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Erratum to "Uniform boundary controllability and homogenization of wave equations"

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Abstract. This erratum corrects the statements and proofs of Theorems 1.1 and 1.2 of [J. Eur. Math. Soc. 24, 3031–3053 (2022)].

There is a flaw in the proof of Theorem 1.2 in our paper [1]. More specifically, from (3.11) in [1], it is not possible to deduce the next inequality displayed with the same constant C on both sides and carry out an iteration argument. Instead, we simply take out the iteration argument and modify Theorems 1.1 and 1.2 as follows.

Theorem 0.1. *Assume $A = A(y)$ satisfies conditions* (1.3)–(1.5). *Also assume that there exists $M > 0$ such that*

$$|A(x) - A(y)| \le M|x - y| \quad \text{for any } x, y \in \mathbb{R}^d. \tag{0.1}$$

Let Ω be a bounded C^3 domain in $\mathbb{R}^d$. Let u_ε be a solution of (1.8) *with initial data $(\varphi_{\varepsilon,0}, \varphi_{\varepsilon,1}) \in \mathcal{A}_N \times \mathcal{A}_N$. If*

$$N \le C_0 T^{-1} \varepsilon^{-1/2} \quad \text{for some } C_0 > 0,$$

then the inequality (1.9) *holds with a constant C depending only on d, μ, C_0, M and Ω. Moreover, there exist $c_0 > 0$ and $T_0 > 0$, depending only on d, μ, M and Ω, such that if*

$$N \le c_0 T^{-1} \varepsilon^{-1/2} \quad \text{and} \quad T \ge T_0,$$

then (1.10) *holds with a constant c depending only on d, μ, M and Ω.*

Theorem 0.2. *Assume $A = A(y)$ satisfies conditions* (1.3)–(1.5). *Let u_ε be a weak solution of* (1.17), *where Ω is a bounded Lipschitz domain in $\mathbb{R}^d$. Let*

$$w_\varepsilon = u_\varepsilon - u_0 - (\Phi_{\varepsilon,k} - x_k)\frac{\partial u_0}{\partial x_k}, \tag{0.2}$$

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where u_0 is the solution of (1.18). *Then for any $t \in (0, T]$,*

$$\left(\int_\Omega (|\nabla w_\varepsilon(x,t)|^2 + |\partial_t w_\varepsilon(x,t)|^2)\, dx\right)^{1/2}$$
$$\begin{aligned}
&\leq C\{\|\mathcal{L}_\varepsilon(\varphi_{\varepsilon,0}) - \mathcal{L}_0(\varphi_0)\|_{H^{-1}(\Omega)} + \|\varphi_{\varepsilon,1} - \varphi_1\|_{L^2(\Omega)}\} \\
&\quad + C\varepsilon\{\|\nabla^2\varphi_0\|_{L^2(\Omega)} + \|\nabla\varphi_1\|_{L^2(\Omega)}\} \\
&\quad + C\varepsilon \sup_{t\in(0,T]} \|\nabla^2 u_0(\cdot,t)\|_{L^2(\Omega)} \\
&\quad + C\varepsilon T \sup_{t\in(0,T]} \big\| |\partial_t \nabla^2 u_0(\cdot,t)| + |\partial_t^2 \nabla u_0(\cdot,t)| \big\|_{L^2(\Omega)}, \qquad (0.3)
\end{aligned}$$

where C depends only on d and μ.

Note that in Theorem 0.1, we have replaced the condition $N \leq C_0 T^{-2/3}\varepsilon^{-2/3}$ of [1, Theorem 1.1] by a stronger condition $N \leq C_0 T^{-1}\varepsilon^{-1/2}$. In Theorem 0.2 we have replaced the last term on the right-hand side of (1.21) in [1, Theorem 1.2],

$$C\varepsilon\sqrt{T} \sup_{t\in(0,T]} \big\| |\partial_t \nabla^2 u_0(x,t)| + |\partial_t^2 \nabla u_0(\cdot,t)| \big\|_{L^2(\Omega)}^{1/2} \sup_{t\in(0,T]} \|\nabla^2 u_0(\cdot,t)\|_{L^2(\Omega)}^{1/2}$$

by

$$C\varepsilon T \sup_{t\in(0,T]} \big\| |\partial_t \nabla^2 u_0(\cdot,t)| + |\partial_t^2 \nabla u_0(\cdot,t)| \big\|_{L^2(\Omega)}.$$

The modified results in Theorems 0.1 and 0.2 are somewhat weaker than those stated in [1]. But the nature of the main conclusions is intact, and Theorem 0.1 remains to be the only result on the uniform observability for wave equations with oscillating coefficients $A(x/\varepsilon)$ in higher dimensions.

Let M_0 and M_1 be defined by (3.9) in [1] . To prove Theorem 0.2, we use Lemma 3.1 of [1] to show that for $0 \leq t \leq T$,

$$E_\varepsilon(t;\omega_\varepsilon) \leq E_\varepsilon(0;\omega_\varepsilon) + C\varepsilon(TM_1 + M_0) \sup_{t\in[0,T]} E_\varepsilon(t;\omega_\varepsilon)^{1/2},$$

where C depends only on d and μ. This yields

$$\begin{aligned}
\sup_{t\in[0,T]} E_\varepsilon(t;w_\varepsilon) &\leq E(0;w_\varepsilon) + C\varepsilon(TM_1 + M_0) \sup_{t\in[0,T]} E_\varepsilon(t;w_\varepsilon)^{1/2} \\
&\leq E_\varepsilon(0;w_\varepsilon) + C\varepsilon^2(T^2M_1^2 + M_0^2) + \frac{1}{2}\sup_{t\in[0,T]} E_\varepsilon(t;w_\varepsilon),
\end{aligned}$$

which, together with Lemma 3.2 of [1], gives the estimate (0.3).

Theorem 0.1 follows from Theorem 0.2 in the same manner as in [1] with a few obvious modifications:

- The second term on the right-hand side of (4.9) in [1, Lemma 4.2] should be replaced by

$$CT^3\varepsilon\{\|\varphi_0\|_{H^3(\Omega)}^2 + \|\varphi_1\|_{H^2(\Omega)}^2\}.$$

- The third term on the right-hand side of (4.12) in [1, Theorem 4.3] should be replaced by
$$CT^3\varepsilon\{\|\mathcal{L}_\varepsilon(\varphi_{\varepsilon,0})\|^2_{H^1(\Omega)} + \|\mathcal{L}_\varepsilon(\varphi_{\varepsilon,1})\|^2_{L^2(\Omega)}\}.$$
- The third term on the right-hand side of (4.16) in [1, Theorem 4.4] should be replaced by
$$CT^3\varepsilon\{\|\mathcal{L}_\varepsilon(\varphi_{\varepsilon,0})\|^2_{H^1(\Omega)} + \|\mathcal{L}_\varepsilon(\varphi_{\varepsilon,1})\|^2_{L^2(\Omega)}\}.$$
- In the proof of Theorem 1.1 in [1], the term $T\varepsilon N^{3/2}$ should be replaced by $T^2\varepsilon N^2$, which is small if $N \le CT^{-1}\varepsilon^{-1/2}$.

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References

[1] Lin, F., Shen, Z.: Uniform boundary controllability and homogenization of wave equations. J. Eur. Math. Soc. **24**, 3031–3053 (2022)