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Mixed Contact Problems in Plane Elasticity

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Der Beitrag behandelt das gemischte Kontaktproblem der isotropen elastischen Ebene mit einem einfach zusammenhängenden elastischen Einschluß aus anderem Material im Rahmen der Elastostatik. Die Kontaktkurve L ist zerlegt in 9 paarweise disjunkte Kurvensysteme, auf denen 9 verschiedene Kontaktbedingungen vorgeschrieben sind. Es werden verschiedene Regularitätskonzeptionen eingeführt (sogenannte *-Regularität, ε^*-Regularität und ε-Regularität), aus denen sich Forderungen an die Kontaktdaten ergeben, unter anderem gewisse Kompatibilitätsbedingungen. Unter Verwendung von Greenschen Formeln wird das Eindeutigkeitsproblem untersucht und in einigen Fällen Lösbarkeitsbedingungen angegeben. Mit Hilfe des elastischen Einfachschichtpotentials wird ein Integralgleichungssystem aufgestellt, in dessen rechte Seite zwei willkürliche Konstanten eingehen. Die gesuchte Lösung Φ des Integralgleichungssystems ist einer integralen Zusatzbedingung unterworfen. Durch Anwendung eines geeigneten Differentialoperators Ω_p wird ein singuläres Integralgleichungssystem mit stückweise stetigen Koeffizienten erhalten. Die Dimension des linearen Raumes ker Ω_p wird ermittelt. In einem weiteren Beitrag dieser Zeitschrift soll das Studium des formulierten Kontaktproblems durch die vollständige Untersuchung des singulären Integralgleichungssystems komplettiert werden.

Рассматривается контактная проблема упругой плоскости с одним односвязным включением из другого упругого материала в рамках эластостатики изотропных сред. Контактная кривая является объединением 9 непересекающихся систем кривых, вдоль которых задаются 9 различных контактных условий. Определяются некоторые понятия регулярности (так называемая *-регулярность, ε^*-регулярность и ε-регулярность), вследствие которых контактные данные должны удовлетворять некоторым требованиям, в том числе и некоторым условиям совместимости. Методом формул Грина изучается проблема единственности, а в некоторых частных случаях также выводятся необходимые условия разрешимости. С помощью упругого потенциала простого слоя контактная задача приводится к системе интегральных уравнений, в правой части которой содержится две произвольных постоянных. Решение этой системы интегральных уравнений подчиняется дополнительному интегральному условию. Посредством дифференциального оператора Ω_p получается система сингулярных интегральных уравнений с разрывными коэффициентами. Определяется размерность линейного пространства ker Ω_p. Заключительное изучение контактной проблемы полученной системы сингулярных интегральных уравнений будет проведено в последующей работе, опубликуемой в этом же журнале.

The paper is concerned with the contact problem of the isotropic elastic plane with a simply connected elastic inclusion of different material in the frame of elastostatics. The contact curve is dissected into 9 pairwise disjoint curve systems, at which 9 different contact conditions are prescribed. Some regularity concepts are defined (so-called *-regularity, ε^*-regularity and ε-regularity), which imply certain restrictions for the contact data, for instance certain compatibility conditions. Using Green formulas, the problem of uniqueness is studied and, in certain cases, some necessary conditions are given. By the aid of elastic potential of single layer, a system of integral equations is obtained, containing two arbitrary constants on the right-hand side. The solution Φ to be determined is subjected to an additional integral relation. By application of a suitable differential operator Ω_p, a system of singular integral equations with discontinuous coefficients is obtained. The dimension of the linear space $\ker \Omega_p$ is calculated. In a following note in this journal, the investigation of the contact problem will be continued by detailed study of the singular integral equation system.

§ 1 Introduction

The present paper is concerned with a class of plane mixed boundary value problems in linear elastostatics for bodies with inclusions of other elastic materials. On the common boundary curves of the inclusions and the environmental media, the displacement vector and the stresses must fulfil suitable relations, which depend on the actual physical kind of contact. In our paper, these relations are briefly referred to as contact conditions, in accordance with usual terminologies [9, 28, 17].

The considerations have been confined to the study of linear contact conditions in consequence of the singular integral equation method being used. However, we consider the case of mixed contact conditions, which as far as we know has not yet been studied in other papers, at least for general domains.

Problems of such kind have importance for some topics in mechanics. For instance, some problems of fracture mechanics can be interpreted as mixed contact problems.

The fundamental differential equations of plane elasticity in terms of displacements are given by

$$\Delta^* \mathbf{u} \equiv \mu \Delta \mathbf{u} + (\lambda + \mu) \operatorname{grad} \operatorname{div} \mathbf{u} = -\mathbf{F}, \tag{1.1}$$

$\mathbf{u} = (u_1, u_2) = \big(u_1(x_1, x_2), u_2(x_1, x_2)\big)$ — displacement vector field, λ, μ — Lamé modules, x_1, x_2 — Cartesian coordinates of the point $\mathbf{x}$ in the plane $\mathbf{R}^2$; $\mathbf{F}$ — vector field of volume forces. The modules λ, μ are supposed to be piecewise constant in the considered domains. Furthermore, we make the natural assumptions λ, $\mu > 0$.

Using the elastic volume potential [29, 17], a particular solution of equation (1.1) can be obtained by quadrature. Consequently, without loss of generality, we will assume $\mathbf{F} = \mathbf{0}$ in the sequel.

In some papers of L. Jentsch on contact problems of elasticity and thermoelasticity [6, 7, 10], the useful concept of contact fundamental solution (gekoppelte Grundlösungsmatrix) was established. This concept allows to solve in two steps a general boundary value problem for bodies with inclusions. First, a pure contact problem (i.e. a problem in the whole plane having inclusions but not having cavities) is considered, in order to construct the so-called contact fundamental solution. Secondly, this contact fundamental solution permits to study problems with boundary conditions at cavities in complete analogy to the elastic homogeneous case. In addition, that idea also leads itself to the treatment of problems with inclusions having inclusions and cavities themselves [25, 7].

Taking into account these results, the present paper deals with the pure contact problem only. For ease of exposition, we consider the elastic plane $\mathbf{R}^2$ with one inclusion of in general different material; the considerations might immediately be generalized to the case of n inclusions.

Let D_1 be a given simply connected bounded domain of $\mathbf{R}^2$ and $D_0 = \mathbf{R}^2 \setminus \overline{D}_1$. Let $L = \partial D_0 = \partial D_1$ and $L \in C^{2,\beta}$ $(0 < \beta \leqq 1)$. Suppose that D_0 and D_1 are occupied by two elastic bodies in their natural configuration. Let λ_0, μ_0 and λ_1, μ_1 be the values of λ, μ in the domains D_0 and D_1, respectively.

Let L be dissected into m pairwise disjoint non-empty single open curves $S_1, \ldots, S_m$ $(m \geqq 2)$, which are arranged in counter-clockwise sense on L, and let $L = \overline{S}_1 \cup \overline{S}_2 \cup \ldots \cup \overline{S}_m$. In the following, the common end points $\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_m$ of the curves S_m and

S_1, S_1 and $S_2, \ldots, S_{m-1}$ and S_m will be called *nodes*. At times, we will make use of the notations $S_1 = (\mathbf{a}_1, \mathbf{a}_2)$, $S_2 = (\mathbf{a}_2, \mathbf{a}_3), \ldots, S_m = (\mathbf{a}_m, \mathbf{a}_{m+1}) = (\mathbf{a}_m, \mathbf{a}_1)$ (the nodes $\mathbf{a}_i$, $\mathbf{a}_{i+1}$ do not belong to S_i).

Furthermore, let the set of the curves $S_1, \ldots, S_m$ be divided into 9 pairwise disjoint curve systems $L_1, L_2, \ldots, L_9$. Let

$$L_\nu = \bigcup_{\mu=1}^{m_\nu} S_{\nu_\mu} = \bigcup_{\mu=1}^{m_\nu} (\mathbf{a}_{\nu_\mu}, \mathbf{a}_{\nu_\mu+1}), \qquad \sum_{\nu=1}^{9} m_\nu = m, \qquad m_\nu \geqq 0 \quad (\nu = 1, 2, \ldots, 9). \tag{1.2}$$

Then we have $L = \bigcup_{i=1}^{m} S_i = \bigcup_{\nu=1}^{9} L_\nu$. Let, in addition, each of the nodes $\mathbf{a}_i$ $(i = 1, \ldots, m)$ be an encountering point of two different curve systems L_ν and $L_\varkappa$ $(\nu \neq \varkappa)$.

We consider the following contact problem: to determine two displacement fields $\mathbf{u}^k$ $(k = 0, 1)$ belonging to the classes $C^2(D_k) \cap C^0(\overline{D}_k)$ $(k = 0, 1)$, respectively, which solve the equations (1.1)

$$\Delta^* \mathbf{u}^k = \mathbf{0} \qquad (k = 0, 1) \tag{1.3}$$

in the domains D_0, D_1, respectively. The first partial derivatives $\frac{\partial \mathbf{u}^k}{\partial x_j}$ $(j = 1, 2)$ are required to be continuous in the points of L with exception of the nodes $\mathbf{a}_i$ $(i = 1, \ldots, m)$. Furthermore, the displacements $\mathbf{u}^k$ and the stresses $\mathcal{T}(\mathbf{n})\, \mathbf{u}^k$ are supposed to satisfy the following contact conditions on L:

$$\mathbf{u}^1(\mathbf{z}) - \mathbf{u}^0(\mathbf{z}) = \mathbf{f}(\mathbf{z}), \qquad \mathcal{T}(\mathbf{n})\, \mathbf{u}^1(\mathbf{z}) - \mathcal{T}(\mathbf{n})\, \mathbf{u}^0(\mathbf{z}) = \mathbf{g}(\mathbf{z}) \quad \text{for} \quad \mathbf{z} \in L_1; \tag{1.4a}$$

$$\begin{aligned} &\mathbf{s} \cdot \mathcal{T}(\mathbf{n})\, \mathbf{u}^k(\mathbf{z}) = h_k(\mathbf{z}) \ \ (k = 0, 1), \qquad \mathbf{n} \cdot \big(\mathbf{u}^1(\mathbf{z}) - \mathbf{u}^0(\mathbf{z})\big) = f(\mathbf{z}), \\ &\mathbf{n} \cdot \big(\mathcal{T}(\mathbf{n})\, \mathbf{u}^1(\mathbf{z}) - \mathcal{T}(\mathbf{n})\, \mathbf{u}^0(\mathbf{z})\big) = g(\mathbf{z}) \quad \text{for} \quad \mathbf{z} \in L_2; \end{aligned} \tag{1.4b}$$

$$\begin{aligned} &\mathbf{s} \cdot \mathbf{u}^k(\mathbf{z}) = l_k(\mathbf{z}) \ \ (k = 0, 1), \qquad \mathbf{n} \cdot \big(\mathbf{u}^1(\mathbf{z}) - \mathbf{u}^0(\mathbf{z})\big) = f(\mathbf{z}), \\ &\mathbf{n} \cdot \big(\mathcal{T}(\mathbf{n})\, \mathbf{u}^1(\mathbf{z}) - \mathcal{T}(\mathbf{n})\, \mathbf{u}^0(\mathbf{z})\big) = g(\mathbf{z}) \quad \text{for} \quad \mathbf{z} \in L_3; \end{aligned} \tag{1.4c}$$

$$\begin{aligned} &\mathbf{n} \cdot \mathbf{u}^k(\mathbf{z}) = l_k(\mathbf{z}) \ \ (k = 0, 1), \qquad \mathbf{s} \cdot \big(\mathbf{u}^1(\mathbf{z}) - \mathbf{u}^0(\mathbf{z})\big) = f(\mathbf{z}), \\ &\mathbf{s} \cdot \big(\mathcal{T}(\mathbf{n})\, \mathbf{u}^1(\mathbf{z}) - \mathcal{T}(\mathbf{n})\, \mathbf{u}^0(\mathbf{z})\big) = g(\mathbf{z}) \quad \text{for} \quad \mathbf{z} \in L_4; \end{aligned} \tag{1.4d}$$

$$\begin{aligned} &\mathbf{n} \cdot \mathcal{T}(\mathbf{n})\, \mathbf{u}^k(\mathbf{z}) = h_k(\mathbf{z}) \ \ (k = 0, 1), \qquad \mathbf{s} \cdot \big(\mathbf{u}^1(\mathbf{z}) - \mathbf{u}^0(\mathbf{z})\big) = f(\mathbf{z}), \\ &\mathbf{s} \cdot \big(\mathcal{T}(\mathbf{n})\, \mathbf{u}^1(\mathbf{z}) - \mathcal{T}(\mathbf{n})\, \mathbf{u}^0(\mathbf{z})\big) = g(\mathbf{z}) \quad \text{for} \quad \mathbf{z} \in L_5; \end{aligned} \tag{1.4e}$$

$$\mathbf{u}^k(\mathbf{z}) = \mathbf{f}_k(\mathbf{z}) \quad (k = 0, 1) \quad \text{for} \quad \mathbf{z} \in L_6; \tag{1.4f}$$

$$\mathcal{T}(\mathbf{n})\, \mathbf{u}^k(\mathbf{z}) = \mathbf{g}_k(\mathbf{z}) \quad (k = 0, 1) \quad \text{for} \quad \mathbf{z} \in L_7; \tag{1.4g}$$

$$\mathbf{s} \cdot \mathbf{u}^k(\mathbf{z}) = f_k(\mathbf{z}), \qquad \mathbf{n} \cdot \mathcal{T}(\mathbf{n})\, \mathbf{u}^k(\mathbf{z}) = g_k(\mathbf{z}) \quad (k = 0, 1) \quad \text{for} \quad \mathbf{z} \in L_8; \tag{1.4h}$$

$$\mathbf{n} \cdot \mathbf{u}^k(\mathbf{z}) = f_k(\mathbf{z}), \qquad \mathbf{s} \cdot \mathcal{T}(\mathbf{n})\, \mathbf{u}^k(\mathbf{z}) = g_k(\mathbf{z}) \quad (k = 0, 1) \quad \text{for} \quad \mathbf{z} \in L_9. \tag{1.4i}$$

In this formulas $\mathbf{n}$, $\mathbf{s}$ mean the unit vectors of the (outward) normal and tangent of L, respectively. $\mathcal{T}(\mathbf{n})$ is the operator of stresses given by

$$\mathcal{T}(\mathbf{n})\, \mathbf{u} = 2\mu \frac{\partial \mathbf{u}}{\partial \mathbf{n}} + \lambda \mathbf{n} \operatorname{div} \mathbf{u} + \mu \mathbf{n} \times \operatorname{rot} \mathbf{u}. \tag{1.5}$$

Of course, the stresses $\mathcal{T}(\mathbf{n})\, \mathbf{u}^0$ and $\mathcal{T}(\mathbf{n})\, \mathbf{u}^1$ in the expressions (1.4a)—(1.4i) must be calculated with the Lamé modules λ_0, μ_0 and λ_1, μ_1, respectively. The quantities

$\mathfrak{f}_1, f, \mathfrak{f}_k, f_k, \mathfrak{g}, g_1, \mathfrak{g}_k, g_k, h_k, l_k$ $(k = 0, 1)$ denote certain vector fields or functions, defined on corresponding parts of L and satisfying suitable properties of smoothness discussed later.

