

Ancient Indian mathematics needs an honorific place in modern mathematics celebration

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The Indian tradition in mathematics is long and glorious. It dates to the earliest times, and indeed many of the Indian discoveries from a period starting 5000 years ago correspond rather naturally to modern mathematical results. Celebration of Indian mathematics needs to consider the personalities among ancient mathematicians who laid a solid foundation for modern thinking. Our main purpose here is, by presenting very briefly some of the main contributions of ancient Indian mathematicians and astronomers, to argue and convince the reader that before the great Ramanujan, there have been thousands of years of rich mathematical discoveries in India and those personalities' work also needs to be honored on Indian Mathematics Day.

The government of India announced in 2012 that every year, Indian mathematician Srinivasa Ramanujan's birthday, December 22, will be celebrated as national mathematics day.¹ The year 2012 was the great Ramanujan's 125th birth anniversary. The government of India released a commemorative stamp on that occasion as well. Brilliant contributions in number theory and combinatorics by Ramanujan are well known [1, 2, 13, 16]. However, deep astronomical and mathematical developments in India are several thousand years older than Ramanujan. In this comment, we try to recollect a few gems of the ancient Indian mathematics and its

mathematicians who did fundamental work in number systems, mathematics of astronomy, calculus, etc., over more than 5000 years.

Our main purpose for writing this article is to argue and convince that, while giving Ramanujan's brilliant achievements during the past 125 years their due place, reducing the Mathematics Day in India to the celebration of Ramanujan's birthday (who was born in the 19th century) is somewhat short-sighted. Our goal is to make sure Indian Mathematics Day is seen as a celebration of thousands of years of deep-rooted mathematical thought processes and discoveries since the times of *Shulba sutra*. Moreover, it should also be devoted to celebrating many very strong mathematicians, such as, say, Harish Chandra, who have come since Ramanujan's time.

The origins of the mathematics that emerged in the Indian subcontinent can be seen around the *Shulba sutra* period, around 1200 BCE to 500 BCE. During this period the numbers up to 10^{12} were counted (in Vedic Sanskrit this number was referred to as *Paradham*). The Vedic period mathematics was confined to the geometry of fire-altars and astronomy, and these concepts were used to perform rituals by the priests. Some of the famous names from that era are Baudhayana, Apastamba, and Katyayana. In Table 1 we describe Sanskrit sounds and their corresponding English numerals. Indian mathematics also introduced the decimal number system that is in use today and the concept of zero as a number. The concepts of sine (written as *jaya* in Sanskrit) and cosine (*cojaya*), negative numbers, arithmetic, and algebra were found in ancient Indian mathematics [6]. The mathematics developed in India was later translated and transmitted to China, East Asia, West Asia, Europe, and

¹ Prime Minister's speech at the 125th Birth Anniversary Celebrations of Ramanujan at Chennai: <https://archive.is/20120729041631/http://pmindia.nic.in/speech-details.php?nodeid=1117> (accessed on April 15, 2023).

Ancient Indian number	०	१	२	३	४	५	६	७	८	९
Sound	shunya	ekah	dva	triyoh	caturah	panca	sat	sapta	astah	nava
English number	0	1	2	3	4	5	6	7	8	9
Sound	zero	one	two	three	four	five	six	seven	eight	nine

Table 1. Sanskrit and English numerals 0 to 9 and their sounds.

Saudi Arabia. The classical period of Indian mathematics was often attributed to the interval from 200 CE to 1400 CE, during which works of several well-known mathematicians, like Varahamihira, Aryabhata, Brahmagupta, Bhaskara, and Madhava have been translated into other languages and transmitted outside the sub-continent.

The number systems present since the Vedic days, especially since the Sukla Yajurveda and their Sanskrit sounds, were as follows: 1 (*Eka*), 10 (*Dasa*), 100 (*Sata*), 1000 (*Sahasra*), 10^4 (*Aayuta*), 10^5 (*Laksa* or *Niyuta*), 10^7 (*Koti*), 10^{12} (*Sanku* or *Paraardha*), 10^{17} (*Maha Sanku*), 10^{22} (*Vrnda*), 10^{52} (*Samudra*), 10^{62} (*Maha-ogha*).

In addition to the number systems of ancient India, still today in India are heard the popular “*Vishnu Sahasra Nama Stotra*,” which dates back to the Mahabharata epic. In this, there is a verse that sounds like Sahasra “*Koti Yugadharine Namah*.” If we translate this verse, then, as we saw above, *Sahasra* means 1000, and *Koti* means 10^7 , so a simple translation of the phrase *Sahasra Koti* could mean 10^{10} . The entire phrase has been interpreted in different ways. We do not list here all possible interpretations and confine ourselves to number systems.

The deep investigations in astronomy and the solar system, geometry, and ground-breaking mathematical calculations by ancient and medieval great scholars in India, for example, Baudhayana, Varahamihira, Aryabhata, Bhaskara I & II, Pingala, Madhava, and many more, are well known (see [5, 7, 10] and [17, p. 423]).

It seems that celebrating national mathematics day in India only as part of Ramanujan’s birthday is confining the glory and celebration of Indian mathematics to a little over 100 years of the past. Schools and colleges across India have celebrated Ramanujan’s birthday for many decades, but that is different from exclusively limiting national day only to the great Ramanujan.

A good deal can be written on ancient scholar’s work from India; material on this can be found, for example in [3, 6, 8, 12, 15]. In this opinion piece, we highlight only a few of them.

