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Qi Lü · Jiongmin Yong · Xu Zhang

Erratum to "Representation of Itô integrals by Lebesgue/Bochner integrals"

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In this erratum, the authors would like to correct an error in the characterization of the dual of the Banach space of some vector-valued stochastic processes having different integrability with respect to the time variable and the probability measure, which appeared in [2].

In [2, Lemma 2.1], we presented the following result (see [2] for the notation used below):

Lemma 1. *Let H be a Banach space, $(X_1, \mathcal{M}_1, \mu_1)$ and $(X_2, \mathcal{M}_2, \mu_2)$ be finite measure spaces, $\mathcal{M}$ be a sub-σ-field of $\mathcal{M}_1 \otimes \mathcal{M}_2$, and let $1 \le p, q < \infty$. Then H^* has the Radon–Nikodým property with respect to $(X_1 \times X_2, \mathcal{M}, \mu_1 \times \mu_2)$ if and only if for any $F \in L^p_{\mathcal{M}}(X_1; L^q(X_2; H))^*$, there exists a unique $g \in L^{p'}_{\mathcal{M}}(X_1; L^{q'}(X_2; H^*))$ such that*

$$F(f) = \int_{X_1 \times X_2} (f(x_1, x_2), g(x_1, x_2))_{H,H^*}\, d\mu_1\, d\mu_2, \quad \forall f \in L^p_{\mathcal{M}}(X_1; L^q(X_2; H)),$$

and

$$\|F\|_{L^p_{\mathcal{M}}(X_1;L^q(X_2;H))^*} = \|g\|_{p',q',H^*}.$$

It turns out that for the conclusion to be true, a further assumption is needed (see [1] for a counterexample). The reason is that the function f constructed in Cases 1–4 in the proof of the necessity of [2, Lemma 2.1] for $H = \mathbb{R}$ (i.e., in [2, Subsection 2.2]) might not be $\mathcal{M}$-measurable. To avoid this, we need to introduce the following assumption:

Condition 2. *For any $\mathcal{M}$-measurable, nonnegative and bounded function ξ, the following function Ξ (defined on $X_1 \times X_2$) is $\mathcal{M}$-measurable:*

$$\Xi(x_1, x_2) = \left(\int_{X_2} \xi(x_1, s)\, d\mu_2(s) \right)^{p'/q'-1}, \quad (x_1, x_2) \in X_1 \times X_2.$$

Once Condition 2 is assumed, there is no gap in the proof of [2, Lemma 2.1].

Q. Lü, X. Zhang: School of Mathematics, Sichuan University, Chengdu 610064, China;
e-mail: lu@scu.edu.cn, zhang_xu@scu.edu.cn

J. Yong: Department of Mathematics, University of Central Florida, Orlando, FL 32816, USA;
e-mail: jyong@mail.ucf.edu

Note that in [2, proofs of Theorems 3.1–3.2] we only applied the above lemma to the space $L^r_{\mathbb{F}}(0, s; L^p(\Omega; H))$ (with $r = 1$ and $p \in [1, \infty)$ for Theorem 3.1, and $r \in (1, \infty]$ and $p \in (1, \infty)$ for Theorem 3.2), for which Condition 2 automatically holds. Indeed, if $\xi \in L^r_{\mathbb{F}}(0, s; L^p(\Omega; H))$, then $\mathbb{E}\xi \in L^r_{\mathbb{F}}(0, s; H) \subset L^r_{\mathbb{F}}(0, s; L^p(\Omega; H))$. Elsewhere in [2], the above lemma was not used. Hence, the main results in [2] remain true. More precisely, the answers to our Problems (E), (R) and (C) remain true without assuming Condition 2, or with this condition holding automatically.

References

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