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Which 3-manifold groups are Kähler groups?

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Abstract. The question in the title, first raised by Goldman and Donaldson, was partially answered by Reznikov. We give a complete answer, as follows: if G can be realized as both the fundamental group of a closed 3-manifold and of a compact Kähler manifold, then G must be finite—and thus belongs to the well-known list of finite subgroups of O(4), acting freely on S^3.

Keywords. Kähler manifold, 3-manifold, fundamental group, cohomology ring, resonance variety, isotropic subspace

1. Introduction

1.1. As is well-known, every finitely presented group G occurs as the fundamental group of a smooth, compact, connected, orientable 4-dimensional manifold M. As shown by Gompf [14], the manifold M can be chosen to be symplectic. Requiring a complex structure on M is no more restrictive, as long as one is willing to go up to complex dimension 3 (see Taubes [32]).

Suppose now G is the fundamental group of a compact Kähler manifold M. Groups arising this way are called *Kähler groups* (or, *projective groups*, if M is actually a smooth projective variety). The Kähler condition puts strong restrictions on what G can be. For instance, the first Betti number, $b_1(G)$, must be even, by classical Hodge theory. Moreover, G must be 1-formal, by work of Deligne, Griffiths, Morgan, and Sullivan [9]. Also, G cannot split non-trivially as a free product, by a result of Gromov [17]. On the other hand, every finite group is a projective group, by a classical result of Serre [29]. We refer to [1] for a comprehensive survey of Kähler groups, and to the recent work of Delzant–Gromov [11], Napier–Ramachandran [25], and Delzant [10] for further geometric restrictions imposed by the Kähler condition on a group G.

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Requiring that M be a 3-dimensional compact, connected manifold also puts severe restrictions on $G = \pi_1(M)$. For example, if G is abelian, then G is either $\mathbb{Z}/n\mathbb{Z}$, $\mathbb{Z}$, $\mathbb{Z} \oplus \mathbb{Z}_2$, or $\mathbb{Z}^3$ (see [20]).

1.2. A natural question—raised by Goldman and Donaldson in 1989, and independently by Reznikov in 1993—is then: what are the 3-manifold groups which are Kähler groups?

In [28], Reznikov proved the following result, which Simpson [31] calls "one of the deepest restrictions" on the homotopy types that may occur for Kähler manifolds: *Let M be an irreducible, atoroidal* 3*-manifold, and suppose there is a homomorphism $\rho\colon \pi_1(M) \to \mathrm{SL}(2,\mathbb{C})$ with Zariski dense image. Then $G = \pi_1(M)$ is not a Kähler group.* The same conclusion was reached by Hernández-Lamoneda in [19], under the assumption that M is a geometrizable 3-manifold, with all pieces hyperbolic.

In this note, we answer the above question for all 3-manifold groups, as follows.

Theorem 1.1. *Let G be the fundamental group of a compact, connected* 3*-manifold. If G is a Kähler group, then G is finite.*

By the 3-dimensional spherical space-form conjecture, now established by Perelman [26, 27], a closed 3-manifold M has finite fundamental group if and only if it admits a metric of constant positive curvature (for a detailed proof, see Morgan and Tian [24, Corollary 0.2]). Thus, $M = S^3/G$, where G is a finite subgroup of O(4), acting freely on S^3. The list of such finite groups (essentially due to Hopf) is given by Milnor in [23].

1.3. The paper is organized as follows. In §2, we discuss the characteristic and resonance varieties of a group G, and two notions of isotropy. In §3, we recall the Isotropic Subspace Theorem of Catanese, and a correspondence due to Beauville. In §4, we use these tools to prove a key result, tying the first resonance variety of a Kähler manifold to the rank of the cup-product map in low degrees. In §5, we investigate the first resonance variety of a closed, oriented 3-manifold; Poincaré duality and properties of Pfaffians yield a very different conclusion in this setting.

All this works quite well, provided the first Betti number of G is positive. To deal with the remaining case, we need two theorems of Reznikov and Fujiwara, relating the Kähler, respectively the 3-manifold condition on a group to Kazhdan's property T; we recall those in §6. Finally, we put everything together in §7, and give a proof of Theorem 1.1.