In consequence of the continuity of the displacements one gets immediately some compatibility conditions for the data $\mathfrak{f}$, f, $\mathfrak{f}_k$, f_k and l_k.

Indeed, in the nodes $\mathbf{a}_i$ of the type $L_1 - L_2$ the equation

$$\lim_{L_1 \ni z \to a_i} \mathfrak{n}(z) \cdot \mathfrak{f}(z) = \lim_{L_2 \ni z \to a_i} f(z) \tag{1.6a}$$

is necessary. Further the following relationships must be taken into account:

for $\mathbf{a}_i$ of the type $L_1 - L_3$: $\lim\limits_{L_1 \ni z \to a_i} \mathfrak{f}(z) = \lim\limits_{L_3 \ni z \to a_i} \{(l_1(z) - l_0(z))\, \mathfrak{s}(z) + f(z)\, \mathfrak{n}(z)\}$. (1.6b)

of the type $L_1 - L_4$: $\lim\limits_{L_1 \ni z \to a_i} \mathfrak{f}(z) = \lim\limits_{L_4 \ni z \to a_i} \{f(z)\, \mathfrak{s}(z) + (l_1(z) - l_0(z))\, \mathfrak{n}(z)\}$, (1.6c)

of the type $L_1 - L_5$: $\lim\limits_{L_1 \ni z \to a_i} \mathfrak{s}(z) \cdot \mathfrak{f}(z) = \lim\limits_{L_5 \ni z \to a_i} f(z)$, (1.6d)

of the type $L_1 - L_6$: $\lim\limits_{L_1 \ni z \to a_i} \mathfrak{f}(z) = \lim\limits_{L_6 \ni z \to a_i} (\mathfrak{f}_1(z) - \mathfrak{f}_0(z))$, (1.6e)

of the type $L_1 - L_8$: $\lim\limits_{L_1 \ni z \to a_i} \mathfrak{s}(z) \cdot \mathfrak{f}(z) = \lim\limits_{L_8 \ni z \to a_i} (f_1(z) - f_0(z))$, (1.6f)

of the type $L_1 - L_9$: $\lim\limits_{L_1 \ni z \to a_i} \mathfrak{n}(z) \cdot \mathfrak{f}(z) = \lim\limits_{L_9 \ni z \to a_i} (f_1(z) - f_0(z))$, (1.6g)

of the type $L_2 - L_3$: $\lim\limits_{L_2 \ni z \to a_i} f(z) = \lim\limits_{L_3 \ni z \to a_i} f(z)$, (1.6h)

of the type $L_2 - L_4$: $\lim\limits_{L_2 \ni z \to a_i} f(z) = \lim\limits_{L_4 \ni z \to a_i} (l_1(z) - l_0(z))$, (1.6i)

of the type $L_2 - L_6$: $\lim\limits_{L_2 \ni z \to a_i} f(z) = \lim\limits_{L_6 \ni z \to a_i} \mathfrak{n}(z) \cdot (\mathfrak{f}_1(z) - \mathfrak{f}_0(z))$, (1.6j)

of the type $L_2 - L_9$: $\lim\limits_{L_2 \ni z \to a_i} f(z) = \lim\limits_{L_9 \ni z \to a_i} (f_1(z) - f_0(z))$, (1.6k)

of the type $L_3 - L_4$:

$$\lim_{L_3 \ni z \to a_i} \{(l_1(z) - l_0(z))\, \mathfrak{s}(z) + f(z)\, \mathfrak{n}(z)\} = \lim_{L_4 \ni z \to a_i} \{f(z)\, \mathfrak{s}(z) + (l_1(z) - l_0(z))\, \mathfrak{n}(z)\}, \tag{1.6l}$$

of the type $L_3 - L_5$: $\lim\limits_{L_3 \ni z \to a_i} (l_1(z) - l_0(z)) = \lim\limits_{L_5 \ni z \to a_i} f(z)$, (1.6m)

of the type $L_3 - L_6$:

$$\lim_{L_3 \ni z \to a_i} l_k(z) = \lim_{L_6 \ni z \to a_i} \mathfrak{s}(z) \cdot \mathfrak{f}_k(z) \quad (k = 0, 1) \quad \text{and} \quad \lim_{L_3 \ni z \to a_i} f(z) = \lim_{L_6 \ni z \to a_i} \mathfrak{n}(z) \cdot (\mathfrak{f}_1(z) - \mathfrak{f}_0(z)), \tag{1.6n}$$

of the type $L_3 - L_8$: $\lim\limits_{L_3 \ni z \to a_i} l_k(z) = \lim\limits_{L_8 \ni z \to a_i} f_k(z) \quad (k = 0, 1)$, (1.6o)

of the type $L_3 - L_9$: $\lim\limits_{L_3 \ni z \to a_i} f(z) = \lim\limits_{L_9 \ni z \to a_i} (f_1(z) - f_0(z))$, (1.6p)

of the type $L_4 - L_5$: $\lim\limits_{L_4 \ni z \to a_i} f(z) = \lim\limits_{L_5 \ni z \to a_i} f(z)$, (1.6q)

of the type $L_4 - L_6$:

$$\lim_{L_4 \ni z \to a_i} l_k(z) = \lim_{L_6 \ni z \to a_i} \mathfrak{n}(z) \cdot \mathfrak{f}_k(z) \quad (k = 0, 1) \quad \text{and} \quad \lim_{L_4 \ni z \to a_i} f(z) = \lim_{L_6 \ni z \to a_i} \mathfrak{s}(z) \cdot (\mathfrak{f}_1(z) - \mathfrak{f}_0(z)), \tag{1.6r}$$

of the type $L_4 - L_8$: $\lim\limits_{L_4 \ni z \to a_i} f(z) = \lim\limits_{L_8 \ni z \to a_i} (f_1(z) - f_0(z))$, (1.6s) of the type $L_4 - L_9$: $\lim\limits_{L_i \ni z \to a_i} l_k(z) = \lim\limits_{L_i \ni z \to a_i} f_k(z) \quad (k = 0, 1)$, (1.6t)

of the type $L_5 - L_6$: $\lim\limits_{L_i \ni z \to a_i} f(z) = \lim\limits_{L_i \ni z \to a_i} s(z) \cdot (\mathfrak{f}_1(z) - \mathfrak{f}_0(z))$, (1.6u)

of the type $L_5 - L_8$: $\lim\limits_{L_i \ni z \to a_i} f(z) = \lim\limits_{L_i \ni z \to a_i} (f_1(z) - f_0(z))$, (1.6v)

of the type $L_6 - L_8$: $\lim\limits_{L_i \ni z \to a_i} s(z) \cdot \mathfrak{f}_k(z) = \lim\limits_{L_i \ni z \to a_i} f_k(z) \quad (k = 0, 1)$, (1.6w)

of the type $L_6 - L_9$: $\lim\limits_{L_i \ni z \to a_i} \mathbf{n}(z) \cdot \mathfrak{f}_k(z) = \lim\limits_{L_i \ni z \to a_i} f_k(z) \quad (k = 0, 1)$. (1.6x)

Some of the contact conditions (1.4a)—(1.4i) have an obvious mechanical meaning. For instance, condition (1.4a) expresses for $\mathfrak{f} = \mathfrak{g} = 0$ that the materials are welded along L_1. (1.4a) with $\mathfrak{f} \neq 0$, $\mathfrak{g} = 0$ describes the welding of the two parts D_0 and D_1 in the frame of linear theory if, in the natural configuration, the two boundaries diverge a little from the curve L. (1.4a) with $\mathfrak{f} = 0$, $\mathfrak{g} \neq 0$ can be interpreted as welding of the materials with initial stresses (e.g. thermal stresses).

The meaning of (1.4f)—(1.4i) is evident. Such boundary conditions at inner curves of elastic bodies are of importance in crack problems.

The conditions (1.4b)—(1.4e) are also interesting. (1.4b) implies for $h_k = f = g = 0$ the frictionless sliding of the homogeneous parts without gap along L_2. In principle, inhomogeneities of the data can be explained as initial stresses or as divergence of the boundary of D_0, D_1 in the natural configuration.

The mathematical treatment of (1.4b) was suggested by JENTSCH [5]. In the non-mixed plane case the conditions (1.4b)—(1.4e) have been completely studied in the author's book [29] by the method of potential of single layer. The corresponding spatial problem (1.4b) has been treated by BECKERT and JENTSCH in [1] and [8], respectively, with variational methods. The integral equation approach was established by JENTSCH for (1.4b) [9] and for other relationships also in the spatial case (see [9—11]). The connection of (1.4b) with a more general problem of Signorini type is discussed in [9]. Further non-mixed contact problems in the plane have been studied in [29].

Two special mixed contact problems with the conditions (1.4a) and (1.4b) and, on the other hand, (1.4a) and (1.4h) have been investigated in the dissertation B [30] of the author. The present paper is based on the considerations in [26—30].

It should still be remarked that similarly general boundary value problems of thermoelasticity and micropolar elasticity (homogeneous media) are treated in [28].

In the following considerations a further notation is necessary. Let $A_{\nu\mu}$ ($\nu, \mu = 1, \ldots, 9$) be the number of nodes of the type L_ν—L_μ (it does not characterize the order of L_ν and L_μ). Then we have

$$A_{\nu\nu} = 0, \qquad A_{\nu\mu} = A_{\mu\nu} \quad \text{and} \quad \sum_{\mu=1}^{9}{}' A_{\nu\mu} = 2m, \qquad (\nu = 1, \ldots, 9). \tag{1.7}$$

§ 2 Rigorous statement of the contact problem. Integral theorems

In this paper the points $\mathbf{x}, \mathbf{y}, \mathbf{z}, \ldots$ of the plane $\mathbf{R}^2$ are sometimes identified with complex numbers $t, \tau, \ldots$ In general we apply the notations of singular integral equation theory for functions defined on smooth curves [35, 38].

For instance, a Hölder-continuous complex function φ on L (L smooth curve) with Hölder exponent α, $0 < \alpha \leqq 1$, is called a *function of the class H* (also $H_\alpha(L)$ or $C^{0,\alpha}(L)$). The Hölder condition can refer to the variable point of L or to the arc length of L. Both points of view are equivalent. The class of n times Hölder-continuously differentiable (with respect to the arc length s) functions is denoted by $C^{n,\alpha}(L)$.

In addition, we make use of some function classes on L which are defined one-valued on the curves $S_1, \ldots, S_m$, but not, in general, in the nodes $\mathbf{a}_1, \ldots, \mathbf{a}_m$. Such a function φ belongs to the class H_0, if $\varphi \in H_\alpha(S_i)$ $(i = 1, \ldots, m)$ with a suitable constant $\alpha > 0$. Then the one-sided limits of φ in the nodes $\mathbf{a}_i$ exist. If the function φ satisfies a Hölder condition only on every closed subcurve $[\mathbf{a}_i', \mathbf{a}_i'']$ of $(\mathbf{a}_i, \mathbf{a}_{i+1})$ and, moreover, a formula

$$\varphi(t) = \frac{\varphi_0(t)}{(t - \mathbf{a}_i)^\gamma}, \qquad \varphi_0 \in H_0, \qquad 0 < \operatorname{Re} \gamma < 1 \quad (i = 1, \ldots, m) \tag{2.1}$$

is valid in a neighbourhood of each node $\mathbf{a}_i$, the function φ belongs to the class H^*. If, additionally, a representation (2.1) holds with a constant γ having an arbitrarily small real part $\operatorname{Re} \gamma = \delta > 0$, then the function φ belongs to the class H_ε.

Now let again $L \in C^{2,\beta}$ $(0 < \beta \leqq 1)$ and let D be the bounded domain with $\partial D = L$. We consider a displacement field $\mathbf{u}(\mathbf{x}) = \big(u_1(\mathbf{x}), u_2(\mathbf{x})\big)$ defined for $\mathbf{x} \in D$. The displacement field $\mathbf{u}$ is called *regular* if

$$u_i \in C^2(D) \cap C^1(\overline{D}) \quad (i = 1, 2). \tag{2.2}$$

In connection with mixed problems, having in the sense of § 1 certain nodes $\mathbf{a}_1, \ldots, \mathbf{a}_m$ further regularity conceptions are necessary. Let $D_\varepsilon = D \setminus \bigcup_{i=1}^{m} K_\varepsilon(\mathbf{a}_i)$ with $K_\varepsilon(\mathbf{a}_i) = \{t \in \mathbf{C} \mid |t - a_i| \leqq \varepsilon\}$. The displacement field $\mathbf{u}$ is called **-regular* if

$$u_i \in C^2(D) \cap C^0(\overline{D}) \cap C^1(\overline{D}_\varepsilon) \tag{2.3}$$

for sufficiently small $\varepsilon > 0$ and if the in neighbourhood of the nodes $\mathbf{a}_i$ the estimates

$$\left|\frac{\partial u_i}{\partial x_k}\right| = \mathcal{O}(|\mathbf{x} - \mathbf{a}_i|^{-\delta}) \tag{2.4}$$

are valid for a fixed δ, $0 < \delta < 1$, and $i, k = 1, 2$.

$\mathbf{u}$ is called *ε-regular* if $\mathbf{u}$ is *-regular and the estimates (2.4) hold for every $\delta > 0$. $\mathbf{u}$ is called *ε*-regular* if $\mathbf{u}$ is *-regular and satisfies the estimate (2.4) with every $\delta > 0$ for certain (but in general not for all) nodes $\mathbf{a}_i$.

To define suitable regularity conceptions for the case of the unbounded domain D with $\partial D = L$ (L is located in a boundad part of $\mathbf{R}^2$ and has the above-mentioned properties), additional conditions for large $|\mathbf{x}|$ must be required. We demand

$$|u_i(\mathbf{x})| = \mathcal{O}(1) \quad \text{and} \quad \left|\frac{\partial u_i}{\partial x_k}\right| = \mathcal{O}(|\mathbf{x}|^{-1-\eta}), \qquad \eta > 0 \tag{2.5}$$

for large $|\mathbf{x}|$. Now a solution $\mathbf{u}$ of the homogeneous equation (1.1) is called regular, *-regular, ε-regular or ε*-regular if $\mathbf{u}$, besides the above-mentioned properties, satisfies condition (2.5).

The just defined regularity conceptions allow the rigorous statement of the general contact problem of § 1. By the problems C^*, C_ε and C_ε^* we agree to understand the problem (1.2), (1.4a)—(1.4i), (1.6a)—(1.6x) stated in § 1) in the class of *-regular, ε-regular and ε*-regular displacement fields, respectively. Of course, for ε-*regular vectors the set of nodes $\mathbf{a}_i$ must be specified, in the neighbourhood of which the estimates (2.4) hold with arbitrary $\delta > 0$.

The given data are assumed to satisfy the following additional restrictions:

a) $\mathbf{f}, f, \mathbf{f}_k, f_k, l_k \in H$ on the corresponding curves S_i,

b) $\dfrac{d}{ds}\mathbf{f},\ \dfrac{d}{ds}f,\ \dfrac{d}{ds}\mathbf{f}_k,\ \dfrac{d}{ds}f_k,\ \dfrac{d}{ds}l_k;\ \mathbf{g}, g, \mathbf{g}_k, g_k, h_k \in H_\varepsilon \quad (k = 0, 1).$

The necessity of these assumptions follows from the integral equation method being implemented.

