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Sébastien Boucksom · Tomoyuki Hisamoto · Mattias Jonsson

Erratum to
"Uniform K-stability and asymptotics of energy functionals in Kähler geometry"

Abstract. The goal of this note is to indicate a gap in the proof of Theorem 5.6 of [J. Eur. Math. Soc. 21, 2905–2944 (2019)], and the consequences it has on other results in the same paper. Let us stress that the main result (Theorem A), which expresses the slopes at infinity of functionals in algebro-geometric terms, is independent of the flawed result, and thus remains valid.

1. The flawed statement

We first repeat the statement in question [1, Theorem 5.6].

Theorem 1.1. *Let G be a complex reductive group with a linear action on a finite dimensional complex vector space U. If the (Zariski) closure of the G-orbit of a point $x \in \mathbb{P}(U)$ meets a G-invariant Zariski closed subset $Z \subset \mathbb{P}(U)$, then some $z \in Z \cap \overline{G \cdot x}$ can be reached by a 1-parameter subgroup $\lambda : \mathbb{C}^* \to G$, i.e. $\lim_{t \to 0} \lambda(t) \cdot x = z$.*

Set $X := \mathbb{P}(U)$, $K := \mathbb{C}((t))$ and $R := \mathbb{C}[[t]]$. As in [2], our approach was based on the Iwasawa decomposition theorem, which states that each double coset in $G(K)$ modulo $G(R)$ is represented by a 1-PS of G (viewed as an element of $G(K)$).

By properness of X, each $\xi \in X(K)$ has a reduction $\tilde{\xi} \in X(\mathbb{C})$, to be interpreted as $\lim_{t \to 0} \xi(t)$. The problem with the proof of [1, Theorem 5.6] is the claim that for any 1-PS λ of G and $\xi \in X(K)$, the reduction of $\lambda \cdot \xi$ only depends on $\tilde{\xi}$.

This claim is indeed incorrect, as shown by the following simple counterexample, kindly provided to us by Yan Li.

Sébastien Boucksom: CNRS-CMLS, École Polytechnique, 91128 Palaiseau Cedex, France;
sebastien.boucksom@polytechnique.edu

Tomoyuki Hisamoto: Graduate School of Mathematics, Nagoya University, Furocho, Chikusa, Nagoya, Japan; hisamoto@math.nagoya-u.ac.jp

Mattias Jonsson: Department of Mathematics, University of Michigan, Ann Arbor, MI 48109-1043, USA; mattiasj@umich.edu

Example 1.2. Consider $\xi := [1 : 0]$, $\xi' := [1 : t]$ in $\mathbb{P}^1(K)$ and the 1-PS $\lambda := \mathrm{diag}(t^2, t^{-2})$. Then $\tilde{\xi} = \tilde{\xi}' = [1 : 0] \in \mathbb{P}^1(\mathbb{C})$, but $\lambda \cdot \xi = [t^2 : 0] = [1 : 0]$, $\lambda \cdot \xi' = [t^2 : t^{-1}] = [t^3 : 1]$, and hence $\widetilde{\lambda \cdot \xi} = [1 : 0] \neq [0 : 1] = \widetilde{\lambda \cdot \xi'}$.

Remark 1.3. While our proof of Theorem 1.1 is definitely incorrect, it is nevertheless possible, to the best of our knowledge, that the statement itself remains valid.

2. Other affected results

The main result of [1] affected by Theorem 5.6 is Theorem 5.4, which itself affects Theorem C and Corollaries D and E. However, Theorem A and Corollary B, which are independent of Theorem 5.6, remain valid. This is more generally the case of all results of [1] up to and including Section 4.

References

[1] Boucksom, S., Hisamoto, T., Jonsson, M.: Uniform K-stability and asymptotics of energy functionals in Kähler geometry. J. Eur. Math. Soc. **21**, 2905–2944 (2019)   Zbl 07117723   MR 3985614

[2] Mumford, D., Fogarty, J., Kirwan, F.: Geometric Invariant Theory. 3rd ed., Ergeb. Math. Grenzgeb. (2) 34, Springer, Berlin (1994)   Zbl 0797.14004   MR 1304906