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Erratum to "Phase transitions on the Markov and Lagrange dynamical spectra"

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Abstract. We acknowledge an error in the proof of the last statement of Theorem D of Lima and Moreira (2021).

There is a mistake in the proof of the last statement of [1, Theorem D] by the first two authors, where we use $\mathrm{Diff}^2_*(M)$ for conservative diffeomorphisms of class C^2, which states the following:

Theorem D. *Given $\varphi \in \mathrm{Diff}^2_*(M)$ with a horseshoe Λ, $HD(\Lambda) > 1$ and $k \geq 2$, there are an open set $\mathrm{Diff}^2_*(M) \supset \mathcal{U} \ni \varphi$ and a residual set $\mathcal{U}^{**} \subset \mathcal{U}$ such that for every $\psi \in \mathcal{U}^{**}$ there is a C^k-residual subset $\mathcal{R}_\psi \subset C^k(M, \mathbb{R})$ such that if $f \in \mathcal{R}_\psi$ then*

$$\mathrm{Leb}(M_f(\Lambda_\psi) \cap (-\infty, a-\delta)) = 0 = \mathrm{Leb}(L_f(\Lambda_\psi) \cap (-\infty, a-\delta))$$

but

$$\mathrm{int}(L_f(\Lambda_\psi) \cap (-\infty, a+\delta)) \neq \emptyset \neq \mathrm{int}(M_f(\Lambda_\psi) \cap (-\infty, a+\delta))$$

for all $\delta > 0$, where $a = a(\varphi, f) = \sup\{t \in \mathbb{R} : HD(\Lambda_t) < 1\}$.

Moreover,

$$HD(M_f(\Lambda_\psi) \cap (-\infty, a)) = HD(L_f(\Lambda_\psi) \cap (-\infty, a)) = 1.$$

More precisely, the mistake is in the following statement, which is used in the proof that $\lim_{\delta \to 0} HD(\Lambda_{a-\delta}) = 1$:

If $\delta > 0$ is sufficiently small we have that

$$K^s_{a+\delta} \subset K^s_{a-\delta} \cup \left(\bigcup_{i=1}^{M} I^s_{(\underline{b}^i)^t \underline{a}^i} \right),$$

which is not necessarily true, since, even for $\delta > 0$ small, the set $\Lambda_{a+\delta} \cap \{p \in \Lambda; f(p) \leq a - \delta\}$ is not necessarily close, in the Hausdorff metric, to $\Lambda_{a-\delta}$.

However, in [2, Theorem 2] we prove the following result:

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Theorem 2. *Let $\varphi_0 \in \mathrm{Diff}^2_*(M)$ with a mixing horseshoe Λ_0 and $\mathcal{U}$ a C^2-sufficiently small neighbourhood of φ_0 in $\mathrm{Diff}^2_*(M)$ such that Λ_0 admits a continuation Λ $(= \Lambda(\varphi))$ for every $\varphi \in \mathcal{U}$. There exists a residual set $\widetilde{\mathcal{U}} \subset \mathcal{U}$ such that for every $\varphi \in \widetilde{\mathcal{U}}$ and $r \geq 2$ there exists a C^r-residual set $\widetilde{\mathcal{R}}_{\varphi,\Lambda} \subset C^r(M, \mathbb{R})$ such that for any $f \in \widetilde{\mathcal{R}}_{\varphi,\Lambda}$ the functions*

$$t \mapsto d_u(t) := HD(K_t^u) \quad \text{and} \quad t \mapsto d_s(t) := HD(K_t^s)$$

are continuous and, in fact, they are equal to

$$HD(\Lambda_t) = d_u(t) + d_s(t) = 2d_u(t)$$

and

$$\min\{1, HD(\Lambda_t)\} = HD(L_{\varphi,f} \cap (-\infty, t)) = HD(M_{\varphi,f} \cap (-\infty, t)).$$

Using this result it is not difficult to complete the proof that $\lim_{\delta \to 0} HD(\Lambda_{a-\delta}) = 1$ (so correcting the proof of [1, Theorem D], where our horseshoes are assumed to be mixing). Indeed, by the definition of $a = a(\varphi, f)$, for every $\delta > 0$, $HD(\Lambda_{a-\delta}) < 1$ and $HD(\Lambda_{a+\delta}) \geq 1$. Since $HD(\Lambda_t) = 2d_u(t)$ is a continuous function, we should have $HD(\Lambda_a) = \lim_{\delta \to 0+} HD(\Lambda_{a-\delta}) \leq 1$ and also $HD(\Lambda_a) = \lim_{\delta \to 0+} HD(\Lambda_{a+\delta}) \geq 1$, and we conclude that $\lim_{\delta \to 0} HD(\Lambda_{a-\delta}) = HD(\Lambda_a) = 1$.

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