Now let D be bounded or unbounded with $\partial D \in C^1$, $\mathbf{u}$ a regular solution of $\Delta^*\mathbf{u} = 0$ and $\mathbf{v}$ an arbitrary regular vector field. Then we have the following well-known integral theorem

$$\int_D E(\mathbf{u}, \mathbf{v})\, d\mathbf{x} = \int_{\partial D} \mathbf{v} \cdot \mathscr{T}(\mathbf{n})\, \mathbf{u}\, ds, \tag{2.6}$$

where $\mathbf{n}$ is the outward normal, and

$$E(\mathbf{u}, \mathbf{v}) = \frac{\mu}{2} \sum_{i,j=1}^{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) + \lambda \left(\sum_{j=1}^{2} \frac{\partial u_j}{\partial x_j}\right)\left(\sum_{j=1}^{2} \frac{\partial v_j}{\partial x_j}\right). \tag{2.7}$$

$E(\mathbf{u}, \mathbf{v})$ is a symmetric bilinear form. The positiveness of the corresponding quadratic form is evident. Obviously, the formula (2.6) remains valid for *-regular vectors $\mathbf{u}$, $\mathbf{v}$. For proof one can apply formula (2.6) in the domain D_ε. In virtue of (2.4) the proposition is obtained for $\varepsilon \to 0$. The symmetric relation

$$\int_D \{\mathbf{u} \cdot \Delta^*\mathbf{v} - \mathbf{v} \cdot \Delta^*\mathbf{u}\}\, d\mathbf{x} = \int_{\partial D} \{\mathbf{u} \cdot \mathscr{T}(\mathbf{n})\, \mathbf{v} - \mathbf{v} \cdot \mathscr{T}(\mathbf{n})\, \mathbf{u}\}\, ds \tag{2.8}$$

holds for arbitrary *-regular displacement vector fields, provided that $\Delta^*\mathbf{u}$ and $\Delta^*\mathbf{v}$ are summable in D.

Let D be a bounded domain and $\mathbf{u}$ a given regular (*-regular) vector field. Then from $E(\mathbf{u}, \mathbf{u}) = 0$ in D we can conclude by simple arguments that $\mathbf{u}$ belongs to the linear space generated by the three vectors

$$\mathbf{c}^1 = (1, 0), \qquad \mathbf{c}^2 = (0, 1), \qquad \mathbf{c}^3 = (-x_2, x_1). \tag{2.9}$$

Under the same assumptions the vector $\mathbf{u}$ in an unbounded domain must be a linear combination of $\mathbf{c}^1$ and $\mathbf{c}^2$.

§ 3 Uniqueness theorem

The uniqueness of the considered contact problems C^*, C_ε and C_ε^* is determined by the corresponding homogeneous contact problems C^*, C_ε and C_ε^* allowing nontrivial solutions, or not. Therefore in the sequel we are concerned with the homogeneous contact problems only. First we deal with the homogeneous problem C^*.

The considerations turn out by the following general pattern. Let $\mathbf{u}^0$, $\mathbf{u}^1$ be *-regular solutions of the homogeneous problem C^*. Substituting $\mathbf{u} = \mathbf{v} = \mathbf{u}^k$ $(k = 0, 1)$ into formula (2.6) we obtain

$$\int_{D_0} E(\mathbf{u}^0, \mathbf{u}^0)\, d\mathbf{x} = \int_L \mathbf{u}^0 \cdot \mathscr{T}(\tilde{\mathbf{n}})\, \mathbf{u}^0\, ds, \tag{*}$$

$$\int_{D_1} E(\mathbf{u}^1, \mathbf{u}^1)\, d\mathbf{x} = \int_L \mathbf{u}^1 \cdot \mathscr{T}(\mathbf{n})\, \mathbf{u}^1\, ds. \tag{**}$$

$\tilde{\mathbf{n}}$ in $(*)$ is the inside normal with respect to L, but $\mathbf{n}$ in $(**)$ is the outward normal. By replacing $\tilde{\mathbf{n}}$ by $\mathbf{n}$ in $(*)$, the sign of the line integral is altered. Summing $(*)$ and $(**)$ we get

$$\int_{D_0} E(\mathbf{u}^0, \mathbf{u}^0)\, d\mathbf{x} + \int_{D_1} E(\mathbf{u}^1, \mathbf{u}^1)\, d\mathbf{x} = \int_L [\mathbf{u}^1 \cdot \mathscr{T}(\mathbf{n})\, \mathbf{u}^1 - \mathbf{u}^0 \cdot \mathscr{T}(\mathbf{n})\, \mathbf{u}^0]\, ds. \tag{3.1}$$

On account of the homogeneous contact conditions it is not difficult to see that the expression in the square brackets vanishes on each of the curve systems $L_1, \ldots, L_9$. Therefore we have

$$\int_{D_0} E(\mathbf{u}^0, \mathbf{u}^0)\, dx + \int_{D_1} E(\mathbf{u}^1, \mathbf{u}^1)\, dx = 0.$$

In consequence of the positiveness of $E(\mathbf{u}, \mathbf{u})$ it follows that $E(\mathbf{u}^0, \mathbf{u}^0) = E(\mathbf{u}^1, \mathbf{u}^1) = 0$. Bearing in mind § 2, one can deduce

$$\mathbf{u}^0 \in \mathfrak{L}\{\mathbf{c}^1, \mathbf{c}^2\}, \qquad \mathbf{u}^1 \in \mathfrak{L}\{\mathbf{c}^1, \mathbf{c}^2, \mathbf{c}^3\}; \tag{3.2}$$

the symbol $\mathfrak{L}\{\ldots\}$ marks the linear space generated by the vectors in brackets.

We have still to check, which of the vectors (3.2) satisfy the homogeneous contact conditions of the problem C^*. For that reason we first discuss each of the contact conditions (1.4a), ..., (1.4i) as independent of the other ones. For this purpose we make use of the relation

$$\mathcal{T}(\mathbf{n})\, \mathbf{c} = \mathbf{0} \quad \text{for every } \mathbf{n} \text{ and for } \mathbf{c} \in \mathfrak{L}\{\mathbf{c}^1, \mathbf{c}^2, \mathbf{c}^3\},$$

which is easily verified. Thus, the homogeneous contact condition (1.4a) allows only the solutions

$$\mathbf{u}^0 = \mathbf{u}^1 = \mathbf{c} \in \mathfrak{L}\{\mathbf{c}^1, \mathbf{c}^2\}. \tag{3.3a}$$

The vectors (3.3a) also satisfy the conditions (1.4b) Additionally, if L consists only of such single curves which are located on circles with fixed centre $\mathbf{y} = (y_1, y_2)$, then the vectors

$$\mathbf{u}^0 = \mathbf{0}, \qquad \mathbf{u}^1 \in \mathfrak{L}\{\mathbf{c}_y{}^3\}, \tag{3.3b}$$

where

$$\mathbf{c}_y{}^3(\mathbf{x}) = (-x_2 + y_2, x_1 - y_1), \tag{3.4}$$

fulfil the homogeneous conditions (1.4b).

A) Solutions that are independent of geometrical shape:

contact condition	solutions of homogeneous cont. conditions
(1.4a)	$\mathbf{u}^0 = \mathbf{u}^1 = \mathbf{c} \in \mathfrak{L}\{\mathbf{c}^1, \mathbf{c}^2\}$
(1.4b)	$\mathbf{u}^0 = \mathbf{u}^1 = \mathbf{c} \in \mathfrak{L}\{\mathbf{c}^1, \mathbf{c}^2\}$
(1.4c)	$\mathbf{u}^0 = \mathbf{u}^1 = \mathbf{0}$
(1.4d)	$\mathbf{u}^0 = \mathbf{u}^1 = \mathbf{0}$
(1.4e)	$\mathbf{u}^0 = \mathbf{u}^1 = \mathbf{c} \in \mathfrak{L}\{\mathbf{c}^1, \mathbf{c}^2\}$
(1.4f)	$\mathbf{u}^0 = \mathbf{u}^1 = \mathbf{0}$
(1.4g)	$\mathbf{u}^0 \in \mathfrak{L}\{\mathbf{c}^1, \mathbf{c}^2\}, \quad \mathbf{u}^1 \in \mathfrak{L}\{\mathbf{c}^1, \mathbf{c}^2, \mathbf{c}^3\}$
(1.4h)	$\mathbf{u}^0 = \mathbf{u}^1 = \mathbf{0}$
(1.4i)	$\mathbf{u}^0 = \mathbf{u}^1 = \mathbf{0}$

In a similar way, we can also consider the contact conditions (1.4c)—(1.4i). Doing this, one gets both such solutions which are independent of the geometrical shape of the corresponding curve system L_i (e.g. the vectors (3.3a)), and other ones, which are only met for special geometrical shape of L_i (e.g. the vectors (3.3b)). For lucidity of exposition, the possible solutions are listed in the tables A) and B).

B) Additional solutions for special geometrical shape of L_i:

Ba) L_i consists of parts of circles with a common centre $\mathbf{y} = (y_1, y_2)$

contact condition	additional solutions
(1.4b)	$\mathbf{u}^0 = \mathbf{0}, \quad \mathbf{u}^1 \in \mathfrak{L}\{\mathbf{c}_y{}^3\}$
(1.4i)	$\mathbf{u}^0 = \mathbf{0}, \quad \mathbf{u}^1 \in \mathfrak{L}\{\mathbf{c}_y{}^3\}$

Bb) L_i consists of parts of straight lines with one and the same direction

$$\mathbf{c}_\omega = \cos \omega \mathbf{c}^1 + \sin \omega \mathbf{c}^2. \tag{3.5}$$

Moreover, we define

$$\mathbf{c}_\omega{}^\perp = -\sin \omega \mathbf{c}^1 + \cos \omega \mathbf{c}^2. \tag{3.6}$$

contact condition	additional solutions
(1.4b)	$\mathbf{u}^0 = \mathbf{0}, \quad \mathbf{u}^1 \in \mathfrak{L}\{\mathbf{c}_\omega\}$
(1.4c)	$\mathbf{u}^0 = \mathbf{u}^1 = \mathbf{c} \in \mathfrak{L}\{\mathbf{c}_\omega{}^\perp\}$
(1.4d)	$\mathbf{u}^0 = \mathbf{u}^1 = \mathbf{c} \in \mathfrak{L}\{\mathbf{c}_\omega\}$
(1.4e)	$\mathbf{u}^0 = \mathbf{0}, \quad \mathbf{u}^1 \in \mathfrak{L}\{\mathbf{c}_\omega{}^\perp\}$
(1.4h)	$\mathbf{u}^0 = \mathbf{c} \in \mathfrak{L}\{\mathbf{c}_\omega{}^\perp\}, \quad \mathbf{u}^1 = \mathbf{d} \in \mathfrak{L}\{\mathbf{c}_\omega{}^\perp\}$
(1.4i)	$\mathbf{u}^0 = \mathbf{c} \in \mathfrak{L}\{\mathbf{c}_\omega\}, \quad \mathbf{u}^1 = \mathbf{d} \in \mathfrak{L}\{\mathbf{c}_\omega\}$

Bc) L_i consists of straight lines, which intersect in the finite point $\mathbf{v} = (v_1, v_2)$

contact condition	additional solutions
(1.4e)	$\mathbf{u}^0 = \mathbf{0}, \quad \mathbf{u}^1 = \mathbf{c} \in \mathfrak{L}\{\mathbf{c}_v{}^3\}$
(1.4h)	$\mathbf{u}^0 = \mathbf{0}, \quad \mathbf{u}^1 = \mathbf{c} \in \mathfrak{L}\{\mathbf{c}_v{}^3\}$

For a given real situation, the solutions of the homogeneous problem C^* are easy to determine. For this purpose, the intersection of vectors satisfying the homogeneous contact condition at the L_i ($i = 1, \ldots, 9$) (see the above stated tables) has to be defined. Because of the multiplicity of possible cases we do not try to give a complete specification of the last ones. Instead of that, only a few interesting examples shall be considered.

For instance, the homogeneous problem C^* allows only the trivial solution, if one of the following assumptions holds:

1. L_6 is not empty.
2. $L_3 \cup L_4 \cup L_8$ is not empty and does not consist of parts of straight lines with one and the same direction.

The dimension of the linear space of solutions of the homogeneous contact problem C^* is equal to one if for example $L = L_1 \cup L_4 \cup L_7$ holds, provided that L_4 consists of parts of straight lines with direction $\mathbf{c}_\omega$. The general solution of the homogeneous problem C^* in that case is $\mathbf{u}^0 = \mathbf{u}^1 \in \mathfrak{L}\{\mathbf{c}_\omega\}$ (see fig. 1).

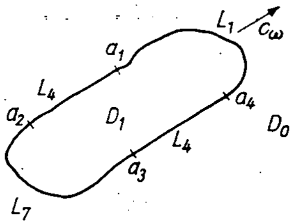

Fig. 1

A further interesting example of dimension one is $L = L_2 \cup L_5 \cup L_7$ with the solution $\mathbf{u}^0 = \mathbf{0}$, $\mathbf{u}^1 = \mathbf{c} \in \mathfrak{L}\{\mathbf{c}_\mathbf{y}{}^3\}$, provided that the shape of L_2 and L_5, e.g., is that of figure 2 (L_2 are circular arcs with the centre in $\mathbf{y}$ and L_5 are parts of straight lines intersecting in $\mathbf{y}$).

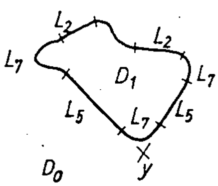

Fig. 2

The homogeneous problem C^* has exactly the two linear independent solutions $\mathbf{u}^0 = \mathbf{u}^1 = \mathbf{c} \in \mathfrak{L}\{\mathbf{c}^1, \mathbf{c}^2\}$ if $L = L_1 \cup L_2 \cup L_5$ and L_1 are not empty. Another example for dimension two is $L = L_2 \cup L_7 \cup L_8$, if L_2 and L_8 have, e.g., the shape of figure 3. (L_2 are circular arcs with the centre in $\mathbf{y}$). Here the solutions are $\mathbf{u}^0 = \mathbf{u}^1 = \mathbf{c} \in \mathfrak{L}\{\mathbf{c}_\omega{}^\perp\}$ and $\mathbf{u}^0 = \mathbf{0}$, $\mathbf{u}^1 \in \mathfrak{L}\{\mathbf{c}_\mathbf{y}{}^3\}$ where $\mathbf{y}$ is the centre of the circular arcs of L_2.

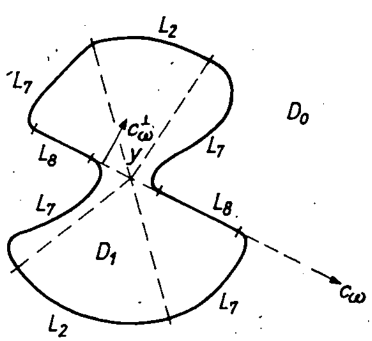

Fig. 3

The dimension is three, for instance, in the case where $L = L_2 \cup L_7$, provided that L_2 has, e.g. one of the two configurations of figure 4 (L_2 are circular arcs with the centre in $\mathbf{y}$ or parts of straight lines, respectively). Here the solutions are $\mathbf{u}^0 = \mathbf{u}^1 = \mathbf{c} \in \mathfrak{L}\{\mathbf{c}^1, \mathbf{c}^2\}$ and, additionally, $\mathbf{u}^0 = \mathbf{0}$, $\mathbf{u}^1 = \mathbf{d} \in \mathfrak{L}\{\mathbf{c}_\mathbf{y}{}^3\}$ in the first case, but $\mathbf{u}^0 = \mathbf{0}$, $\mathbf{u}^1 = \mathbf{e} \in \mathfrak{L}\{\mathbf{c}_\omega\}$ in the other one.

