar{m}\ x\ n\ \bar{x}\ m\ x\ \bar{n}\ \bar{x}\ m$$
$$ml^{\nu} = 1$$
$$l = m\ a\ \bar{n}\ \bar{a}\ \bar{m}\ \bar{a}\ \bar{m}\ a\ \bar{x}\ m\ x\ \bar{m}\ x\ n\ \bar{x}\ m\}.$$

$$A \sim \begin{pmatrix} 0 & 0 & -1+a & 1-a & 0 & a \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1+x & 1-x & 0 & x \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & \nu & 0 \\ 0 & 0 & \bar{a}-\bar{n} & a-x & 1 & 0 \end{pmatrix}$$

$$\sim\begin{pmatrix} 0 & 0 & 1 & \nu \\ 0 & 0 & \bar{a}-\bar{x}+a-x & 1 \end{pmatrix}.$$

Then, $\Delta^{(2)}(a, x)=\nu(ax^2-a^2x)+ax+\nu(a-x)$.

$$\pi_1((W''-\operatorname{int}V'')\underset{\varphi''}{\cup}S^1\times D^2)=\{a, x, n, m, s, l:$$



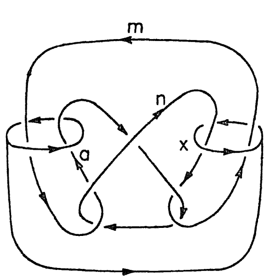
$$x\,\bar{s}\,m\,s\,\bar{x}\,a\,x\,\bar{s}\,\bar{m}\,s\,\bar{x}=n\,s\,a\,\bar{n}$$
$$n\,m\,\bar{n}\,x\,s\,n\,\bar{m}\,\bar{n}=m\,s\,x\,\bar{s}\,\bar{m}$$
$$\bar{n}\,x\,\bar{s}\,n\,m\,\bar{n}\,s\,\bar{x}\,n=s\,a\,\bar{n}\,\bar{a}\,\bar{s}\,\bar{m}\,n\,m\,s\,a\,n\,\bar{a}\,\bar{s}$$
$$s\,a\,x\,\bar{s}\,m\,s\,\bar{x}\,\bar{a}\,\bar{s}=\bar{m}\,n\,m\,s\,a\,n\,\bar{a}\bar{s}\,\bar{m}\,\bar{n}\,m$$
$$m\,l^{\nu}=1$$
$$l=\bar{n}\,s\,\bar{x}\,n\,s\,a\,\bar{n}\,\bar{a}\,\bar{s}\,\bar{m}\,\bar{a}\,\bar{s}\,\bar{m}\,\bar{n}\,m\,s\,a\,x\,\bar{s}\,m^3\}.$$

$$A\sim\begin{pmatrix} 0 & 0 & x-ax & -1+a & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & x & -1 & 0 & 0 \\ 0 & 0 & ax & -a & 0 & 0 \\ 0 & 0 & 1 & 0 & \nu & 0 \\ 0 & 0 & \bar{x}-3 & \bar{a}\bar{x}+a\bar{x}-\bar{x}+1 & 1 & 0 \end{pmatrix}$$

$$\sim\begin{pmatrix} 0 & 0 & 1 & \nu \\ 0 & 0 & x-3+\bar{a}+a-1+x & 1 \end{pmatrix}.$$

Then $\Delta^{(2)}(a, x)=\nu(a^2x+ax^2)+(1-4\nu)ax+\nu(x+a)$.

$$\pi_1((W''-\operatorname{int}V'')\underset{\varphi''}{\cup}S^1\times D^2)=\{a, x, m, n, s, l:$$



$$m\,a\,s\,\bar{m}=a\,n\,\bar{a}\,m\,a\,\bar{n}\,a\,n\,\bar{a}\,\bar{m}\,a\,\bar{n}\,\bar{a}$$
$$m\,x\,s\,\bar{m}=n\,x\,\bar{n}$$
$$\bar{x}\,n\,x=\bar{n}\,a\,n\,\bar{a}\,m\,a\,\bar{n}\,\bar{a}\,n\,\bar{x}\,m\,x\,\bar{n}\,a$$
$$\times\,n\,\bar{a}\,\bar{m}\,a\,\bar{n}\,\bar{a}\,n$$
$$\bar{a}\,\bar{m}\,a\,n\,\bar{a}\,m\,a=\bar{x}\,\bar{m}\,x\,\bar{n}\,a\,n\,\bar{a}\,m\,a\,\bar{n}\,\bar{a}$$
$$\times\,n\,\bar{x}\,m\,x$$
$$ml^{\nu}=1$$
$$l=x\,\bar{n}\,a\,n\,\bar{a}\,\bar{m}\,a\,\bar{n}\,\bar{a}\,n\,\bar{x}\,\bar{a}\,m\,a\,\bar{x}\,\bar{m}\,x$$
$$\times\,\bar{n}\,a\,n\,\bar{a}\,m\}.$$

$$A \sim \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & a \\ 0 & 0 & 1-x & -1+x & 0 & x \\ 0 & 0 & -\bar{x} & \bar{x} & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & \nu & 0 \\ 0 & 0 & -\bar{a}+\bar{x}-1+x & 1-a & 1 & 0 \end{pmatrix}$$

$$\sim \begin{pmatrix} 0 & 0 & 1 & \nu \\ 0 & 0 & -\bar{a}+\bar{x}-a+x & 1 \end{pmatrix}.$$

Then $\Delta^{(2)}(a, x)=\nu(a^2x-ax^2)+ax+\nu(x-a)$.

In every case, the second normalized polynomial $\Delta^{(2)}$ can not have a constant term.

This is a contradiction, i.e. the first assumption is not true. Then the inclusion map from $\pi_1(\partial W'')$ into $\pi_1((W''-\mathrm{int}\,V'')\underset{\varphi''}{\cup} S^1\times D^2)$ is a monomorphism.

This completes the proof.

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