A natural question arises out of this work: Which 3-manifold groups are quasi-Kähler? (A group G is *quasi-Kähler* if $G = \pi_1(M \setminus D)$, where M is a compact Kähler manifold and D is a divisor with normal crossings.) We have some partial results in this direction; those results will be presented elsewhere.

2. Cohomology jumping loci and isotropic subspaces

2.1. Let X be a connected CW-complex with finitely many cells in each dimension. Let $G = \pi_1(X)$ be the fundamental group of X, and $\mathbb{T} = \mathrm{Hom}(G, \mathbb{C}^*)$ its character variety.

Every character $\rho \in \mathbb{T}$ determines a rank 1 local system, $\mathbb{C}_\rho$, on X. The *characteristic varieties* of X are the jumping loci for cohomology with coefficients in such local systems:

$$V^i_d(X) = \{\rho \in \mathbb{T} \mid \dim H^i(X, \mathbb{C}_\rho) \geq d\}. \tag{1}$$

The varieties $V_d(X) = V^1_d(X)$ depend only on $G = \pi_1(X)$, so we sometimes denote them as $V_d(G)$.

2.2. Consider now the cohomology algebra $A = H^*(X, \mathbb{C})$. Left multiplication by an element $x \in A^1$ yields a cochain complex $(A, x)\colon A^0 \xrightarrow{x} A^1 \xrightarrow{x} A^2 \to \cdots$. The *resonance varieties* of X are the jumping loci for the homology of this complex:

$$R^i_d(X) = \{x \in A^1 \mid \dim H^i(A, x) \geq d\}. \tag{2}$$

The varieties $R_d(X) = R^1_d(X)$ depend only on $G = \pi_1(X)$, so we sometimes denote them by $R_d(G)$. By definition, an element $x \in A^1$ belongs to $R_d(X)$ if and only if there exists a subspace $W \subset A^1$ of dimension $d + 1$ such that $x \cup y = 0$ for all $y \in W$.

Fix bases $\{e_1, \ldots, e_n\}$ for A^1 and $\{f_1, \ldots, f_m\}$ for A^2. Writing the cup-product as $e_i \cup e_j = \sum_{k=1}^m \mu_{i,j,k} f_k$, we may define an $m \times n$ matrix Δ of linear forms in variables $x_1, \ldots, x_n$, with entries

$$\Delta_{k,j} = \sum_{i=1}^n \mu_{i,j,k} x_i. \tag{3}$$

It is readily seen that $R_d(X) = V(E_d(\Delta))$, where E_d denotes the ideal of $(n-d) \times (n-d)$ minors. Note also that $x \cup x = 0$ for all $x \in A^1$ implies $\Delta \cdot \vec{x} = 0$, where $\vec{x}$ is the column vector with entries $x_1, \ldots, x_n$.

2.3. Foundational results on the structure of the cohomology support loci for local systems on compact Kähler manifolds were obtained by Beauville [2], Green–Lazarsfeld [15], Simpson [30], and Campana [5]: if G is the fundamental group of such a manifold, then $V_d(G)$ is a union of (possibly translated) subtori of the algebraic group $\mathbb{T}$.

In addition, Theorem A from [12] establishes a strong relationship between the characteristic and resonance varieties of a Kähler group G: the tangent cone to $V_d(G)$ at the identity of $\mathbb{T}$ equals $R_d(G)$ for all $d \geq 1$.

2.4. A non-zero subspace $E \subset H^1(X, \mathbb{C})$ is *(totally) isotropic* if the restriction of the cup-product map $\cup_X\colon H^1(X, \mathbb{C}) \wedge H^1(X, \mathbb{C}) \to H^2(X, \mathbb{C})$ to $E \wedge E$ is identically zero. By analogy, we say E is 1-*isotropic* if the restriction of $\cup_X$ to $E \wedge E$ has 1-dimensional image.

Note that these properties of E depend only on $G = \pi_1(X)$. Indeed, let $h\colon X \to K(G, 1)$ be a classifying map. Then $h_*\colon H_1(X, \mathbb{Z}) \to H_1(G, \mathbb{Z})$ is an isomorphism, and $h_*\colon H_2(X, \mathbb{Z}) \to H_2(G, \mathbb{Z})$ is an epimorphism. Using Kronecker duality and the functoriality of the cup-product, it is readily seen that E is a (1-) isotropic subspace of $H^1(G, \mathbb{C})$ for $\cup_G$ if and only if $h^*(E)$ is a (1-) isotropic subspace of $H^1(X, \mathbb{C})$ for $\cup_X$.

