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Errata: A Correction to "The Completeness Theorems for Some Intuitionistic Logics in Terms of Interval Semantics"

By

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Reading my paper [1], Mr. Shin'ichi Yokota has taught me that the proof of Theorem 10 in it is wrong.

Hence I correct it as follows.

If $\nvdash_{IET} A$, then we have $M \not\models_x A$ for a counter IET-model $M = \langle W, N, \subseteq, R, V \rangle$ and an $x \in W$ by the completeness theorem.

We apply the filtration method to this IET-model M.

Let Φ_0 be the union of the set of all subformulas of A and $\{\Box T\}$, where T is a tautology. And we define the set Φ of formulas:

$$\Phi = \Phi_0 \cup \{\Box\Box T' \mid \Box T' \in \Phi_0 \text{ and } T' \text{ is a tautology}\}.$$

By the definition of Φ, it is clear that Φ is a finite set and that it has the property: If T' is a tautology and $\Box T' \in \Phi$, then $\Box\Box T' \in \Phi$. This is an important property to prove the decidability of the IET-system.

We shall define a filtration model M' of M through Φ.

For every $x, y \in W$, we define $x \equiv y$ when $M \models_x B$ iff $M \models_y B$ for every formula B in Φ.

$$[x] = \{y \in W \mid x \equiv y\}$$

is an equivalence class of x under $\equiv$, and we put

$$W' = \{[x] \mid x \in W\}.$$

For any $[x]$, $[y] \in W'$, we define N', $\subseteq'$, R', and V' one by one.

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$N' \ni [x]$ iff $M \models_x \Box T'$ for some $\Box T' \in \Phi$, where T' is a tautology

$[y] \subseteq' [x]$ iff if $M \models_x B$ then $M \models_y B$ for every formula $B \in \Phi$

$[x] R' [y]$ iff if $M \models_x \Box B$ then $M \models_y B$ for every formula of the form $\Box B \in \Phi$. And for every propositional variable $p \in \Phi$,

$$V'(p, [x]) = 1 \quad \text{iff } V(p, x) = 1.$$

Let M' be the structure $\langle W', N', \subseteq', R', V' \rangle$, which is called a filtration of M through Φ.

It is evident that these definitions are well-defined. We note that N' is not empty, because Φ has at least one element of the form $\Box T$, where T is a tautology.

For that structure M', we have to show that it is indeed an *IET*-model in our sence in [1]. We only show that it satisfies the condition (*IET*), that is, if $[x] \in N'$ and $[x] R' [y]$ then $[y] \in N'$.

Suppose that $[x] \in N'$ and $[x] R' [y]$. Since $[x] \in N'$, we have $M \models_x \Box T'$ for some $\Box T' \in \Phi$, where T' is a tautology. Since $\Box T' \rightarrow \Box\Box T'$ is provable in the *IET*-system, we obtain $M \models_x \Box\Box T'$. The assumption and the property of Φ yield that $M \models_y \Box T'$ and hence that $[y] \in N'$.

For these *IET*-models M and M', we shall establish the next lemma, which corresponds to Lemma 7 in [1].

Lemma. *For every $x \in W$ and formula $B \in \Phi$, we have that*

$$M \models_x B \text{ iff } M' \models_{[x]} B.$$

Proof. (by induction on the length of B) We only consider the case of $\Box B$.

For $\Box B$, suppose that $M' \models_{[x]} \Box B$ but $M \not\models_x \Box B$. Since $M' \models_{[x]} \Box B$, $[x]$ is in N'. Hence there exists a formula $\Box T$ in Φ such that $M \models_x \Box T$. Thus x is in N. The assumption $M \not\models_x \Box B$ means that there are $y \subseteq x$ and z such that yRz and $M \not\models_z B$. Clearly $[y] \subseteq' [x]$, $[y] R' [z]$. And I. H. (induction hypothesis) implies $M' \not\models_{[z]} B$. Hence we have $M' \not\models_{[x]} \Box B$. This contradicts our assumption.

Conversely, suppose that $M\models_x\Box B$ but $M'\not\models_{[x]}\Box B$. If $[x]$ is not in N', then $M\not\models_x\Box T$ for every formula $\Box T\in\Phi$, where T is a tautology. Since x is in N, there are $y\subseteq x$ and z such that yRz and $M\not\models_z T$. But this is a contradiction because T is a tautology. Therefore $[x]$ is in N'.

Since $M'\not\models_{[x]}\Box B$ and $[x]\in N'$, there are $[y]\subseteq'[x]$ and $[z]$ such that $[y]R'[z]$ and $M'\not\models_{[z]}B$. We have $M\not\models_z B$ by I. H.. The definitions of $\subseteq'$ and of R' imply $M\not\models_x\Box B$, but this is a contradiction.

This lemma is proved.

Using this lemma, we can prove that the *IET*-system has the finite model property (Theorem 10 in [1]).

Theorem. *The IET-system has the finite model property.*

I would like to express my gratitude to Mr. S. Yokota for his valuable suggestions.

Reference

[1] Kondo, M., The completeness theorems for some intuitionistic epistemic logics in terms of interval semantics, *Publ. RIMS, Kyoto Univ.*, **20** (1984), 671-681.