A Remark on a Paper by D. E. Edmunds, A. Kufner and J. Rákosník "Embeddings of Sobolev Spaces with Weights of Power Type"1)

H. Triebel

Die Arbeit, ist Approximationseigenschaften für gewichtete Sobolev-Slobodeckij-Räume gewidmet.

Работа посвящена свойствам аппроксимации в пространствах типа Соболева-Слободецкого с весами.

The paper deals with approximation properties in weighted spaces of Sobolev-Slobodeckij type.

1. Definitions and Results

Let M be a non-empty closed subset of the Euclidean n-space \mathbf{R}_n . Let Ω be a domain in \mathbf{R}_n with $\varOmega \subset \mathbf{R}_n - M$ (we may assume, without restriction of generality, that M is a subset of the boundary of Ω , i.e. $M \subset \partial \Omega = \overline{\Omega} - \Omega$; however this is not important in the sequel). If $x \in \mathbb{R}_n$ then dist $(x, M) = \inf |x - y|$ is the usual distance, where the infimum is taken over all $y \in M$. We mollify dist (x, M) : There exists a positive function $d_M(x) \in C^\infty(\mathbf{R}_n - M)$ and two positive numbers c_1 and c_2 with

$$
c_1d_M(x) \leq \text{dist}(x, M) \leq c_2d_M(x), \quad x \in \mathbf{R}_n - M,
$$

cf. the construction in [2: Remark 3.2.3/1], where $d_M(x)$ coincides essentially with $\varrho^{-1}(x)$ (in contrast to the construction given there we do not care about the behaviour of $d_M(x)$ if $|x|$ tends to infinity, but this does not affect our arguments). Let $1 < p < \infty$ and $-\infty < \varepsilon < \infty$. If s is a non-negative integer then $W_p^s(Q, \varepsilon)$ is the collection of all complex distributions $f \in D'(\Omega)$ such that

$$
||f|W_p^{s}(\Omega,\,\varepsilon)||=\sum_{|\alpha|\leq s}\left(\int\limits_{\Omega}|(D^{\alpha}f)\,(x)|^p\,d_M^{s-(s-|\alpha|)p}(x)\,dx\right)^{1/p}<\infty\tag{1}
$$

holds. If $0 < s = [s] + \{s\}$, where [s] is an integer and $0 < s < 1$ then $W_s^s(\Omega, \varepsilon)$ is the collection of all complex distributions $f \in D'(Q)$ such that

$$
||f||W_p^s(\Omega, \varepsilon)||
$$

=
$$
\sum_{|\alpha| \leq |s|} \left(\int_{\Omega \times \Omega} \frac{|d_M^{\varepsilon/p - [s] + |\alpha|}(x) D^{\alpha} f(x) - d_M^{\varepsilon/p - [s] + |\alpha|}(y) D^{\alpha} f(y)|^p}{|x - y|^{n + \{s\}p}} dx dy \right)^{1/p}
$$

+
$$
||f||W_p^{[s]}(\Omega, \varepsilon - \{s\}p)|| < \infty
$$
 (2)

holds. These weighted Sobolev-Slobodeckij spaces are near to the spaces $W_n^{\bullet}(\Omega, \rho^{\mu}, \rho^{\nu})$ in [2], where $\varrho^{-1}(x)$ is essentially $d_M(x)$, $\mu = -\varepsilon$ and $\nu = \mu + sp$: Formula (3.2.3/11)

1) The preceding paper of this issue.

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(where the misprint $B_{p,q}^s$ must be corrected by $B_{p,p}^s$), Theorems 1, 2, and 3 in 3.2.4.

The differences between the above spaces $W_p^s(\Omega, \varepsilon)$ and the spaces $W_p^s(\Omega, \varrho^{\mu}, \varrho^{\nu})$

are characteriz The difference of *B*_{*b*,*e*} *(where the misprint* $B_{p,q}^s$ *must be corrected by* $B_{p,p}^s$ *), Theorems 1, 2, and 3 in 3.2.4.* The differences between the above spaces $\tilde{W}_{p}^{s}(\tilde{\Omega}, \varepsilon)$ and the spaces $W_{p}^{s}(\Omega, \varrho^{\mu}, \varrho^{\nu})$ are characterized by the different behaviour of $\rho(x)$ and $d_M^{-1}(x)$ at infinity and that we have $\partial\Omega$ instead of M in [2]. However in what follows these differences are immaterial. We give a full proof of our main result and take over only some technicalities from $[2: 3.2.3$ and $3.2.4]$. Obviously, $W_p^s(Q, \varepsilon)$ is a Banach space. ⁹⁶