3. The Isotropic Subspace Theorem

By a *fibration* we mean a surjective morphism $f: M \to N$ with connected fibers between two compact complex manifolds M and N. Two fibrations $f: M \to C$ and $f': M \to C'$ over projective curves C and C' are said to be *equivalent* if there is an isomorphism $\phi: C \to C'$ such that $f' = \phi \circ f$. We denote by $\mathcal{E}(M)$ the set of equivalence classes of fibrations $f: M \to C$, with C a projective curve of genus $g \geq 2$.

Let M be a compact Kähler manifold. Beauville's work [2] establishes a bijection between the set $\mathcal{E}(M)$ and the set of irreducible components of the first characteristic variety $V_1(M)$ passing through the identity of the algebraic group $\mathbb{T} = \mathrm{Hom}(\pi_1(M), \mathbb{C}^*)$. In particular, the set $\mathcal{E}(M)$ must be finite.

The Isotropic Subspace Theorem, due to Catanese [6, Theorem 1.10], establishes a relation between the set of equivalence classes of fibrations of a Kähler manifold M over curves of genus $g \geq 2$, and the maximal isotropic subspaces in $H^1(M, \mathbb{C})$.

Theorem 3.1 (Catanese [6]). *Let M be a compact Kähler manifold. Then, for any maximal isotropic subspace $E \subset H^1(M, \mathbb{C})$ of dimension $g \geq 2$, there is a fibration $f: M \to C$ onto a smooth curve of genus g and a maximal isotropic subspace $E' \subset H^1(C, \mathbb{C})$ such that $E = f^* E'$.*

For more information on this correspondence, see [7].

4. The first resonance variety of a Kähler manifold

Theorem 4.1. *Let M be a compact Kähler manifold with $b_1(M) \neq 0$. If $R_1(M) = H^1(M, \mathbb{C})$, then $H^1(M, \mathbb{C})$ is 1-isotropic.*

Proof. By Hodge theory, we must have $b_1(M) \geq 2$. The equality $R_1(M) = H^1(M, \mathbb{C})$ says that, for any non-zero cohomology class $x \in H^1(M, \mathbb{C})$, there is a class $y \in H^1(M, \mathbb{C}) \setminus \mathbb{C} \cdot x$ such that $x \cup y = 0$. Consequently, the vector space spanned by x and y is a (2-dimensional) isotropic subspace containing x.

Let U_x be a maximal isotropic subspace of $H^1(M, \mathbb{C})$ containing x; we must then have $\dim U_x \geq 2$. Thus, by Theorem 3.1, there is a fibration $f_x: M \to C_x$ onto a smooth projective curve C_x of genus $g_x = \dim U_x$, with $x \in f_x^*(H^1(C_x, \mathbb{C}))$.

Recall now that the set $\mathcal{E}(M)$ of equivalence classes of fibrations of M over curves of genus at least 2 is finite. Thus, we may write the first cohomology group of M as a finite union of linear subspaces,

$$H^1(M, \mathbb{C}) = \bigcup_{[f] \in \mathcal{E}(M)} f^*(H^1(C_f, \mathbb{C})), \tag{4}$$

where $f = f_x$ for some $x \in H^1(M, \mathbb{C})$, and $C_f := C_x$. This is possible only if there is a fibration $f_1: M \to C_1$ such that $H^1(M, \mathbb{C}) = f_1^*(H^1(C_1, \mathbb{C}))$.

Since f_1 is a fibration, the induced morphism $f_1^*: H^1(C_1, \mathbb{C}) \to H^1(M, \mathbb{C})$ is injective. The defining property of f_1 implies that $f_1^*: H^1(C_1, \mathbb{C}) \to H^1(M, \mathbb{C})$ is an isomorphism.