Shulba sutras were believed to have started in India around 2000 BCE through verbal usage. Their compilation in Sanskrit started perhaps 1000 years later by Baudhayana then by Manava, Apastamba, Katyayana and consisted of geometric-shaped fire-altars for performing ancient Indian rituals [3, 12]. Some of these sutras also contain the statements of Pythagorean theorems and triples. For example, Apastamba provided the following triples:

$$\left\{ (3, 4, 5), (5, 12, 13), (15, 8, 17), \right. \\ \left. (12, 35, 37), (36, 15, 39) \right\}$$

for constructing fire-altars [15].

These sutras can be used to find the approximate value of $\sqrt{2}$ [12, 15], using the expression

$$\sqrt{2} \approx 1 + \frac{1}{3} + \frac{1}{3 \cdot 4} - \frac{1}{3 \cdot 4 \cdot 34} \approx 1.41421568627451.$$

६ (6)	१ (1)	८ (8)
७ (7)	५ (5)	३ (3)
२ (2)	९ (9)	४ (4)

६ +n	१ +n	८ +n
७ +n	५ +n	३ +n
२ +n	९ +n	४ +n

Figure 1. Magic squares (*anka-yantra*) for the Sun (top) and for the other eight planets (bottom) taken from ancient Indian literature.

In the Vedic period astrology (*Jyotisha*) of India, the magic squares (*anka-yantra*) were used to please and worship nine planets of the solar system [9, 18]. Figure 1 is about ancient magic squares for the Sun and the other eight planets in our solar system.

In the 5th century CE, Aryabhata calculated, among many other things, that the moon orbit takes 27.396 days, the value of $\pi = 3.1416$, etc. He is believed to have started the study of properties of sine and cosine in trigonometry.

According to [12], the Leibniz infinite series

$$4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots\right)$$

was known in the works of Indian mathematician Madhava, who lived three centuries before Leibniz.

In the 12th century CE, Bhaskara described in his famous book *Bijaganita* the rules of algebraic operations on positive, negative signs, rules of zero (*shunya*), and infinity (*anantam*). His book also shows how to obtain solutions to intermediate equations of the first degree [14]. Bhaskara’s book titled *Siddhanta siromani* provided a detailed account of Indian astronomy and its development. Computations of the planetary movements, shapes of planets, rotation axis, lunar month days, etc., were explained in detail. See Figure 2. Pavuluri Mallana translated Mahavira’s *Ganitasarasam-graha* from Sanskrit in the 11th century to another ancient indian language, *Telugu*; Joseph [11] thinks that this stood as a role model for other subsequent translations. Bhaskara’s *Lilavati Ganitam* was for the first time translated from Sanskrit to *Telugu* in the 12th century by Eluganti Peddana [4], and into English first in 1816 by John Taylor, then in 1817 by Henry Thomas Holbrooke, who was considered as the first European Sanskrit scholar.

What we advocate in this piece is for an exposition of deep-rooted mathematical knowledge in India, and not an exhaustive

Bhāskarācārya, b. 1114

Līlāvāṭī gaṇitamū

Vijayanagarām, 1936

Bhāskarācārya, b. 1114.

Bhāskarācārya virācitamāgu vāsanaḥbhāṣya sahita Līlāvāṭī gaṇitamū / Pīḍamartī Kṛṣṇamūrtīsāstrī prāṇītamāgu Āṇḍhravyākhyānamū. - 1 kūrpu. - Vijayanagarām : Vidyamudrāśāraśāla, 1936.

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పరిమితసంఖ్యలపై పనియందు అంకగణితము (Arithmetic) అనుశీర్షికక్రింద చర్చింపబడు విషయములన్నియు సంగ్రహముగ శ్రీలహరియందు వర్ణింపబడియున్నవి. అవి క్రమముగ పరికర్మాస్తకము Four rules of Arithmetic, Square, Square root, Cube, (cube root) భిన్నములు (Fractions) వాని పరియోగములు (application to problems of daily life) త్రైరాశికము (Rule of Three) తేత్రవ్యవహారము (Mensuration), ఇంతియగాక ఉచ్చ బీజగణితమునకుజేరిన శ్రేణీవ్యవహారము (Progressions) వ్యక్తి భేద, అంకపాత వ్యవహారములు (Combination and Permutations) కుట్టక వ్యవహారము (Solutions of Indeterminate Equations) కూడ సీగంధమునందు వర్ణింపబడియున్నవి. కుట్టకవ్యవహారసందర్భమున విలతభిన్న (Continued Fractions) అసన్నమాన (Convergent) అనయనములు సంగ్రహముగ సూచింపబడియున్నవి. శూన్యపరికర్మసిద్ధాంతములు కూడ వర్ణితములై యున్నవి.

Figure 2. Screenshots from the book “Lilavati Ganitamu” written by Bhaskara (also known as Bhaskaracharya) in the year 1114 CE. Top: Bhaskara wrote the book in Sanskrit. These screenshots were taken from its translated version into another ancient Indian language, Telugu, by Pīḍamartī Kṛṣṇamūrtī Sāstrī in 1936, published by Maharaja College, Vizianagaram, Andhra Pradesh, India.² Bottom: Here the contents of the book are mentioned in Telugu, as well as in English.

account of all possible results and conclusions. Several of the ancient texts in the language *Sanskrit* are either lost or preserved in museums.

Srinivasa Ramanujan’s work undoubtedly shines as part of modern Indian mathematics but thousands of years ancient mathematical discoveries, the introduction of various branches of pure and applied mathematics needs a proper representation in any celebration of India’s contribution to world mathematics.

We hope that this short list of significant examples will convince the readers as well as the decision makers of the need to incorpo-

rate and celebrate all the rich past and contemporary history of Indian mathematics during the Indian Mathematics Day.

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² This book is available for free at <https://upload.wikimedia.org/wikipedia/commons/7/75/Lilavatiganitamu00bhassher.pdf>. © Sundarayya Vignana Kendram, Bagh Lingampally, Hyderabad, India

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