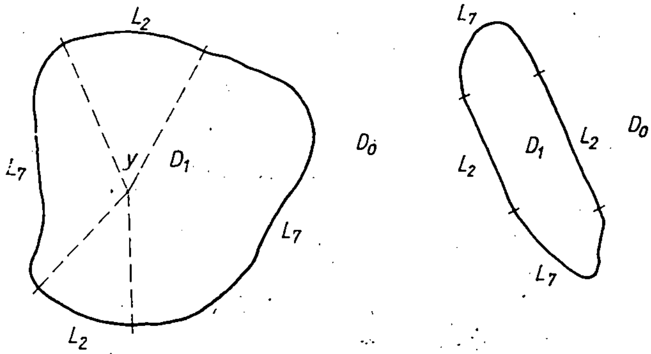

Fig. 4

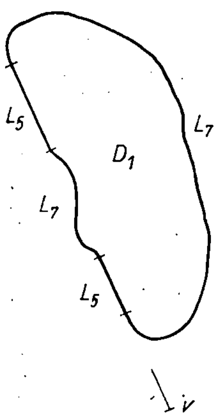

Fig. 5

An example for dimension four is $L = L_5 \cup L_7$ in the following geometrical configuration (see fig. 5). Solutions here are $\mathbf{u}^0 = \mathbf{u}^1 = \mathbf{c} \in \mathfrak{L}\{\mathbf{c}^1, \mathbf{c}^2\}$ and, additionally, $\mathbf{u}^0 = \mathbf{0}$, $\mathbf{u}^1 = \mathbf{d} \in \mathfrak{L}\{\mathbf{c}_\omega\}$ and $\mathbf{u}^0 = \mathbf{0}$, $\mathbf{u}^1 = \mathbf{e} \in \mathfrak{L}\{\mathbf{c}_\mathbf{v}{}^3\}$.

The considerations show that the set of solutions of the homogeneous problem C^* is a subset of the regular vectors (3.2). Consequently, the results for the investigation of the homogeneous problems C_ε and $C_\varepsilon{}^*$ are the same as for C^*. In part II of our paper the existence of *-regular solutions of the inhomogeneous problem C^* will be proved, provided that the homogeneous problem C^* has no nontrivial solutions. If the homogeneous problem C^* has nontrivial solutions, then the inhomogeneous one has solvability conditions. The latter ones can be found, in usual manner, by the aid of (2.8). In the next, they are derived for the above-mentioned situations with nontrivial solutions.

Let $\mathbf{u}^0, \mathbf{u}^1$ be the solutions of the inhomogeneous problems to consider. Setting in (2.8) $\mathbf{v} = \mathbf{u}^0$, $\mathbf{u} = \mathbf{c}_\omega$, $D = D_0$ and $\mathbf{v} = \mathbf{u}^1$, $\mathbf{u} = \mathbf{c}_\omega$, $D = D_1$, respectively, we obtain for the first considered case of one nontrivial solution the following relations

$$0 = \int_{L_1} \mathbf{c}_\omega \cdot \mathcal{J}(\tilde{\mathbf{n}})\, \mathbf{u}^0 \, ds + \int_{L_4} [(\mathbf{n} \cdot \mathbf{c}_\omega)\, (\mathbf{n} \cdot \mathcal{J}(\tilde{\mathbf{n}})\, \mathbf{u}^0) + (\mathbf{s} \cdot \mathbf{c}_\omega)\, (\mathbf{s} \cdot \mathcal{J}(\tilde{\mathbf{n}})\, \mathbf{u}^0)] \, ds - \int_{L_7} \mathbf{c}_\omega \cdot \mathbf{g}_0 \, ds$$

and

$$0 = \int_{L_1} \mathbf{c}_\omega \cdot \mathcal{J}(\mathbf{n})\, \mathbf{u}^1 \, ds + \int_{L_4} [(\mathbf{n} \cdot \mathbf{c}_\omega)\, (\mathbf{n} \cdot \mathcal{J}(\mathbf{n})\, \mathbf{u}^1) + (\mathbf{s} \cdot \mathbf{c}_\omega)\, (\mathbf{s} \cdot \mathcal{J}(\mathbf{n})\, \mathbf{u}^1)] \, ds + \int_{L_7} \mathbf{c}_\omega \cdot \mathbf{g}_1 \, ds.$$

Because of $\mathbf{n} \cdot \mathbf{c}_\omega = 0$ the terms containing $\mathbf{n} \cdot \mathcal{T}(\mathbf{n})\, \mathbf{u}^i$ $(i = 1, 0)$ vanish. By summing we get the solvability condition

$$0 = \int_{L_1} \mathbf{c}_\omega \cdot \mathbf{g}\, ds + \int_{L_4} (\mathbf{s} \cdot \mathbf{c}_\omega)\, g\, ds + \int_{L_7} \mathbf{c}_\omega \cdot (\mathbf{g}_1 - \mathbf{g}_0)\, ds, \tag{3.7}$$

which is, of course, necessary for the existence of a *-regular solution of C^* in the considered special case. In the second example of dimension one we obtain by setting $\mathbf{v} = \mathbf{u}^1$, $\mathbf{u} = \mathbf{c}_{\mathbf{y}}{}^3$ the solvability condition

$$0 = \int_{L_2} (\mathbf{s} \cdot \mathbf{c}_{\mathbf{y}}{}^3)\, h_1\, ds + \int_{L_5} (\mathbf{n} \cdot \mathbf{c}_{\mathbf{y}}{}^3)\, h_1\, ds + \int_{L_7} \mathbf{c}_{\mathbf{y}}{}^3 \cdot \mathbf{g}_1\, ds. \tag{3.8}$$

The first example of dimension two leads us to the conditions

$$\int_{L_1} \mathbf{c}^i \mathbf{g}\, ds + \int_{L_3} \mathbf{c}^i [(h_1 - h_0)\, \mathbf{n} + g\mathbf{s}]\, ds + \int_{L_2} \mathbf{c}^i \cdot [(h_1 - h_0)\, \mathbf{s} + g\mathbf{n}]\, ds = 0 \quad (i = 1, 2), \tag{3.9}$$

but in the other one we get the conditions

$$\int_{L_2} [(\mathbf{s} \cdot \mathbf{c}_\omega{}^\perp)\, (h_1 - h_0) + (\mathbf{n} \cdot \mathbf{c}_\omega{}^\perp)\, g]\, ds + \int_{L_7} \mathbf{c}_\omega{}^\perp \cdot (\mathbf{g}_1 - \mathbf{g}_0)\, ds + \int_{L_8} (\mathbf{n} \cdot \mathbf{c}_\omega{}^\perp)\, (g_1 - g_0)\, ds = 0 \tag{3.10}$$

and

$$\int_{L_2} (\mathbf{s} \cdot \mathbf{c}_{\mathbf{y}}{}^3)\, h_1\, ds + \int_{L_7} \mathbf{c}_{\mathbf{y}}{}^3 \cdot \mathbf{g}_1\, ds + \int_{L_8} (\mathbf{n} \cdot \mathbf{c}_{\mathbf{y}}{}^3)\, g_1\, ds = 0. \tag{3.11}$$

In both cases of dimension three we have

$$\int_{L_2} \mathbf{c}^i \cdot [\mathbf{n}g + (h_1 - h_0)\, \mathbf{s}]\, ds + \int_{L_7} \mathbf{c}^i \cdot (\mathbf{g}_1 - \mathbf{g}_0)\, ds = 0 \qquad (i = 1, 2), \tag{3.12}$$

and either

$$\int_{L_2} (\mathbf{s} \cdot \mathbf{c}_{\mathbf{y}}{}^3)\, h_1\, ds + \int_{L_7} \mathbf{c}_{\mathbf{y}}{}^3 \cdot \mathbf{g}_1\, ds = 0 \tag{3.13}$$

or

$$\int_{L_2} (\mathbf{s} \cdot \mathbf{c}_\omega)\, h_1\, ds + \int_{L_7} \mathbf{c}_\omega \cdot \mathbf{g}_1\, ds = 0. \tag{3.14}$$

Finally, in the example of figure 5 one obtains the conditions

$$\int_{L_5} \mathbf{c}^i \cdot [(h_1 - h_0)\, \mathbf{n} + g\mathbf{s}]\, ds + \int_{L_7} \mathbf{c}^i \cdot (\mathbf{g}_1 - \mathbf{g}_0)\, ds = 0 \qquad (i = 1, 2), \tag{3.15}$$

$$\int_{L_5} (\mathbf{n} \cdot \mathbf{c}_\omega{}^\perp)\, h_1\, ds + \int_{L_7} \mathbf{c}_\omega{}^\perp \cdot \mathbf{g}_1\, ds = 0 \tag{3.16}$$

and

$$\int_{L_5} (\mathbf{n} \cdot \mathbf{c}_{\mathbf{y}}{}^3)\, h_1 ds + \int_{L_7} \mathbf{c}_{\mathbf{y}}{}^3 \cdot \mathbf{g}_1 ds = 0. \tag{3.17}$$

The physical meaning of the solvability conditions derived in such a way with the solutions of the corresponding homogeneous problem by the aid of formula (2.8) is the equilibrium of surface forces and their moments. In the sequel it is proved that these physical conditions with respect to the boundary data are sufficient for the existence of a *-regular solution. Moreover, the problems will be studied in the smaller classes of ε-regular and ε^*-regular vectors. In these cases the existence of some additional conditions for solvability is proved. However, the latter ones cannot be derived explicitly by the aid of known physical principles, as it might be expected because of the more quantitative than qualitative difference between *-regularity and ε- or ε^*-regularity.

§ 4 Fundamental solution and the potential of single layer

The Kelvin-Somigliana matrix

$$\Gamma(\mathrm{x}-\mathrm{y}) = [\Gamma_{ij}(\mathrm{x}-\mathrm{y})] = \left[a \ln \frac{k}{|\mathrm{x}-\mathrm{y}|}\,\delta_{ij} + b\,\frac{(x_i-y_i)(x_j-y_j)}{|\mathrm{x}-\mathrm{y}|^2}\right]_{i,j=1,2} \tag{4.1}$$

where

$$a = \frac{\lambda+3\mu}{2\mu(\lambda+2\mu)}, \qquad b = \frac{\lambda+\mu}{2\mu(\lambda+2\mu)}, \qquad k > 0$$

is a fundamental solution of (1.1). Let L be a curve of the class $C^{1,\beta}$. Then we have

$$\left[\frac{d}{ds_x}\,\Gamma_{ij}(\mathrm{x}-\mathrm{y})\right] = \left[a\,\frac{d}{ds_x}\,\ln\frac{1}{|\mathrm{x}-\mathrm{y}|}\,\delta_{ij}\right] + R_1(\mathrm{x}-\mathrm{y}) \quad \text{for} \quad \mathrm{x},\mathrm{y} \in L \tag{4.2}$$

with a matrix $R_1(\mathrm{x}-\mathrm{y})$ of the order $\mathcal{O}(|\mathrm{x}-\mathrm{y}|^{-1+\eta})$ $(\eta > 0)$ (see [29]). Moreover, the representation

$$\mathcal{T}_x(\mathrm{n})\,\Gamma(\mathrm{x}-\mathrm{y}) = c\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\frac{d}{ds_x}\,\ln\frac{1}{|\mathrm{x}-\mathrm{y}|} + R_2(\mathrm{x}-\mathrm{y}), \qquad c = \frac{\mu}{\lambda+2\mu},$$

$$\mathrm{x},\mathrm{y} \in L, \tag{4.3}$$

holds (Here, the operator $\mathcal{T}_x(\mathrm{n})$ acts columnwise with respect to x; $\mathrm{n} = \mathrm{n}(\mathrm{x})$ is the normal at $\mathrm{x} \in L$). The matrix $R_2(\mathrm{x}-\mathrm{y})$ is of the order $\mathcal{O}(|\mathrm{x}-\mathrm{y}|^{-1+\eta})$ $(\eta > 0)$ (see [29]). By more sophisticated considerations it can be proved, that the components r_i^l $(l = 1, 2)$ of matrices $R_l(\mathrm{x}-\mathrm{y})$ allow a representation

$$r_{ij}^l(\mathrm{x}-\mathrm{y}) = \frac{p_{ij}^l(\mathrm{x}-\mathrm{y})}{|\mathrm{x}-\mathrm{y}|^{1-\eta}} \qquad (l, i, j = 1, 2)$$

with $p_{ij}^l \in C^{0,\beta-\eta}(L \times L)$ for every $0 < \eta < \beta$. On the assumption that $L \in C^{2,\beta}$ we additionally obtain $r_{ij}^l(\mathrm{x}-\mathrm{y}) \in H$. The elements of $\Gamma(\mathrm{x}-\mathrm{y})$ are of the order $\mathcal{O}(|\ln|\mathrm{x}-\mathrm{y}||)$ for $\mathrm{x} \to \mathrm{y}$ and also for $|\mathrm{x}| \to \infty$, but their first partial derivatives $\mathcal{O}(|\mathrm{x}-\mathrm{y}|^{-1})$. We still remark the formulas ([29])

$$\mathrm{c}^i(\mathrm{z}) + \frac{1}{\pi}\int_{\partial D}[\mathcal{T}(\mathrm{n})\,\Gamma(\mathrm{z}-\mathrm{y})]^T\,\mathrm{c}^i(\mathrm{y})\,ds_y = 0 \qquad (i = 1, 2, 3). \tag{4.4}$$

Here the operator $\mathcal{T}_y(\mathrm{n})$ also acts columnwise with $\mathrm{n} = \mathrm{n}(\mathrm{y})$ with respect to the variable y.

Now let D be a bounded or unbounded domain with $\partial D = L \in C^{1,\beta}$. Let us consider the potential of single layer

$$\mathrm{V}(\mathrm{x};\varphi) = \frac{1}{\pi}\int_L \Gamma(\mathrm{x}-\mathrm{y})\,\varphi(\mathrm{y})\,ds_y \tag{4.5}$$

with a given vector field $\varphi \doteq (\varphi_1, \varphi_2)$. From the results in [17, 29] we can deduce

Theorem 1: *Let $\varphi \in C^{0,\alpha}(L)$ $(0 < \alpha < \beta \leqq 1)$. Then the following propositions hold.*

1. *If D is a bounded domain, then $\mathrm{V}(\mathrm{x};\varphi)$ is a regular solution of the equation* (1.1) *in D.*
2. *Let D be unbounded. Then $\mathrm{V}(\mathrm{x};\varphi)$ is regular if and only if the relation*

$$\int_L \varphi(\mathrm{y})\,ds_y = 0 \tag{4.6}$$

is satisfied. If (4.6) *is fulfilled, then*

$$\lim_{|\mathbf{x}|\to\infty} \mathbf{V}(\mathbf{x}, \boldsymbol{\varphi}) = 0, \tag{4.7}$$

but in the opposite case $\mathbf{V}(\mathbf{x}; \boldsymbol{\varphi})$ *is unbounded for* $|\mathbf{x}| \to \infty$.