On the other hand, the induced morphism $f_1^*\colon H^2(C_1,\mathbb{C}) \to H^2(M,\mathbb{C})$ is also injective. To prove this claim, first note that any cohomology class in $H^1(M,\mathbb{C})$ is primitive. Using the Hodge–Riemann bilinear relations (see e.g. [16, p. 123]), it follows that, for any non-zero (1, 0)-class $\alpha \in H^1(M,\mathbb{C})$, the product $\beta = \sqrt{-1}\,\alpha \cup \overline{\alpha}$ is a non-zero, real, (1, 1)-class in $H^2(M,\mathbb{C})$. Since $f_1^*\colon H^1(C_1,\mathbb{C}) \to H^1(M,\mathbb{C})$ is an isomorphism, there is an element $a \in H^1(C_1,\mathbb{C})$ such that $f_1^*(a) = \alpha$. Hence, $f_1^*(\sqrt{-1}\,a \wedge \overline{a}) = \beta$, and the claim is proved.

Consider now the commuting diagram

$$\begin{array}{ccc} H^1(M,\mathbb{C}) \wedge H^1(M,\mathbb{C}) & \xrightarrow{\ \cup_M\ } & H^2(M,\mathbb{C}) \\ {\scriptstyle f_1^*\wedge f_1^*}\uparrow & & \uparrow{\scriptstyle f_1^*} \\ H^1(C_1,\mathbb{C}) \wedge H^1(C_1,\mathbb{C}) & \xrightarrow{\ \cup_{C_1}\ } & H^2(C_1,\mathbb{C}) \end{array} \tag{5}$$

As we saw above, the left arrow is an isomorphism, and the right one is an injection. Since $\cup_{C_1}$ surjects onto $H^2(C_1,\mathbb{C}) = \mathbb{C}$, we conclude that $\cup_M$ has 1-dimensional image. □

Remark 4.2. An alternative way to prove Theorem 4.1 is by using the much more general Theorem C from [12], which guarantees that *every* positive-dimensional component of $R_1(M)$ is an 1-isotropic subspace of $H^1(M,\mathbb{C})$. This is the argument we had in an earlier version of this paper; at the urging of one of the referees, we came up with the above, more self-contained proof.

5. The first resonance variety of a 3-manifold

Let M be a compact, connected, orientable 3-manifold. Fix an orientation on M, that is, pick a generator $[M] \in H^3(M,\mathbb{Z}) \cong \mathbb{Z}$. With this choice, the cup-product on M determines an alternating 3-form $\mu = \mu_M$ on $H^1(M,\mathbb{Z})$, given by

$$\mu(x,y,z) = \langle x \cup y \cup z, [M]\rangle, \tag{6}$$

where $\langle\ ,\ \rangle$ is the Kronecker pairing. In turn, the cup-product map $\cup_M\colon H^1(M,\mathbb{Z}) \wedge H^1(M,\mathbb{Z}) \to H^2(M,\mathbb{Z})$ is determined by μ, via $\langle x \cup y, \gamma\rangle = \mu(x,y,z)$, where $z = \mathrm{PD}(\gamma)$ is the Poincaré dual of $\gamma \in H_2(M,\mathbb{Z})$.

Now fix a basis $\{e_1,\dots,e_n\}$ for $H^1(M,\mathbb{C})$, and choose as basis for $H^2(M,\mathbb{C})$ the set $\{e_1^\vee,\dots,e_n^\vee\}$, where $e_i^\vee$ denotes the Kronecker dual of the Poincaré dual of e_i. Then

$$\mu(e_i,e_j,e_k) = \Big\langle \sum_{1\le m\le n} \mu_{i,j,m} e_m^\vee, \mathrm{PD}(e_k)\Big\rangle = \mu_{i,j,k}. \tag{7}$$

Recall from (3) the $n \times n$ matrix with entries $\Delta_{k,j} = \sum_{i=1}^n \mu_{i,j,k} x_i$. Since μ is an alternating form, Δ is a skew-symmetric matrix.