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deterized by the different behaviour of $\varrho(x)$ and $d_M^{-1}(x)$
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are characterized by the different b

Theorem: Let $1 < p < \infty$, $s > 0$ and $-\infty < \varepsilon < \infty$. Then

$$
\{f \mid f \in W_{p}^{s}(\Omega, \varepsilon), \operatorname{supp} f \cap M = \emptyset\}
$$

Remark 1: We recall that we may assume that *M* is a subset of $\partial\Omega.$ If s is a natural number then $W_p^s(Q, \varepsilon)$ coincides with the spaces $H^{s,p}(Q; d_M, \varepsilon)$ from [1]. In [1] it has

 (3)

$$
C_M^{\infty}(\Omega) = \{w \mid w \in C^{\infty}(\overline{\Omega}), \text{supp } w \cap M = \emptyset\}
$$

is dense in $H^{s,p}(\Omega, d_M, \varepsilon)$. Our theorem (restricted to natural *s*) is not a new proof of this assertion, however-it shows that the weighted case can be reduced to the unweighted ease, and this simplifies the task considerably. On the other hand, in contrast to [1], we deal also with fractional spaces (and only for that purpose we mollified dist (x, M)).

Remark 2: Let Ω be a bounded domain and $M = \partial \Omega$. Then we have

$$
W_p^{s}(\Omega,\varepsilon) = W_p^{s}(\Omega,\varrho^{-\varepsilon},\varrho^{-\varepsilon+sp})
$$

with $\varrho^{-1}(x) = d_M(x)$ if $x \in \Omega$, where $1 < p < \infty$, $s \geq 0$ and $-\infty < \varepsilon < \infty$, cf. [2: Theorem 3.2.4/3] and Remark 3 at the end of this paper. By $[2:$ Theorem 3.2.4/1], $C_0^{\infty}(\Omega)$ is dense in $W_p^s(\Omega, \varepsilon)$. This extends Theorem 1.1 in [1] to the fractional spaces, at least if Ω is bounded. with $\varrho^{-1}(x) = d_M(x)$ if $x \in \Omega$, where $1 < p < \infty$, s

Theorem 3.2.4/3] and Remark 3 at the end of this
 $C_0^{\infty}(\Omega)$ is dense in $W_p^s(\Omega, \varepsilon)$. This extends Theorem

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2. Proof of the Theorem
 $\begin{aligned}\n\mathbf{w}_1 &= \mathbf{w}_1 \mathbf{w}_2 \mathbf{w}_1 \mathbf{w}_2 \mathbf{w}_2 \mathbf{w}_3 \\
&= \mathbf{w}_1 \mathbf{w}_2 \mathbf{w}_3 \mathbf{w}_4 \mathbf{w}_5 \mathbf{w}_5 \mathbf{w}_6 \mathbf{w}_7 \mathbf{w}_7 \mathbf{w}_8\n\end{aligned}$ (a) is dense in $W_p^s(Q, \varepsilon)$. This extends Theorem 1.1 in [1] to the fractiona

2. Proof of the Theorem

2. Proof of the Theorem \overline{W} we prove the above theorem in two steps;

Step 1: Let *s* be a natural number. Let

of the Theorem
\nthe above theorem in two steps.
\nLet s be a natural number. Let
\n
$$
M^{(j)} = \{x \mid x \in \mathbf{R}_n, d_M(x) > 2^{-j}\} \quad \text{if} \quad j = 1, 2, ...
$$
\n(5)

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$$
M_1 = M^{(3)}, \quad M_k = M^{(k+2)} - \overline{M^{(k-1)}} \quad \text{if} \quad k = 2, 3, ...
$$
 (6)

We assume $M^{(1)} \cap \Omega = \emptyset$ (without restriction of generality). Let $\psi = {\psi_i(x)}_{i=1}^{\infty}$ be

$$
M^{(j)} = \{x \mid x \in \mathbb{R}_n, d_M(x) > 2^{-j}\} \quad \text{if} \quad j = 1, 2, \ldots
$$
\n(5)