3. *For* $\boldsymbol{\varphi} \in C^{0,\alpha}(L)$ *and* (4.6) *there exists a constant* C *with*

$$\|\mathbf{V}(\mathbf{x}; \boldsymbol{\varphi})\|_{C^{1,\alpha}(D)} = C\, \|\boldsymbol{\varphi}\|_{C^{0,\alpha}(L)}. \tag{4.8}$$

The constant C *depends only on the domain* D.

In generalization we prove

Theorem 2: *Let* D *be bounded or unbounded with* $L = \partial D \in C^{1,\beta}$, $\boldsymbol{\varphi} \in H^*$. *In the case of an unbounded domain* D *assume that* (4.6) *is fulfilled.*
Then

1. $\mathbf{V}(\mathbf{x}; \boldsymbol{\varphi})$ *is a *-regular solution of* (1.1).
2. *If* $\boldsymbol{\varphi} \in H_\varepsilon$, *then* $\mathbf{V}(\mathbf{x}; \boldsymbol{\varphi})$ *is* ε*-regular.*
3. *If* $\boldsymbol{\varphi}$ *belongs to* H^* *and to* H_ε *only in the neighbourhood of certain nodes, then* $\mathbf{V}(\mathbf{x}; \boldsymbol{\varphi})$ *is a* ε^**-regular solution of* (1.1).

First we remark that most of the propositions of Theorem 2 follow by simple considerations from Theorem 1. We have to prove only the estimates (2.4) for the first partial derivatives. For proof of (2.4) we can suppose that the point $\mathbf{x}$ is located within a standard circle $K_\varrho(\mathbf{a}_i)$ with centre $\mathbf{a}_i$ (see [35]). We set $2\eta = |\mathbf{x} - \mathbf{a}_i|$ $(2\eta < \varrho)$ and split the components of the density vector $\boldsymbol{\varphi} = (\varphi_1, \varphi_2)$ as follows

$$\varphi_j(\mathbf{y}) = \psi_j^\eta(\mathbf{y}) + \chi_j^\eta(\mathbf{y}) \qquad (j = 1, 2).$$

Here $\psi_j^\eta(\mathbf{y})$ is defined by

$$\psi_j^\eta(\mathbf{y}) = \begin{cases} \varphi_j(\mathbf{y}) & \text{for} \quad \mathbf{y} \in L \setminus K_\eta(\mathbf{a}_i), \\ \varphi_j(\mathbf{y}^1) + \dfrac{|\mathbf{y} - \mathbf{y}^1|}{|\mathbf{y}^2 - \mathbf{y}^1|}\left(\varphi_j(\mathbf{y}^2) - \varphi_j(\mathbf{y}^1)\right) & \text{for} \quad \mathbf{y} \in L \cap K_\eta(\mathbf{a}_i). \end{cases} \tag{4.9}$$

In this formula $\mathbf{y}^1$ and $\mathbf{y}^2$ mean the two points of intersection of the curve L with $\partial K_\eta(\mathbf{a}_i)$. $\chi_j^\eta(\mathbf{y})$ is completely defined by $\chi_j^\eta(\mathbf{y}) = \varphi_j(\mathbf{y}) - \psi_j^\eta(\mathbf{y})$. A simple consequence is $\chi_j^\eta(\mathbf{y}) = 0$ for every $\mathbf{y} \notin L \cap K_\eta(\mathbf{a}_i)$. Because the singularity behaviour of the first partial derivatives of $\mathbf{V}(\mathbf{x}; \boldsymbol{\varphi})$ near the node $\mathbf{a}_i$ is determined only by the values of $\boldsymbol{\varphi}$ in the neighbourhood of $\mathbf{a}_i$, one can assume without loss of generality that $\mathbf{a}_i$ is the only node at L.

In virtue of $\boldsymbol{\varphi} \in H^*$ there exist constants A_1, A_2 for which the estimates

$$|\varphi_j(\mathbf{y})| \leqq \frac{A_j}{|\mathbf{y} - \mathbf{a}_i|^\delta} \qquad (j = 1, 2), \qquad \delta = \operatorname{Re}\gamma \tag{4.10}$$

hold. That implies

$$|\psi_j^\eta(\mathbf{y})| \leqq \frac{A_j}{\eta^\delta} \tag{4.11}$$

and

$$|\chi_j^\eta(\mathbf{y})| \leqq \frac{2A_j}{|\mathbf{y} - \mathbf{a}_i|^\delta}. \tag{4.12}$$

Now we set

$$\psi_j^\eta(\mathbf{y}) = \frac{\tilde{\psi}_j^\eta(\mathbf{y})}{(\eta)^{\delta'}} \quad \text{with a suitable } \delta' \colon \delta < \delta' < 1.$$

It will be shown that the $C^{0,\delta'-\delta}$-norm of the vector family $\{\tilde{\psi}_j^\eta(\mathbf{y})\}$ with parameter η is bounded. For this purpose, first the maximum norm of $\tilde{\psi}_j^\eta(\mathbf{y})$ is proved to be bounded. That follows immediately from (4.11). Secondly, we have to prove that the vectors $\tilde{\psi}_j^\eta(\mathbf{y})$ satisfy a Hölder condition with Hölder exponent $\delta' - \delta$ and a uniformly bounded Hölder coefficient. Additionally, we assume the constant δ' chosen in such a way that $\delta' - \delta$ is not greater than the Hölder exponent of the denominator of $\varphi_j(\mathbf{y})$ corresponding to the representation (2.1). Therefore, the uniformity of the Hölder coefficient on the part $L \setminus K_\eta(\mathbf{a}_i)$ is evident. Now let $\mathbf{y}' \cdot \mathbf{y}'' \in K_\eta(\mathbf{a}_i)$. Then we have

$$\begin{aligned} |\tilde{\psi}_j^\eta(\mathbf{y}') - \tilde{\psi}_j^\eta(\mathbf{y}'')| &\leqq \eta^{\delta'} |\psi_j^\beta(\mathbf{y}') - \psi_j^\eta(\mathbf{y}'')| \leqq \eta^{\delta'} \frac{|\mathbf{y}' - \mathbf{y}''|}{|\mathbf{y}^2 - \mathbf{y}^1|} |\psi_j^\eta(\mathbf{y}^2) - \psi_j^\eta(\mathbf{y}^1)| \\ &\leqq 2A_j\eta^{\delta'-\delta} \frac{1}{|\mathbf{y}^2 - \mathbf{y}^1|} |\mathbf{y}' - \mathbf{y}''| \\ &\leqq KA_j\eta^{-(1-\delta'+\delta)} |\mathbf{y}' - \mathbf{y}''|^{(1-\delta'+\delta)+(\delta'-\delta)} \leqq KA_j |\mathbf{y}' - \mathbf{y}''|^{\delta'-\delta} \end{aligned}$$

with a constant $K > 0$ independent of $\mathbf{a}_i$ and η. Consequently, the uniform boundedness of the family $\{\tilde{\psi}_j^\eta(\mathbf{y})\}$ with respect to the $C^{0,\delta'-\delta}$-norm is proved. There exists a fixed constant C_1 with

$$\|\tilde{\psi}_j^\eta\|_{C^{0,\delta'-\delta}} \leqq C_1 \tag{4.13}$$

for every $\eta : 0 < 2\eta \leqq \varrho$.

Now we verify the estimate (2.4). Let $\frac{\partial}{\partial x_l} \mathbf{V}(\mathbf{x};\varphi)_j$ be the partial derivative of the j-component of $\mathbf{V}(\mathbf{x};\varphi)$ with respect to the variable x_l. Let ψ_j^η, $\tilde{\psi}_j^\eta$, χ_j^η be the above defined functions and $\boldsymbol{\psi}^\eta$, $\tilde{\boldsymbol{\psi}}^\eta$, $\boldsymbol{\chi}^\eta$ the corresponding vectors. Then we have for the point $\mathbf{x}$, $|\mathbf{x} - \mathbf{a}_i| = 2\eta$:

$$\begin{aligned} \left|\frac{\partial}{\partial x_l} \mathbf{V}(\mathbf{x};\varphi)_j\right| &\leqq \left|\frac{1}{\pi}\int_L \frac{\partial}{\partial x_l} \Gamma_j(\mathbf{x}-\mathbf{y})\, \boldsymbol{\psi}^\eta(\mathbf{y})\, ds\right| + \left|\frac{1}{\pi}\int_L \frac{\partial}{\partial x_l} \Gamma_j(\mathbf{x}-\mathbf{y})\, \boldsymbol{\chi}^\eta(\mathbf{y})\, ds\right| \\ &= \frac{1}{\eta^{\delta'}} \left|\frac{1}{\pi}\int_L \frac{\partial}{\partial x_l} \Gamma_j(\mathbf{x}-\mathbf{y})\, \tilde{\boldsymbol{\psi}}^\eta(\mathbf{y})\, ds\right| + \left|\frac{1}{\pi}\int_{L\cap K_\eta(a_i)} \frac{\partial}{\partial x_l} \Gamma_j(\mathbf{x}-\mathbf{y})\, \boldsymbol{\chi}^\eta(\mathbf{y})\, ds\right|. \end{aligned}$$

Here the j-th row of $\boldsymbol{\Gamma}(\mathbf{x}-\mathbf{y})$ is denoted by $\Gamma_j(\mathbf{x}-\mathbf{y})$. Using (4.8) and (4.13), the first integral on the right-hand side can be estimated by CC_1. Because the first derivatives $\frac{\partial}{\partial x_l}\Gamma_j(\mathbf{x}-\mathbf{y})$ are of the order $|\mathbf{x}-\mathbf{y}|^{-1}$, we get

$$\begin{aligned} \left|\frac{\partial}{\partial x_l} \mathbf{V}(\mathbf{x};\varphi)_j\right| &\leqq \frac{CC_1}{\eta^{\delta'}} + C_2 \int_{L\cap K_\eta(a_i)} \frac{1}{|\mathbf{x}-\mathbf{y}|} \frac{ds_y}{|\mathbf{y}-\mathbf{a}_i|^\delta} \leqq \frac{CC_1}{\eta^{\delta'}} + \frac{C_3}{\eta}\int_{-\eta}^{\eta} \frac{dr}{r^\delta} \\ &\leqq \frac{CC_1}{\eta^{\delta'}} + \frac{C_3}{\eta}\,\frac{2}{1-\delta}\,\eta^{1-\delta} \leqq C_4\eta^{-\delta'} \leqq 2^{\delta'}C_4 |\mathbf{x}-\mathbf{a}_i|^{-\delta'}. \end{aligned}$$

This estimate completes the proof of proposition 1.

If the vector φ belongs to the class H_ε in the neighbourhood of $\mathbf{a}_i$, then the estimate can be derived for every $\delta' > 0$. That implies the propositions 2 and 3. The theorem is proved ∎

Both the following theorems are known for densities of the class H (see, e.g., [29]). Their validity for $\varphi \in H^*$ at the ordinary points of L (except the nodes) is immediately clear.

Theorem 3: *Let the assumptions of Theorem 2 be satisfied.* $\mathbf{n}$ *let be the outward normal of* L. *Let* $\mathbf{z} \in L$ *be an ordinary point of* L. *Then*

$$\mathscr{T}(\mathbf{n})\,\mathbf{V}(\mathbf{z};\varphi)^{\pm} = \lim_{\substack{\mathbf{x}\in D^{\pm}\\ \mathbf{x}\to\mathbf{z}}} \mathscr{T}(\mathbf{n}_z)\,\frac{1}{\pi}\int_L \Gamma(\mathbf{x}-\mathbf{y})\,\varphi(\mathbf{y})\,ds = \pm\varphi(\mathbf{z})$$

$$+\frac{1}{\pi}\int_L \mathscr{T}_z(\mathbf{n})\,\Gamma(\mathbf{z}-\mathbf{y})\,\varphi(\mathbf{y})\,ds_y. \qquad (4.14)$$

The sign + is taken for a bounded domain D, the − for an unbounded one. The integral on the right-hand side exists in the sense of Cauchy principal value.

Theorem 4: *Let the assumption of Theorem 3 be satisfied. Then the tangential derivatives of* $\mathbf{V}(\mathbf{x};\varphi)$ *on* L *exist in the ordinary points; further they can be calculated by differentiation under the integral.*

Let D be a bounded domain. In [29] the following properties of the single layer potential $\mathbf{V}(\mathbf{x},\varphi)$ were proved; the relations $\mathbf{V}(\mathbf{x},\varphi) = \mathbf{0}$ for every $\mathbf{x} \in D$ and $\varphi \in H$ involve $\varphi(\mathbf{z}) = 0$ for every $\mathbf{z} \in L$, provided that the constant k in (4.1) does not coincide with an exceptional value. Moreover, there are two exceptional values, at most. The proofs of these properties are based on certain facts with respect to homogeneous singular integral equation system of the second boundary value problem. KHVEDELIDZE has proved [14] that every L_p-solution of a homogeneous regular-type integral equation system with coefficients of the class H belongs to the class H. Using this well-known result, the validity of the above-mentioned proposition can be proved also for $\varphi \in H^*$: The relations $\mathbf{V}(\mathbf{x};\varphi) = \mathbf{0}$ for every $\mathbf{x} \in D$ and $\varphi \in H^*$ involve $\varphi(\mathbf{z}) = \mathbf{0}$ for every ordinary point $\mathbf{z} \in L$. In the sequel, that property will be called *equivalence*. Unless stated otherwise, the potential $\mathbf{V}(\mathbf{x};\varphi)$ is always assumed to be equivalent, i.e. k does not coincide with an exceptional value.

For the unbounded domain the following result [29: p. 68 Hilfssatz 15.2] is important:

The integral equation system

$$\frac{1}{\pi}\int_L \Gamma(\mathbf{z}-\mathbf{y})\,\varphi(\mathbf{y})\,ds_y = A_1\mathbf{c}^1 + A_2\mathbf{c}^2; \quad \mathbf{z}\in L,\ A_1, A_2 \text{ arbitrary constants}, \qquad (4.15)$$

has only the trivial solution in the class of densities belonging to H^* and satisfying the additional condition

$$\int_L \varphi(\mathbf{y})\,ds_y = \mathbf{0}. \qquad (4.16)$$

Here the constant $k > 0$ in the matrix $\Gamma(\mathbf{z}-\mathbf{y})$ is arbitrary; especially it can be chosen $k = 1$.

§ 5 Integral equations for the problem C^*, C_ε and C_ε^*

The customary setup for treating nonmixed plane contact problems [29] is

$$\mathbf{u}^0(\mathbf{x}) = \mathbf{V}^0(\mathbf{x}; \varphi^0) + A_1\mathbf{c}^1 + A_2\mathbf{c}^2 = \frac{1}{\pi}\int_L \Gamma^0(\mathbf{x}-\mathbf{y})\,\varphi^0(\mathbf{y})\,ds_y + A_1\mathbf{c}^1 + A_2\mathbf{c}^2,$$

$$\mathbf{u}^1(\mathbf{x}) = \mathbf{V}^1(\mathbf{x}; \varphi^1) = \frac{1}{\pi}\int_L \Gamma^1(\mathbf{x}-\mathbf{y})\,\varphi^1(\mathbf{y})\,ds_y \tag{5.1}$$

(the upper indices by Γ^i and $\mathbf{V}^i$ refer to the modules λ_i, μ_i) with the additional condition

$$\int_L \varphi^0(\mathbf{y})\,ds_y = 0. \tag{5.2}$$

(We remark that in [29] the term $A_1\mathbf{c}^1 + A_2\mathbf{c}^2$ is added to the potential $\mathbf{V}^1(\mathbf{x}; \varphi^1)$; but this difference is not essential with regard to the results on the first boundary value problem obtained also in [29]).