Proposition 5.1. *Let M be a closed, orientable 3-manifold. Then:*

(1) *$H^1(M,\mathbb{C})$ is not 1-isotropic.*
(2) *If $b_1(M)$ is even, then $R_1(M) = H^1(M,\mathbb{C})$.*

Proof. To prove (1), suppose $\dim \operatorname{im}(\cup_M) = 1$. This means there is a hyperplane $E \subset H := H^1(M,\mathbb{C})$ such that $x \cup y \cup z = 0$ for all $x, y \in H$ and $z \in E$. Hence, the skew 3-form $\mu\colon \bigwedge^3 H \to \mathbb{C}$ factors through a skew 3-form $\bar{\mu}\colon \bigwedge^3(H/E) \to \mathbb{C}$. But $\dim H/E = 1$ forces $\bar{\mu} = 0$, and so $\mu = 0$, a contradiction.

To prove (2), recall $R_1(M) = V(E_1(\Delta))$. Since Δ is a skew-symmetric matrix of even size, it follows from Buchsbaum–Eisenbud [4, Corollary 2.6] that $V(E_1(\Delta)) = V(E_0(\Delta))$ (see [8, eq. (6.9)]). But $\Delta \cdot \vec{x} = 0$ implies $\det \Delta = 0$, and so $V(E_0(\Delta)) = H$. □

Remark 5.2. As noted by S. Papadima, the following holds. Suppose M is a closed, orientable 3-manifold, with $b_1(M)$ odd. Then $R_1(M) \neq H^1(M,\mathbb{C})$ if and only if μ_M is generic, in the sense of [3].

6. Kazhdan's property T

The following question is due to J. Carlson and D. Toledo (see J. Kollár [22]): For a Kähler group G, is $b_2(G) \neq 0$? This question was answered in the affirmative by A. Reznikov in [28], under an additional assumption, as follows.

Theorem 6.1 (Reznikov [28]). *Let G be a Kähler group. If G does not satisfy Kazhdan's property T, then $b_2(G) \neq 0$.*

Recall that a discrete group G satisfies *Kazhdan's property T* (for short, G is a *Kazhdan group*) if and only if $H^1(G,\mathcal{H}) = 0$ for all orthogonal or unitary representations of G on a Hilbert space $\mathcal{H}$ (see de la Harpe and Valette [18, p. 47]). In particular, if $b_1(G) \neq 0$, then G is not Kazhdan. (For a simple proof of Theorem 6.1 in this case, see [21].)

We will also need the following relationship between 3-manifold groups and Kazhdan's property T, established by K. Fujiwara in [13].

Theorem 6.2 (Fujiwara [13]). *Let G be the fundamental group of a closed, orientable 3-manifold. If G satisfies Kazhdan's property T, then G is finite.*

In fact, the theorem is valid for any subgroup $G < \pi_1(M)$, where M is a compact (not necessarily boundaryless), connected, orientable 3-manifold. Fujiwara further assumes that each piece of the canonical decomposition of M along embedded spheres, disks and tori admits one of the eight geometric structures in the sense of Thurston, but this is now guaranteed by the work of Perelman [26, 27].

7. Kähler 3-manifold groups

We are now in a position to prove Theorem 1.1 from the introduction.

Let G be the fundamental group of a compact, connected 3-manifold M. Suppose G is a Kähler group, and G is not finite.

Step 1. A finite-index subgroup of a Kähler group is again a Kähler group (see [1, Example 1.10]). Passing to the orientation double cover of M if necessary, we may as well assume M is orientable.

Step 2. Since G is an infinite, orientable 3-manifold group, G is not Kazhdan, by Fujiwara's Theorem 6.2. Since G is Kähler and not Kazhdan, $b_2(G) \neq 0$, by Reznikov's Theorem 6.1.

Step 3. Since $b_2(M) \geq b_2(G)$, we must also have $b_2(M) \neq 0$. By Poincaré duality, $b_1(M) = b_2(M)$. Hence, $b_1(G) = b_1(M)$ is not zero.

Step 4. Since G is Kähler, $b_1(G)$ must be even. Since M is a closed, orientable 3-manifold with $G = \pi_1(M)$, Proposition 5.1 tells us that $R_1(G) = H^1(G, \mathbb{C})$ and $H^1(G, \mathbb{C})$ is not 1-isotropic. Since, on the other hand, G is Kähler, Theorem 4.1 tells us that $b_1(G) = 0$.

Our assumptions have led us to a contradiction. Thus, the theorem is proved.

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