\nand

\n
$$
M_1 = M^{(3)}, \quad M_k = M^{(k+2)} - \overline{M^{(k-1)}} \quad \text{if} \quad k = 2, 3, \ldots
$$
\n(6)

\nWe assume $M^{(1)} \cap \Omega \neq \emptyset$ (without restriction of generality). Let $\psi = \{\psi_j(x)\}_{j=1}^{\infty}$ be a system of infinitely differentiable functions on \mathbb{R}_n with the following properties:

\n
$$
0 \leq \psi_j(x) \leq 1, \quad \text{supp } \psi_j \subset M_j, \quad \sum_{j=1}^{\infty} \psi_j(x) = 1 \quad \text{if} \quad x \in \mathbb{R}_n - M; \tag{7}
$$
\nor any multi-index γ there exists a positive number c , with

\n
$$
|D^r \psi_j(x)| \leq c_2 2^{|y|} \quad \text{for all } j = 1, 2, 3, \ldots \text{ and all } x \in \mathbb{R}_n.
$$
\n(8)

or any multi-index γ there exists a positive number c_r with

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Systems of this type exist, we refer to $[2:3.2.3]$. We claim that-

Remark on a Paper by D. E. EDMUNDS, ...
\n37
\n50f this type exist, we refer to [2: 3.2.3]. We claim that
\n
$$
\left(\sum_{j=1}^{\infty} ||\psi_j f| W_p^s(\Omega, \varepsilon)||^p\right)^{1/p} = ||f| W_p^s(\Omega, \varepsilon)||^p
$$
\n(9)
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 1 this type exist, we if
 $\sum_{j=1}^{\infty} ||\psi_j f|| W_p^s(\Omega, \varepsilon) ||^p$
 1 valent norm in $W_p^s(\Omega, \varepsilon)$

m above by $c ||f|| W_p$

at is an equivalent norm in $W_p^s(Q, \varepsilon)$. It is easy to see that $||f|| W_p^s(Q, \varepsilon)||$ can be estimated from above by $c \frac{||f||}{||h|}$ $W_p^s(Q, \varepsilon)$ $||\cdot|$. In order to prove the reverse assertion we remark that

Remark on a Paper by D. E. EDMUNDS, ... 37
\nSystems of this type exist, we refer to [2: 3.2.3]. We claim that\n
$$
\left(\sum_{j=1}^{\infty} ||\psi_j f| W_p^s(\Omega, \varepsilon)||^p\right)^{1/p} = ||f| W_p^s(\Omega, \varepsilon)||^p
$$
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$$
||\psi_j f| W_p^s(\Omega, \varepsilon)||^p \leq c \sum_{|\alpha| \leq s} 2^{-j\epsilon + j p(s - |\alpha|)} \int_{\alpha}^{\beta} |D^s(\psi_j f)(x)|^p dx
$$
\n
$$
\leq c' \sum_{|\alpha| \leq s} 2^{-j\epsilon + j p(s - |\alpha|)} \int_{\alpha} |D^s f(x)|^p dx
$$
\nSummation yields the desired result. Now it follows easily that\n
$$
\left(\sum_{|\alpha| \leq s}^N \psi_j\right) f \to f \quad \text{if } N \to \infty \quad (f \in W_p^s(\Omega, \varepsilon)).
$$
\nThis proves the theorem if s is a non-negative integer.\nSlenin the set of U is a unique integer and $0 < \langle s \rangle < 1$. Again we then that U is a positive integer, so U is a unique integer and $0 < \langle s \rangle < 1$. Again we obtain that U is a positive integer, so U is a unique integer, so U is a positive integer, so U

Summation yields the desired result. Now it follows easily that

$$
|\alpha| \leq s \quad D \cap M,
$$

on yields the desired result. Now it follows easily that

$$
\left(\sum_{j=1}^{N} \psi_j\right) f \to f \quad \text{if } N \to \infty \quad \left(f \in W_p^s(\Omega, \varepsilon)\right).
$$
 (11)
see the theorem if s is a non-negative integer.

This proves the theorem if *s* is a non-negative integer.