It will be convenient to agree upon the following denotations

$$\Phi = \begin{bmatrix}\varphi^0\\ \varphi^1\end{bmatrix} = \begin{bmatrix}\varphi_1{}^0\\ \varphi_2{}^0\\ \varphi_1{}^1\\ \varphi_2{}^1\end{bmatrix}; \quad \mathbf{V}(\mathbf{x}; \Phi) = \begin{cases}\mathbf{V}^0(\mathbf{x}; \varphi^0) & \text{for } \mathbf{x}\in D_0\\ \mathbf{V}^1(\mathbf{x}; \varphi^1) & \text{for } \mathbf{x}\in D_1\end{cases}. \tag{5.3}$$

For treating the considered problems C^*, C_ε, C_ε^* we also start from (5.1), (5.2). The vector $\varphi^0, \varphi^1 \in H^*$ as well as the constants A_1, A_2 have to be defined in order to obtain a *-regular (ε-regular or ε^*-regular) solution of C^* (C_ε or C_ε^*).

On application of Theorem 4.3, the contact conditions (1.4a)—(1.4i) give rise to an integral equation system abbreviated by the symbolic notation

$$\mathcal{A}\Phi = A_1\mathbf{w}_1 + A_2\mathbf{w}_2 + \mathbf{w}. \tag{5.4}$$

The contact data $\mathbf{f}, f, \mathbf{f}_k, \ldots, g_k, h_k$ are represented by $\mathbf{w}$, whereas $\mathbf{w}_1, \mathbf{w}_2$ are the contact data of the vectors $\mathbf{u}^0 = \mathbf{c}^1$, $\mathbf{u}^1 = \mathbf{0}$ and $\mathbf{u}^0 = \mathbf{c}^2$, $\mathbf{u}^1 = \mathbf{0}$, respectively. Let

$$d = \dim \mathfrak{L}\{\mathbf{w}_1, \mathbf{w}_2\}; \tag{5.5}$$

then $1 \leqq d \leqq 2$ is a simple consequence of our assumptions. Both cases $d = 1$ and $d = 2$ are possible.

Now let us define the linear manifolds

$$\mathfrak{A} = \{\Phi \in H^* \mid \mathcal{A}\Phi = A_1\mathbf{w}_1 + A_2\mathbf{w}_2 \quad \text{for any constants } A_1, A_2\}, \tag{5.6}$$

$$\mathfrak{A}_0 = \left\{\Phi \in \mathfrak{A} \;\middle|\; \int_L \varphi^0(\mathbf{y})\,ds_y = 0\right\}. \tag{5.7}$$

The following lemma holds true.

Lemma 1: *Let h be the number of linearly independent solutions of the homogeneous problem C^*. Then*

$$\dim \mathfrak{A} \leqq h + d \quad \textit{and} \quad \dim \mathfrak{A}_0 \leqq h + d - 2. \tag{5.8}$$

Indeed, the linearity of C^* implies that the problem C^* with contact data in the linear manifold $\mathfrak{L}\{\mathbf{w}_1, \mathbf{w}_2\}$ has exactly $h + d$ linearly independent solutions, i.e. the dimension of the linear manifold $\mathfrak{Q}$ of *-regular solutions with contact data in

$\mathfrak{L}\{w_1, w_2\}$ is equal to $h + d$. Especially, $\mathfrak{Q}$ contains the two vectors $\mathbf{u}^0 = \mathbf{c}^1$, $\mathbf{u}^1 = \mathbf{0}$ and $\mathbf{u}^0 = \mathbf{c}^2$, $\mathbf{u}^1 = \mathbf{0}$. For proof of the first proposition assume that $\dim \mathfrak{A} \geq h + d + 1$. Let $\Phi_1, \ldots, \Phi_{h+d+1}$ be linearly independent vectors of $\mathfrak{A}$. Obviously one can assume without loss of generality that the vectors $\Phi_1, \ldots, \Phi_{h+d-1}$ belong to $\mathfrak{A}_0$. Consequently, the $h + d - 1$ potentials $\mathbf{V}(\mathbf{x}; \Phi_i)$ $(i = 1, \ldots, h + d - 1)$ belong to the manifold $\mathfrak{Q}$. Besides, these potentials are linearly independent, which follows from the equivalence of $\mathbf{V}^1(\mathbf{x}; \varphi^1)$ and from the considerations on $\mathbf{V}^0(\mathbf{x}; \varphi^0)$ in connection with (4.15). Taking into account Theorem 4.1 (esp. (4.7)), one gets the linear independence of the $h + d + 1$ vectors $\mathbf{V}(\mathbf{x}; \Phi_i)$ $(i = 1, \ldots, h + d - 1)$ and $\mathbf{u}^0 = \mathbf{c}^1$, $\mathbf{u}^1 = \mathbf{0}$ and $\mathbf{u}^0 = \mathbf{c}^2$, $\mathbf{u}^1 = \mathbf{0}$. But this contradicts $\dim \mathfrak{Q} = h + d$. Consequently, the first inequality of the lemma is proved. The second one is an immediate consequence of the first one ∎

The explicit form of the linear integral operator $\mathcal{A}$ is not interesting. It is easily seen that (5.4) consists of equations of alternative kind. A given equation of (5.4) at a fixed arc S_i is either a singular integral equation of the second kind or a Fredholm equation of the first kind with kernel having logarithmic singularity. The first alternative is given in equations expressing a condition for stresses, but the second one, for displacements.

In order to get a singular integral equation of a type well known in literature, the Fredholm equations of the first kind are submitted to the operator

$$\left(\frac{d}{ds} + p\right), \qquad p = \text{const.} \neq 0. \tag{5.9}$$

The resulting system is symbolically denoted by

$$\Omega_p \mathcal{A}\Phi = A_1 \Omega_p \mathrm{w}_1 + A_2 \Omega_p \mathrm{w}_2 + \Omega_p \mathrm{w}. \tag{5.10}$$

Later, in part II of this paper, it will be proved that (5.10) is a singular integral equation system with coefficients of the class H_0. Moreover, (5.10) is of regular type in the sense of [35, 38]. The index of (5.10) will also be calculated in part II.

The operator Ω_p can also be considered as a linear operator. Its action is to implement the operator (5.9) on some equations of (5.4) at several arcs S_i, while the remaining equation stay unaltered.

The integral operator $\Omega_p\mathcal{A}$ of system (5.10) has the following explicit form

$$\Omega_p\mathcal{A}\Phi = \mathbf{A}(\mathbf{z})\,\Phi(\mathbf{z}) + \frac{1}{\pi}\int_L [\mathbf{K}(\mathbf{z}-\mathbf{y}) + p\mathbf{R}(\mathbf{z}-\mathbf{y})]\,\Phi(\mathbf{y})\,ds_y, \tag{5.11}$$

where

$$\mathbf{K}(\mathbf{z}-\mathbf{y}) = \begin{cases} \begin{bmatrix} -\dfrac{d}{ds}\,\Gamma^0(\mathbf{z}-\mathbf{y}) & \dfrac{d}{ds}\,\Gamma^1(\mathbf{z}-\mathbf{y}) \\ -\mathscr{T}(\mathbf{n})\,\Gamma^0(\mathbf{z}-\mathbf{y}) & \mathscr{T}(\mathbf{n})\,\Gamma^1(\mathbf{z}-\mathbf{y}) \end{bmatrix} & (\mathbf{z} \in L_1), \\[2ex] \begin{bmatrix} 0 & 0 & \mathbf{s}\cdot\mathscr{T}(\mathbf{n})\,\Gamma^1(\mathbf{z}-\mathbf{y}) \\ \mathbf{s}\cdot\mathscr{T}(\mathbf{n})\,\Gamma^0(\mathbf{z}-\mathbf{y}) & 0 & 0 \\ -\dfrac{d}{ds}\left(\mathbf{n}\cdot\Gamma^0(\mathbf{z}-\mathbf{y})\right) & \dfrac{d}{ds}\left(\mathbf{n}\cdot\Gamma^1(\mathbf{z}-\mathbf{y})\right) & \\ -\mathbf{n}\cdot\mathscr{T}(\mathbf{n})\,\Gamma^0(\mathbf{z}-\mathbf{y}) & \mathbf{n}\cdot\mathscr{T}(\mathbf{n})\,\Gamma^1(\mathbf{z}-\mathbf{y}) & \end{bmatrix} & (\mathbf{z} \in L_2), \end{cases} \tag{5.12}$$

$$
\mathbf{K}(\mathrm{z}-\mathrm{y})=\begin{cases}
\begin{bmatrix}
0 & 0 & \frac{d}{ds}\left(\mathrm{s}\cdot\Gamma^1(\mathrm{z}-\mathrm{y})\right) \\
\frac{d}{ds}\left(\mathrm{s}\cdot\Gamma^0(\mathrm{z}-\mathrm{y})\right) & 0 & 0 \\
-\frac{d}{ds}\left(\mathrm{n}\cdot\Gamma^0(\mathrm{z}-\mathrm{y})\right) & & \frac{d}{ds}\left(\mathrm{n}\cdot\Gamma^1(\mathrm{z}-\mathrm{y})\right) \\
-\mathrm{n}\cdot\mathscr{T}(\mathrm{n})\,\Gamma^0(\mathrm{z}-\mathrm{y}) & & \mathrm{n}\cdot\mathscr{T}(\mathrm{n})\,\Gamma^1(\mathrm{z}-\mathrm{y})
\end{bmatrix} & (\mathrm{z}\in L_3), \\[2ex]
\begin{bmatrix}
0 & 0 & \frac{d}{ds}\left(\mathrm{n}\cdot\Gamma^1(\mathrm{z}-\mathrm{y})\right) \\
\frac{d}{ds}\left(\mathrm{n}\cdot\Gamma^0(\mathrm{z}-\mathrm{y})\right) & 0 & 0 \\
-\frac{d}{ds}\left(\mathrm{s}\cdot\Gamma^0(\mathrm{z}-\mathrm{y})\right) & & \frac{d}{ds}\left(\mathrm{s}\cdot\Gamma^1(\mathrm{z}-\mathrm{y})\right) \\
-\mathrm{s}\cdot\mathscr{T}(\mathrm{n})\,\Gamma^0(\mathrm{z}-\mathrm{y}) & & \mathrm{s}\cdot\mathscr{T}(\mathrm{n})\,\Gamma^1(\mathrm{z}-\mathrm{y})
\end{bmatrix} & (\mathrm{z}\in L_4), \\[2ex]
\begin{bmatrix}
0 & 0 & \mathrm{n}\cdot\mathscr{T}(\mathrm{n})\,\Gamma^1(\mathrm{z}-\mathrm{y}) \\
\mathrm{n}\cdot\mathscr{T}(\mathrm{n})\,\Gamma^0(\mathrm{z}-\mathrm{y}) & 0 & 0 \\
-\frac{d}{ds}\left(\mathrm{s}\cdot\Gamma^0(\mathrm{z}-\mathrm{y})\right) & & \frac{d}{ds}\left(\mathrm{s}\cdot\Gamma^1(\mathrm{z}-\mathrm{y})\right) \\
-\mathrm{s}\cdot\mathscr{T}(\mathrm{n})\,\Gamma^0(\mathrm{z}-\mathrm{y}) & & \mathrm{s}\cdot\mathscr{T}(\mathrm{n})\,\Gamma^1(\mathrm{z}-\mathrm{y})
\end{bmatrix} & (\mathrm{z}\in L_5), \\[2ex]
\begin{bmatrix}
\frac{d}{ds}\,\Gamma^0(\mathrm{z}-\mathrm{y}) & 0 & 0 \\
 & 0 & 0 \\
0 \quad 0 & & \frac{d}{ds}\,\Gamma^1(\mathrm{z}-\mathrm{y}) \\
0 \quad 0 & &
\end{bmatrix} & (\mathrm{z}\in L_6), \\[2ex]
\begin{bmatrix}
\mathscr{T}(\mathrm{n})\,\Gamma^0(\mathrm{z}-\mathrm{y}) & 0 & 0 \\
 & 0 & 0 \\
0 \quad 0 & & \mathscr{T}(\mathrm{n})\,\Gamma^1(\mathrm{z}-\mathrm{y}) \\
0 \quad 0 & &
\end{bmatrix} & (\mathrm{z}\in L_7), \\[2ex]
\begin{bmatrix}
\frac{d}{ds}\left(\mathrm{s}\cdot\Gamma^0(\mathrm{z}-\mathrm{y})\right) & 0 & 0 \\
\mathrm{n}\cdot\mathscr{T}(\mathrm{n})\,\Gamma^0(\mathrm{z}-\mathrm{y}) & 0 & 0 \\
0 \quad 0 & & \frac{d}{ds}\left(\mathrm{s}\cdot\Gamma^1(\mathrm{z}-\mathrm{y})\right) \\
0 \quad 0 & & \mathrm{n}\cdot\mathscr{T}(\mathrm{n})\,\Gamma^1(\mathrm{z}-\mathrm{y})
\end{bmatrix} & (\mathrm{z}\in L_8), \\[2ex]
\begin{bmatrix}
\frac{d}{ds}\left(\mathrm{n}\cdot\Gamma^0(\mathrm{z}-\mathrm{y})\right) & 0 & 0 \\
\mathrm{s}\cdot\mathscr{T}(\mathrm{n})\,\Gamma^0(\mathrm{z}-\mathrm{y}) & 0 & 0 \\
0 \quad 0 & & \frac{d}{ds}\left(\mathrm{n}\cdot\Gamma^1(\mathrm{z}-\mathrm{y})\right) \\
0 \quad 0 & & \mathrm{s}\cdot\mathscr{T}(\mathrm{n})\,\Gamma^1(\mathrm{z}-\mathrm{y})
\end{bmatrix} & (\mathrm{z}\in L_9).
\end{cases}
\tag{5.12}
$$

$$
\mathbf{R}(\mathbf{z}-\mathbf{y}) = \begin{cases}
\begin{bmatrix} -\Gamma^0(\mathbf{z}-\mathbf{y}) & & \Gamma^1(\mathbf{z}-\mathbf{y}) & \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & (\mathbf{z} \in L_1), \\
\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\mathbf{n}\cdot\Gamma^0(\mathbf{z}-\mathbf{y}) & & \mathbf{n}\cdot\Gamma^1(\mathbf{z}-\mathbf{y}) & \\ 0 & 0 & 0 & 0 \end{bmatrix} & (\mathbf{z} \in L_2), \\
\begin{bmatrix} 0 & & \mathbf{s}\cdot\Gamma^1(\mathbf{z}-\mathbf{y}) & \\ \mathbf{s}\cdot\Gamma^0(\mathbf{z}-\mathbf{y}) & & 0 & 0 \\ -\mathbf{n}\cdot\Gamma^0(\mathbf{z}-\mathbf{y}) & & \mathbf{n}\cdot\Gamma^1(\mathbf{z}-\mathbf{y}) & \\ 0 & 0 & 0 & 0 \end{bmatrix} & (\mathbf{z} \in L_3), \\
\begin{bmatrix} 0 & 0 & \mathbf{n}\cdot\Gamma^1(\mathbf{z}-\mathbf{y}) & \\ \mathbf{n}\cdot\Gamma^0(\mathbf{z}-\mathbf{y}) & & 0 & 0 \\ -\mathbf{s}\cdot\Gamma^0(\mathbf{z}-\mathbf{y}) & & \mathbf{s}\cdot\Gamma^1(\mathbf{z}-\mathbf{y}) & \\ 0 & 0 & 0 & 0 \end{bmatrix} & (\mathbf{z} \in L_4), \\
\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\mathbf{s}\cdot\Gamma^0(\mathbf{z}-\mathbf{y}) & & \mathbf{s}\cdot\Gamma^1(\mathbf{z}-\mathbf{y}) & \\ 0 & 0 & 0 & 0 \end{bmatrix} & (\mathbf{z} \in L_5), \\
\begin{bmatrix} \Gamma^0(\mathbf{z}-\mathbf{y}) & & 0 & 0 \\ & & 0 & 0 \\ 0 & 0 & \Gamma^1(\mathbf{z}-\mathbf{y}) & \\ 0 & 0 & & \end{bmatrix} \ (\mathbf{z} \in L_6), & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \ (\mathbf{z} \in L_7), \\
\begin{bmatrix} \mathbf{s}\cdot\Gamma^0(\mathbf{z}-\mathbf{y}) & & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{s}\cdot\Gamma^1(\mathbf{z}-\mathbf{y}) & \\ 0 & 0 & 0 & 0 \end{bmatrix} & (\mathbf{z} \in L_8), \\
\begin{bmatrix} \mathbf{n}\cdot\Gamma^0(\mathbf{z}-\mathbf{y}) & & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{n}\cdot\Gamma^1(\mathbf{z}-\mathbf{y}) & \\ 0 & 0 & 0 & 0 \end{bmatrix} & (\mathbf{z} \in L_9).
\end{cases} \tag{5.13}
$$

In the formulas (5.12), (5.13) the vectors $\mathbf{n} = (n_1, n_2)$ and $\mathbf{s} = (-n_2, n_1)$ mean the outward normal and the tangent, respectively, and refer to the point $\mathbf{z} \in L$. $\mathscr{T}(\mathbf{n})$ acts in columns with respect to the variable $\mathbf{z}$, as does the operator $\frac{d}{ds} = \frac{d}{ds_z}$. The matrix $\mathbf{A}(\mathbf{z})$ in (5.11) is given by

$$
\mathbf{A}(\mathbf{z}) \tag{5.14}
$$
$$
= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} (\mathbf{z} \in L_1), \quad \begin{bmatrix} 0 & 0 & -n_2 & n_1 \\ n_2 & -n_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ n_1 & n_2 & n_1 & n_2 \end{bmatrix} (\mathbf{z} \in L_2), \quad \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ n_1 & n_2 & n_1 & n_2 \end{bmatrix} (\mathbf{z} \in L_3),
$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -n_2 & n_1 & -n_2 & n_1 \end{bmatrix} (\mathbf{z} \in L_4), \quad \begin{bmatrix} 0 & 0 & n_1 & n_2 \\ -n_1 & -n_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -n_2 & n_1 & -n_2 & n_1 \end{bmatrix} (\mathbf{z} \in L_5), \quad \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} (\mathbf{z} \in L_6),$$

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} (\mathbf{z} \in L_7), \quad \begin{bmatrix} 0 & 0 & 0 & 0 \\ -n_1 & -n_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & n_1 & n_2 \end{bmatrix} (\mathbf{z} \in L_8), \quad \begin{bmatrix} 0 & 0 & 0 & 0 \\ n_2 & -n_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -n_2 & n_1 \end{bmatrix} (\mathbf{z} \in L_9).$$

§ 6 The kernel of the operator $\boldsymbol{\Omega}_p$

In order to study the connection between the equations (5.4) and (5.10), the linear space ker $\boldsymbol{\Omega}_p$ has to be determined, whereby the domain of the operator $\boldsymbol{\Omega}_p$ is given by the restrictions of § 2 on the contact data.

Obviously, (5.10) has the same solutions as the equation

$$\mathcal{A}\Phi = \mathbf{w} + A_1\mathbf{w}_1 + A_2\mathbf{w}_2 + \mathbf{h} \tag{6.1}$$

with arbitrary $\mathbf{h} \in \ker \boldsymbol{\Omega}_p$. Because the action of the operator $\left(\frac{d}{ds} + p\right)$ to a function $v = v(s)$ given on the arc $(\mathbf{a}_i, \mathbf{a}_{i+1})$ is $\left(\frac{d}{ds} + p\right) v(s) = v'(s) + pv(s)$, the equation $\left(\frac{d}{ds} + p\right) v(s) = 0$ implies

$$v(s) = Ce^{-ps}, \quad C \text{ — arbitrary constant.} \tag{6.2}$$

This remark permits us to establish the general vector of ker $\boldsymbol{\Omega}_p$. Indeed, for $\mathbf{h} \in \ker \boldsymbol{\Omega}_p$ we have

$$\mathbf{h}(s) = \mathbf{h}(\mathbf{z}(s)) = \sum_{\nu=1}^{9} \sum_{\mu=1}^{m_\nu} \sum_{\substack{l=1 \\ l \in T}}^{4} C^{\nu}_{\mu l} \mathbf{v}^{\nu}_{\mu l}\, e^{-ps}. \tag{6.3}$$

Here the $C^{\nu}_{\mu l}$ are arbitrary real constants, and the vectors $\mathbf{v}^{\nu}_{\mu l}$ are given by the formula

$$\mathbf{v}^{\nu}_{\mu l}(\mathbf{z}) = \delta_{\nu\varkappa}\delta_{\mu m} \begin{bmatrix} \delta_{l1} \\ \delta_{l2} \\ \delta_{l2} \\ \delta_{l4} \end{bmatrix} \quad \text{for} \quad \mathbf{z} \in (\mathbf{a}_{\varkappa_m}, \mathbf{a}_{\varkappa_m+1}). \tag{6.4}$$

The restriction $l \in T$ in (6.3) means that the addition should only be extended over such numbers $l = 1, 2, 3, 4$ which correspond to those equations of systems (5.10) to which the operator $\left(\frac{d}{ds} + p\right)$ was applied. Thus, for $\nu = 1$ the symbol $l \in T$ means $l = 1, 2$, for $\nu = 2 : l = 3$, for $\nu = 3 : l = 1, 2, 3$ etc.

Obviously, the vectors $\mathbf{v}^{\nu}_{\mu l}$ are linearly independent. Therefore the linear space generated by the vectors (6.3) is of dimension

$$r = 2m_1 + m_2 + 3m_3 + 3m_4 + m_5 + 4m_6 + 2m_8 + 2m_9. \tag{6.5}$$

However, a more sophisticated consideration shows that the general element $\mathbf{h}$ of $\ker \Omega_p$ can be determined more exactly. Indeed, the assumption $\Phi \in H^*$ implies the continuity of the potential $\mathbf{V}(\mathbf{x}; \Phi)$. Hence the constants $C^{\nu}_{\mu l}$ in (6.3) must satisfy certain linear relations, which can be obtained in the same way as the compatibility conditions (1.6a)—(1.6x).

Let s_i be the arc length at node $\mathbf{a}_i$ ($i = 1, \ldots, m$). We agree to start the numeration from the point $\mathbf{a}_1$. Accordingly, the point $\mathbf{a}_1$ has the arc parameter $s_1^+ = 0$, if the section $S_1 = (\mathbf{a}_1, \mathbf{a}_2)$ is considered. However, in consideration of $(\mathbf{a}_m, \mathbf{a}_1)$ the node $\mathbf{a}_i$ has the arc parameter $s_1^- = L$ (L — arc length of L). Clearly, for the remaining nodes we have $s_i^+ = s_i^- = s_i$ ($i = 2, 3, \ldots, m$).

In order to formulate the above-mentioned linear relations, let us assume that the considered node $\mathbf{a}_i$ is a common end point of the two curve systems L_ν and $L_\varkappa$ ($\nu \neq \varkappa$). Let $\mathbf{a}_i$ belong to the μ-th (m-th) arc $S_{\nu_\mu}(S_{\varkappa_m})$ of $L_\nu(L_\varkappa)$. Then the following relations are necessary:

for $\nu = 1$, $\varkappa = 2$:

$$(n_1 C^1_{\mu 1} + n_2 C^1_{\mu 2})\, e^{-ps_i^{\pm}} = C^2_{m3}\, e^{ps_i^{\mp}}; \tag{6.6a}$$

for $\nu = 1$, $\varkappa = 3$:

$$\begin{aligned} C^1_{\mu 1}\, e^{-ps_i^{\pm}} &= \{-(C^3_{m1} - C^3_{m2})\, n_2 + C^3_{m3} n_1\}\, e^{-ps_i^{\mp}}, \\ C^1_{\mu 2}\, e^{-ps_i^{\pm}} &= \{(C^3_{m1} - C^3_{m2})\, n_1 + C^3_{m3} n_2\}\, e^{-ps_i^{\mp}}; \end{aligned} \tag{6.6b}$$

for $\nu = 1$, $\varkappa = 4$:

$$\begin{aligned} C^1_{\mu 1}\, e^{-ps_i^{\pm}} &= \{(C^4_{m1} - C^4_{m2})\, n_1 - C^4_{m3} n_2\}\, e^{-ps_i^{\mp}}, \\ C^1_{\mu 2}\, e^{-ps_i^{\pm}} &= \{(C^4_{m1} - C^4_{m2})\, n_2 + C^4_{m3} n_1\}\, e^{-ps_i^{\mp}}; \end{aligned} \tag{6.6c}$$

for $\nu = 1$, $\varkappa = 5$:

$$(-n_2 C^1_{\mu 1} + n_1 C^1_{\mu 2})\, e^{-ps_i^{\pm}} = C^5_{m3}\, e^{-ps_i^{\mp}}; \tag{6.6d}$$

for $\nu = 1$, $\varkappa = 6$:

$$C^1_{\mu 1}\, e^{-ps_i^{\pm}} = (C^6_{m3} - C^6_{m1})\, e^{-ps_i^{\mp}}, \qquad C^1_{\mu 2}\, e^{-ps_i^{\pm}} = (C^6_{m4} - C^6_{m2})\, e^{-ps_i^{\mp}}; \tag{6.6e}$$

for $\nu = 1$, $\varkappa = 8$:

$$(-C^1_{\mu 1} n_2' + C^1_{\mu 2} n_1)\, e^{-ps_i^{\pm}} = (C^8_{m3} - C^8_{m1})\, e^{-ps_i^{\mp}}; \tag{6.6f}$$

for $\nu = 1$, $\varkappa = 9$:

$$(C^1_{\mu 1} n_1 + C^1_{\mu 2} n_2)\, e^{-ps_i^{\pm}} = (C^9_{m3} - C^9_{m1})\, e^{-ps_i^{\mp}}; \tag{6.6g}$$

for $\nu = 2$, $\varkappa = 3$:

$$C^2_{\mu 3}\, e^{-ps_i^{\pm}} = C^3_{m3}\, e^{-ps_i^{\mp}}; \tag{6.6h}$$

for $\nu = 2$, $\varkappa = 4$:

$$C^2_{\mu 3}\, e^{-ps_i^{\pm}} = (C^4_{m1} - C^4_{m2})\, e^{-ps_i^{\mp}}; \tag{6.6i}$$

for $\nu = 2$, $\varkappa = 6$:

$$C^2_{\mu 3}\, e^{-ps_i^{\pm}} = \{(C^6_{m3} - C^6_{m1})\, n_1 + (C^6_{m4} - C^6_{m2})\, n_2\}\, e^{-ps_i^{\mp}}; \tag{6.6j}$$

for $\nu = 2$, $\varkappa = 9$:

$$C^2_{\mu 3}\, e^{-ps_i^{\pm}} = (C^9_{m3} - C^9_{m1})\, e^{-ps_i^{\mp}}; \tag{6.6k}$$

for $\nu = 3, \varkappa = 4$:

$$\{C^3_{\mu 3} n_1 - (C^3_{\mu 1} - C^3_{\mu 2}) n_2\} e^{-ps_t \pm} = \{(C^4_{m1} - C^4_{m2}) n_1 - C^4_{m3} n_2\} e^{-ps_t \mp},$$
$$\{C^3_{\mu 3} n_2 + (C^3_{\mu 1} - C^3_{\mu 2}) n_1\} e^{-ps_t \pm} = \{(C^4_{m1} - C^4_{m2}) n_2 + C^4_{m3} n_1\} e^{-ps_t \mp}; \tag{6.6l}$$

for $\nu = 3, \varkappa = 5$:

$$(C^3_{\mu 1} - C^3_{\mu 2}) e^{-ps_t \pm} = C^5_{m3} e^{-ps_t \mp}; \tag{6.6m}$$

for $\nu = 3, \varkappa = 6$:

$$C^3_{\mu 2} e^{-ps_t \pm} = (-n_2 C^6_{m1} + n_1 C^6_{m2}) e^{-ps_t \mp}, \quad C^3_{\mu 1} e^{-ps_t \pm} = (-n_2 C^6_{m3} + n_1 C^6_{m4}) e^{-ps_t \mp},$$
$$C^3_{\mu 3} e^{-ps_t \pm} = \{(C^6_{m3} - C^6_{m1}) n_1 + (C^6_{m4} - C^6_{m2}) n_2\} e^{-ps_t \mp}; \tag{6.6n}$$

for $\nu = 3, \varkappa = 8$:

$$C^3_{\mu 2} e^{-ps_t \pm} = C^8_{m1} e^{-ps_t \mp}, \quad C^3_{\mu 1} e^{-ps_t \pm} = C^8_{m3} e^{-ps_t \mp}; \tag{6.6o}$$

for $\nu = 3, \varkappa = 9$:

$$C^3_{\mu 3} e^{-ps_t \pm} = (C^9_{m3} - C^9_{m1}) e^{-ps_t \mp}; \tag{6.6p}$$

for $\nu = 4; \varkappa = 5$:

$$C^4_{\mu 3} e^{-ps_t \pm} = C^5_{m3} e^{-ps_t \mp}; \tag{6.6q}$$

for $\nu = 4, \varkappa = 6$:

$$C^4_{\mu 1} e^{-ps_t \pm} = (n_1 C^6_{m3} + n_2 C^6_{m4}) e^{-ps_t \mp}, \quad C^4_{\mu 2} e^{-ps_t \pm} = (n_1 C^6_{m1} + n_2 C^6_{m2}) e^{-ps_t \mp}$$
$$C^4_{\mu 3} e^{-ps_t \pm} = \{(C^6_{m4} - C^6_{m2}) n_1 - (C^6_{m3} - C^6_{m1}) n_2\} e^{-ps_t \mp}; \tag{6.6r}$$

for $\nu = 4, \varkappa = 8$:

$$C^4_{\mu 3} e^{-ps_t \pm} = (C^8_{m3} - C^8_{m1}) e^{-ps_t \mp}; \tag{6.6s}$$

for $\nu = 4, \varkappa = 9$:

$$C^4_{\mu 1} e^{-ps_t \pm} = C^9_{m3} e^{-ps_t \mp}, \quad C^4_{\mu 2} e^{-ps_t \pm} = C^9_{m1} e^{-ps_t \mp}; \tag{6.6t}$$

for $\nu = 5, \varkappa = 6$:

$$C^5_{\mu 3} e^{-ps_t \pm} = \{-(C^6_{m3} - C^6_{m1}) n_2 + (C^6_{m4} - C^6_{m2}) n_1\} e^{-ps_t \mp}; \tag{6.6u}$$

for $\nu = 5, \varkappa = 8$:

$$C^5_{\mu 3} e^{-ps_t \pm} = (C^8_{m3} - C^8_{m1}) e^{-ps_t \mp}; \tag{6.6v}$$

for $\nu = 6, \varkappa = 8$:

$$(-n_2 C^6_{\mu 1} + n_1 C^6_{\mu 2}) e^{-ps_t \pm} = C^8_{m1} e^{-ps_t \mp}, \quad (-n_2 C^6_{\mu 3} + n_1 C^6_{\mu 4}) e^{-ps_t \pm} = C^8_{m3} e^{-ps_t \mp}; \tag{6.6w}$$

for $\nu = 6, \varkappa = 9$:

$$(n_1 C^6_{\mu 1} + n_2 C^6_{\mu 2}) e^{-ps_t \pm} = C^9_{m1} e^{-ps_t \mp}, \quad (n_1 C^6_{\mu 3} + n_2 C^6_{\mu 4}) e^{-ps_t \pm} = C^9_{m3} e^{-ps_t \mp}. \tag{6.6x}$$

The equations (6.6a)–(6.6x) form a homogeneous system of linear equations for some constant $C^\nu_{\mu l}$ of (6.3).