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Step 2: Let $0 < s = [s] + \{s\}$, where $[s]$ is an integer and $0 < \{s\} < 1$. Again we claim that $||f| \, W_p^s(Q, \varepsilon)||^p$ from (9) is an equivalent norm in $W_p^s(Q, \varepsilon)$. It is easy to see that $||f|| \overline{W_p}^s(\Omega, \varepsilon)||$ can be estimated from above by $c ||f|| \overline{W_p}^s(\Omega, \varepsilon)||^s$, cf. (2). In order to prove the reverse assertion we use (2) with $\psi_j f$ instead of *f*. The terms $||\psi_i|| \mathbf{W_p}^{[s]}(\Omega, \varepsilon - \{\delta\})||$ can be treated in the same way as in (10). In other words, the problem is reduced to an estimate of *If* $f \rightarrow f$ if $N \rightarrow \infty$ ($f \in W_p^s(\Omega, \varepsilon)$).
 If $N \rightarrow \infty$ ($f \in W_p^s(\Omega, \varepsilon)$).
 I heorem if *s* is a non-negative integer.
 $\langle g \rangle \langle g \rangle = [s] + \{s\}$, where $[s]$ is an integer and $0 \langle s \rangle \langle g \rangle$, $\langle g \rangle = [s]$ **F** from (9) $j=1$ \checkmark *f*
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 \checkmark is the theorem
 $\text{Let } 0 < s = 1$
 $|f | W_p^s(\Omega, \varepsilon)|$
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 $|f | W_p^s(\Omega, \varepsilon)|$
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 \checkmark of $|f$ Let $W_p^s(Q, \varepsilon) \parallel^p$ from (9) is an equivale $f \mid W_p^s(Q, \varepsilon) \parallel^p$ from (9) is an equivale $f \mid W_p^s(Q, \varepsilon) \parallel^p$ from (9) is an equivale $f \mid W_p^s(Q, \varepsilon) \parallel$ can be estimated from a co prove the reverse assertion we use (\frac Can that $||f|| W_p^s(\Omega, \varepsilon)||^p$ from (9) is an equivalent norm in $W_p^s(\Omega, \varepsilon)$
see that $||f|| W_p^s(\Omega, \varepsilon)||^p$ from (9) is an equivalent norm in $W_p^s(\Omega, \varepsilon)$
see that $||f|| W_p^s(\Omega, \varepsilon)||$ can be estimated from above by $c||f|| W_p$

$$
p_{ij}f \mid W_p^{[s]}(\Omega, \varepsilon - \{\delta\} p)\| \text{ can be treated in the same way as in (10). In other words,}
$$
\nthe problem is reduced to an estimate of

\n
$$
\sum_{j=1}^{\infty} \int_{\Omega \times \Omega} \frac{|d_M^{(e/p)-[s]+[s]}(x) D^s(\psi_j f)(x) - d^{(e/p)-[s]+[s]}(y) D^s(\psi_j f)(y)|^p}{|x - y|^{n + \{\delta\} p}} dx dy
$$
\n(12)

\nwith $|\alpha| \leq [s]$. Without restriction of generality we may assume that

\n
$$
\text{dist (supp } \psi_j, \mathbf{R}_n - M_j) \geq c2^{-j}, \tag{13}
$$
\nhere c is an appropriate positive number which is independent of j. Let $\Omega_j = \Omega \cap M_j$.

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$$
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where *c* is an appropriate positive number which is independent of j. Let $\Omega_i = \Omega \cap M_i$.

$$
[s^{\text{1}}](\hat{\Omega}, \varepsilon - \{\hat{s}\} \hat{p})]
$$
 can be treated in the same way as in (10). In other words,
lem is reduced to an estimate of

$$
\sum_{j=1}^{\infty} \int_{\Omega} \frac{|d_M^{(\varepsilon/p)-\{s\}+\{s\}}(x) D^s(\psi_j f)(x) - d^{(\varepsilon/p)-\{s\}+\{s\}}(y) D^s(\psi_j f)(y)|^p}{|x-y|^{n+\{s\}p}} dx dy
$$
(12)

$$
\leq [s].
$$
 Without restriction of generality we may assume that
dist (supp ψ_j , $\mathbf{R}_n - M_j$) $\geq c^{2-j}$,
s an appropriate positive number which is independent of j. Let $\Omega_j = \Omega \cap M_j$.
e sum in (12) can be estimated from above by

$$
2 \sum_{j=1}^{\infty} \left[\int_{\Omega_j \times \Omega_j} \frac{|d_M^{(\varepsilon/p)-\{s\}+\{s\}}(x) D^s(\psi_j f)(x) - d_M^{(\varepsilon/p)-\{s\}+\{s\}}(y) D^s(\psi_j f)(y)|^p}{|x-y|^{n+\{s\}p}} dx dy \right]
$$

$$
+ \int_{\Omega_j} d_M^{s-p(\{s\}-\{s\})} D^s(\psi_j f)(x)|^p \int_{\Omega_j} \frac{dy}{|x-y|^{n+\{s\}p}}.
$$
(14)
the hypothesis (13)' the second terms in (14) can be estimated from above by
 $c \sum_{j=1}^{\infty} \int_{\Omega_j} 2^{-j\epsilon+jp(s)+jp(\{s\}-\{s\})} |D^s(\psi_j f)(x)|^p dx$
 $\leq c ||f|| W_p^{s(\{s\}, \varepsilon) - \{s\} p)||^p \leq c' ||f|| W_p^{s(\{s\}, \varepsilon)||^p},$ (15)

Under the hypothesis $(13)'$ the second terms in (14) can be estimated from above by

$$
c \sum_{j=1}^{\infty} \int_{\Omega_j} 2^{-j\epsilon+jp(s)+jp(\lfloor s \rfloor - \lfloor s \rfloor)} |D^s(\psi_j f)(x)|^p dx
$$

\n
$$
\leq c ||f|| W_p^{(s)}(\Omega, \varepsilon - \frac{\lfloor s \rfloor}{2}) ||^p \leq c' ||f|| W_p^{s}(\Omega, \varepsilon) ||^p,
$$

 $\label{eq:2} \frac{1}{\sqrt{2}}\int_{0}^{\infty}\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^{2}d\theta\,d\theta.$

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where we used inequalities of type (10). The first terms in (14) can be estimated from above by a sum of terms of the form

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\nwhere we used inequalities of type (10). The first terms in (14) can be estimated from
\nabove by a sum of terms of the form
\n
$$
\sum_{j=1}^{\infty} \left[\int_{\alpha_j} |(D^{\beta}f)(x)|^p \int \frac{|d_M^{(s/p)-[s]+[a]}(x) D^{\gamma} \psi_j(x) - d_M^{(s/p)-[s]+[a]}(y) D^{\gamma} \psi_j(y)|^p}{|x-y|^{n+[s]p}} dy dx \right]
$$
\n+
$$
\int_{\alpha_j \times \Omega_j} d_M^{s-p([s]-[s])}(y) |D^{\gamma} \psi_j(y)|^p \frac{|(D^{\beta}f)(x) - (D^{\beta}f)(y)|^p}{|x-y|^{n+[s]p}} dx dy \right]
$$
\nwith $|\beta| + |\gamma| = |\alpha|$. The integral 'over M₁ in (16) can be estimated from above by
\n $c2^{-j\epsilon+j(s)p+jp([s]-[\beta])}$. This follows from [2: Lemma 3.2.4/2] (here we need that
\n $d_M(x)$ is a modified distance). Consequently, the first terms in (16) can be estimated
\nfrom above by c $||f||W_p^{[s]}(Q, \varepsilon - \{s\})p||^p$. The second terms in (16) can be estimated
\nfrom above by c $||f||W_p^{[s]}(Q, \varepsilon - \{s\})p||^p$. The second terms in (16) can be estimated
\nfrom above by
\n
$$
\sum_{j=1}^{\infty} 2^{-j\epsilon+jp([s]-[\beta])} \int_{\alpha_j \times \Omega_j} \frac{|(D^{\beta}f)(x) - (D^{\beta}f)(y)|^p}{|x-y|^{n+[s]p}} dx dy
$$