Using the definition of the numbers $A_{\nu\mu}$ from § 1 (see (1.7)), one can easily see that the above-mentioned linear system consists of exactly q equations, where

$$\begin{aligned} q = [&A_{12} + A_{15} + A_{18} + A_{19} + A_{23} + A_{24} + A_{26} + A_{29} + A_{35} + A_{39} + A_{45} \\ &+ A_{48} + A_{56} + A_{58}] + 2[A_{13} + A_{14} + A_{16} + A_{34} + A_{38} + A_{49} + A_{68} + A_{69}] \\ &+ 3[A_{36} + A_{46}]. \end{aligned} \tag{6.7}$$

The considered linear system is written in matrix form as follows:

$$\mathbf{KC} = \mathbf{0}, \qquad \begin{array}{l} \mathbf{K} \text{ — coefficient matrix} \\ \mathbf{C} \text{ — vector with elements } C_{\mu l}^{\nu}. \end{array} \tag{6.8}$$

In the sequel, it will be proved that (6.8) has the rank q, at least for most values of the constant p.

For proof, some remarks and preparations are necessary.

1. The nodes $\mathbf{a}_i$ are divided into two groups. In this respect, a considered node $\mathbf{a}_i$ is called of first kind if no compatibility condition belongs to the passage from S_{i-1} to S_i. Obviously, $\mathbf{a}_i$ is of first kind if and only if $\mathbf{a}_i$ has one of the following type: $L_j - L_7$ ($j = 1, 2, \dots, 6, 8, 9$), $L_2 - L_5$, $L_2 - L_8$, $L_5 - L_9$, $L_8 - L_9$. The remaining nodes are called of second kind.
2. Suppose that there exist exactly t_1 nodes of first kind and t of second kind ($t_1 + t = m$). Then the system (6.8) is arranged in exactly t groups of linear equations expressing the equations (6.6a)—(6.6x) at the t nodes $\mathbf{a}_i$ of second kind. Each such group consists of exactly k_i linear homogeneous equations ($k_i = 1, 2$ or 3). It is not difficult to see that every constant $C_{\mu l}^{\nu}$ of system (6.8) is met in at most two such groups.
3. The formulas (6.6a)—(6.6x) show that the equation group at the fixed node $\mathbf{a}_i$ (consisting of k_i equations) contains at least k_i different constants $C_{\mu l}^{\nu}$ connected with the arc $S_i = (\mathbf{a}_i, \mathbf{a}_{i+1})$. Above them, with those constants $C_{\mu l}^{\nu}$ (belonging to the mentioned group and connected with the arc S_i) one can always form a nonsingular (k_i, k_i)-block with non-vanishing coefficient determinant. Obviously, the coefficient matrix of such a (k_i, k_i)-block contains the factor e^{-ps_i} and is representable in the form $\mathrm{e}^{-ps_i}\mathbf{A}^{(k_i)}$ with a non-singular (k_i, k_i)-matrix $\mathbf{A}^{(k_i)}$.

Now, a preliminary result is

Lemma 1: *Suppose there exists at least one node of first kind. Then the system (6.8) has the rank q.*

Let us assume that there exists at least one node of second kind (else the proof is superfluous). With the above remarks it is easily seen that the coefficient matrix $\mathbf{K}$ contains a (q, q)-submatrix of the form

$$\mathbf{A} = \begin{bmatrix} \mathrm{e}^{-p\tilde{s}_1}\mathbf{A}^{(\tilde{k}_1)} & & 0 & \\ \mathrm{e}^{-p\tilde{s}_2}\mathbf{B}_{(\tilde{k}_2,\tilde{k}_1)} & \mathrm{e}^{-p\tilde{s}_2}\mathbf{A}^{(\tilde{k}_2)} & & \\ & \ddots & \ddots & \\ & & \mathrm{e}^{-p\tilde{s}_{t-1}}\mathbf{A}^{(\tilde{k}_2)} & \\ 0 & & \mathrm{e}^{-p\tilde{s}_{t-1}}\mathbf{B}_{(\tilde{k}_t,\tilde{k}_{t-1})} & \mathrm{e}^{-p\tilde{s}_t}\mathbf{A}^{(\tilde{k}_t)} \end{bmatrix}. \tag{6.9}$$

Herein, $\mathbf{B}_{(\tilde{k}_l,\tilde{k}_{l-1})}$ are suitable matrices of the format $(\tilde{k}_l, \tilde{k}_{l-1})$, which are uniquely determined by fixed chosen $\mathbf{A}^{(\tilde{k}_l)}$ and $\mathbf{A}^{(\tilde{k}_{l-1})}$. (We remark that the counting of $\tilde{k}_1, \dots, \tilde{k}_t$ and $\tilde{s}_1, \dots, \tilde{s}_t$ does not coincide with the above defined $k_1, \dots, k_m$ and $s_1, \dots, s_m$, respectively, because here we are concerned only with nodes of second kind. The actual counting in (6.9) starts from such a node of second kind the left neighbour of which is of first kind. Of course, the matrix $\mathbf{A}^{(\tilde{k}_l)}$ is equal to the matrix $\mathbf{A}^{(k_j)}$ of remark 3 with a suitable $j = j(i)$). Now, the Laplace theorem implies

$$\det \mathbf{A} = \mathrm{e}^{-p(\tilde{s}_1\tilde{k}_1 + \cdots + \tilde{s}_t\tilde{k}_t)} \prod_{l=1}^{t} \det \mathbf{A}^{(\tilde{k}_l)} \neq 0.$$

The lemma is proved ∎

We remark that Lemma 1 remains valid for $p = 0$. In case of absence of nodes of first kind, the following a little weaker result is proved.

Lemma 2: *The system* (6.8) *has the rank* q, *provided that* p *is not equal to at most three exceptional values.*

Indeed, one can form a (q, q)-submatrix $\mathbf{A}$ of $\mathbf{K}$ as follows:

$$\mathbf{A} = \begin{bmatrix} e^{-ps_1}\mathbf{A}^{(k_1)} & & & & e^{-p(s_1+L)}\mathbf{B}^{(k_1,k_m)} \\ e^{-ps_2}\mathbf{B}_{(k_2,k_1)} & e^{-ps_2}\mathbf{A}^{(k_2)} & 0 & & \\ & \ddots & \ddots & & \\ & & e^{-ps_{m-1}}\mathbf{A}^{(k_{m-1})} & & \\ 0 & & e^{-ps_m}\mathbf{B}_{(k_m,k_{m-1})} & & e^{-ps_m}\mathbf{A}^{(k_m)} \end{bmatrix}. \tag{6.10}$$

(In this formula the constants k_i are defined in accordance with remark 3.) In order to count det $\mathbf{A}$, the factor $e^{-pk_1s_1}$ is taken from the first k_1 rows of det $\mathbf{A}$, the factor $e^{-pk_2s_2}$ from the next k_2 rows, and so on. Using the Laplace theorem with respect to the first k_1 rows of the remaining determinant, one gets

$$\det \mathbf{A} = e^{-p(s_1k_1+\cdots+s_mk_m)}\{a_0 + a_1 e^{-pL} + \cdots + a_k(e^{-pL})^k\} \tag{$*$}$$

with

$$a_0 = \prod_{l=1}^{m} \det \mathbf{A}^{(k_l)} \neq 0, \qquad 1 \leqq k = \min(k_1, k_m) \leqq 3.$$

($*$) is a polynomial of the variable (e^{-pL}) with maximal degree 3. In virtue of $a_0 \neq 0$, this polynomial does not vanish identically. Hence we have det $\mathbf{A} \neq 0$ with exception of at most three values for p. Thus, the lemma is proved ∎

The lemmata 1 and 2 lead us to the following theorem.

Theorem 1: *Suppose that* p *has no exceptional value of Lemma* 2. *Then the linear space* ker $\mathbf{\Omega}_p$ *is of dimension*

$$\begin{aligned} r - q &= 2m_1 + m_2 + 3m_3 + 3m_4 + m_5 + 4m_6 + 2m_8 + 2m_9 \\ &\quad - (A_{12} + A_{15} + A_{18} + A_{19} + A_{23} + A_{24} + A_{26} + A_{29} + A_{35} + A_{39} \\ &\quad + A_{45} + A_{48} + A_{56} + A_{58}) \\ &\quad - 2(A_{13} + A_{14} + A_{16} + A_{34} + A_{38} + A_{49} + A_{68} + A_{69}) - 3(A_{36} + A_{46}) \\ &= \frac{1}{2}\{(A_{12} + A_{13} + A_{14} + A_{15} + A_{27} + A_{29} + A_{36} + A_{38} + A_{46} \\ &\quad + A_{49} + A_{57} + A_{58}) + 2(A_{16} + A_{17} + A_{18} + A_{19} + A_{23} + A_{24} + A_{25} \\ &\quad + A_{34} + A_{35} + A_{45} + A_{68} + A_{69} + A_{78} + A_{79}) \\ &\quad + 3(A_{26} + A_{28} + A_{37} + A_{39} + A_{47} + A_{48} + A_{56} + A_{59}) + 4(A_{67} + A_{89})\}. \end{aligned} \tag{6.11}$$

The first equality follows immediately from (6.5) and (6.7) with consideration of the lemmata 1 and 2. The second equality is a consequence of formula (1.7) ∎

For illustration we consider the linear system (6.8) in two particular cases.

Example 1: Let $L = L_1 \cup L_0$ and L consists of two arcs S_1, S_2. Here system (6.8) is of the form

$$\begin{array}{lllllll}
e^{-ps_1}C_{11}^1 & + & e^{-p(s_1+L)}C_{11}^6 & - & e^{-p(s_1+L)}C_{13}^6 & & = 0\\
& e^{-ps_1}C_{12}^1 & + & e^{-p(s_1+L)}C_{12}^6 & - & e^{-p(s_1+L)}C_{14}^6 & = 0\\
e^{-ps_2}C_{11}^1 & + & e^{-ps_2}C_{11}^6 & - & e^{-ps_2}C_{13}^6 & & = 0\\
& e^{-ps_2}C_{12}^1 & + & e^{-ps_2}C_{12}^6 & - & e^{-ps_2}C_{14}^6 & = 0.
\end{array}$$

A submatrix with maximal rank is

$$\mathbf{A} = \begin{bmatrix} e^{-ps_1} & 0 & e^{-p(s_1+L)} & 0 \\ 0 & e^{-ps_1} & 0 & e^{-p(s_1+L)} \\ e^{-ps_2} & 0 & e^{-ps_2} & 0 \\ 0 & e^{-ps_2} & 0 & e^{-ps_2} \end{bmatrix}.$$

One gets $\det \mathbf{A} = e^{-2p(s_1+s_2)}(e^{-pL} - 1)^2$. Hence we have $\det \mathbf{A} = 0$ only for $p = 0$.

Example 2: Let $L = S_1 \cup S_2 \cup S_3 \cup S_4$ and $S_1 \cup S_3 = L_1$, $S_1 \cup S_4 = L_2$. Here, system (6.8) describes relations between the constants $C_{13}^2, C_{23}^2, C_{11}^1, C_{12}^1, C_{21}^1, C_{22}^1$. The coefficient matrix of (6.8) is

$$\mathbf{K} = \begin{bmatrix} n_1^{(1)} e^{-ps_1} & n_2^{(1)} e^{-ps_1} & 0 & 0 & 0 & -e^{-p(s_1+L)} \\ n_1^{(2)} e^{-ps_2} & n_2^{(2)} e^{-ps_2} & -e^{-ps_2} & 0 & 0 & 0 \\ 0 & 0 & -e^{-ps_3} & n_1^{(3)} e^{-ps_3} & n_2^{(3)} e^{-ps_3} & 0 \\ 0 & 0 & 0 & n_1^{(4)} e^{-ps_4} & n_2^{(4)} e^{-ps_4} & -e^{-ps_4} \end{bmatrix}.$$

It is not difficult to see that the rank of $\mathbf{K}$ is equal to 4, if one of the pairs of normal vectors $\mathbf{n}^{(1)}$, $\mathbf{n}^{(2)}$ and $\mathbf{n}^{(3)}$, $\mathbf{n}^{(4)}$ is linearly independent.

Consider the case where $\mathbf{n}^{(1)} = \mathbf{n}^{(2)}$ and $\mathbf{n}^{(3)} = \pm\mathbf{n}^{(4)}$. Here, a submatrix of maximal rank is of the form

$$\mathbf{A} = \begin{bmatrix} n\, e^{-ps_1} & 0 & 0 & -e^{-p(s_1+L)} \\ n\, e^{-ps_2} & -e^{-ps_2} & 0 & 0 \\ 0 & -e^{-ps_3} & \tilde{n}\, e^{-ps_3} & 0 \\ 0 & 0 & \pm \tilde{n} e^{-ps_4} & -e^{-ps_4} \end{bmatrix}$$

with $n, \tilde{n} \neq 0$. We have

$$\det \mathbf{A} = e^{-2p(s_1+s_2)} n\tilde{n} \begin{vmatrix} 1 & 0 & 0 & -e^{-pL} \\ 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & \pm 1 & -1 \end{vmatrix} = n\tilde{n}\, e^{-2p(s_1+s_2)}\{1 \mp e^{-pL}\}.$$

This implies $\det \mathbf{A} \neq 0$ for every $p \in \mathbf{R}$ in the case $\mathbf{n}^{(3)} = -\mathbf{n}^{(4)}$. For $\mathbf{n}^{(3)} = \mathbf{n}^{(4)}$ we have $\det \mathbf{A} = 0$ if and only if $p = 0$.

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