with $|\beta| + |\gamma| = |\alpha|$. The integral over M_f in (16) can be estimated from above by $c_2-i+1\frac{1}{3}$ ip+ip($|s|-1\beta|$). This follows from [2: Lemma 3.2.4/2] (here we need that $d_M(x)$ is a mollified distance). Consequently, the first terms in (16) can be estimated from above by $c ||f|| W_p^{[s]}(\Omega, \varepsilon - \{\text{s}\} p) ||^p$. The second terms in (16) can be estimated from above by $\begin{array}{l} \mathcal{V}[\,\,=\,|\alpha|. \,\, \text{The integral over M, in (1-2):$} \ \mathcal{V}^{(s_1-|\beta|)}. \,\, \text{This follows from [2: Le-
ollified distance). Consequently, the
by c $\|f\mid W_p^{(s)}(\Omega, \, \varepsilon \, -\, \langle s \rangle \, p)\|^p. \,\, \text{The is} \ \text{by} \ \mathcal{V}^{(s_1-|\beta|)} \quad \int\limits_{\Omega_f \times \Omega_f} \frac{|(D^\beta f)(x) - (D^\beta f)(x) - (D^\beta f)(x) - (D^\beta f)(x)}{|x - y|^{n +$$ $\frac{d}{dx} \left\{ \frac{d}{dx} - \frac{1}{2} \int_{0}^{\pi} \frac{d}{dx} dx \right\}$
 tegral over M_j in (16) can be esti-

blows from [2: Lemma 3.2.4/2]
 $\frac{d}{dx} \left\{ \frac{d}{dx} - \frac{1}{2} \int_{0}^{\pi} \frac{d}{dx} \int_{0}^{\pi} \frac{d}{dx} \int_{0}^{\pi} \frac{d}{dx} \int_{0}^{\pi} \frac{d}{dx} \int_{0}^{\$

$$
\sqrt{1-\frac{1}{2}}\left[\frac{1}{\rho_1} \int_{M_1}^{M_1} \frac{|x-y|^{n+|s|p}}{|x-y|^{n+|s|p}} dx dy\right]
$$
\n
$$
+ \int_{\rho_1 \times \Omega_1}^{1} d_M t^{-p(|s|-|x|)}(y) |D^r \psi_j(y)|^p \frac{|(D^{\beta}f)(x) - (D^{\beta}f)(y)|^p}{|x-y|^{n+|s|p}} dx dy\right]
$$
\nwith $|\beta| + |\gamma| = |\alpha|$. The integral over M_j in (16) can be estimated from above by $c2^{-j\epsilon+j|s|p+jp(|s|-|\beta|)}$. This follows from [2: Lemma 3.2.4/2] (here we need that $d_M(x)$ is a modified distance). Consequently, the first terms in (16) can be estimated from above by $c ||f|| W_p^{[s]}(\Omega, \varepsilon - \{s\} p)||^p$. The second terms in (16) can be estimated from above by\n
$$
\sum_{j=1}^{\infty} 2^{-j\epsilon+jp(|s|-|\beta|)} \int_{\Omega_j \times \Omega_j} \frac{|(D^{\beta}f)(x) - (D^{\beta}f)(y)|^p}{|x-y|^{n+|s|p}} dx dy
$$
\n
$$
\leq c \int_{\Omega \times \Omega} \frac{|d_M^{(\epsilon/p)-[s]+|\beta|}(x) D^{\beta}f(x) - d_M^{(\epsilon/p)-[s]+|\beta|}(y) D^{\beta}f(y)|^p}{|x-y|^{n+|s|p}} dx dy
$$
\n
$$
\leq c ||f|| W_p^{s}(\Omega, \varepsilon)||^p.
$$
\nThis completes the proof that (9) is an equivalent norm in $W_p^{s}(\Omega, \varepsilon)$. Then we have again (11), and this proves the theorem if s is a fractional number \blacksquare
\nRemark 3: At the first glance we would try to replace \sum in (2) by \sum . This seems

This completes the proof that (9) is an equivalent norm in $W_p^s(Q, \varepsilon)$. Then we have again (11), and this proves the theorem if s is a fractional number

to be the better definition of the fractional spaces $W_p^s(Q, \varepsilon)$ and it agrees also completely with our theory in $[2: 3.2.3]$ and 3.2.4. If $M = \partial\Omega$ (this is the case treated in [2]) then such a replacement is possible. However in the general case $M + \partial\Omega$ this is not clear: The difficulties come from the estimate (17) of the terms with $|\beta| < |s|$. $\leq c \int \frac{|\mathcal{L}_M|}{\rho \times \rho}$
 $\leq c ||f || W_p^s(Q, \varepsilon)||^2$

This completes the proof th

again (11), and this proves

Remark 3: At the first g

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in [2]) then such a replac

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 $V = \frac{1}{2}$, we have $V = \frac{1}{2}$

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'Prof. Dr. HANs **TRIEBET**

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