Zeitschrift für Analysis
und ihre Anwendungen
Bd. 4 (3) 1985, S. 193 – 200

Generalization of Cramér's and Linnik's Factorization Theorems in the Continuation Theory of Distribution Functions

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Angeregt durch ein Problem von Kruglov und ein Resultat von Titov gewinnen wir einen elementaren Fortsetzungssatz für Verteilungsfunktionen. Er impliziert die folgende Verallgemeinerung des Theorems von Cramér. Für die nicht ausgearteten Verteilungsfunktionen F_1 und F_2 gelte

$$F_1 * F_2(x) = \Phi_{a,\sigma}(x), \qquad x \leqq x_0,$$

wo $x_0 \in \mathbf{R}_1$ und $\Phi_{a,\sigma}$ die Normalverteilung $N(a, \sigma^2)$ bezeichnet; wenn die charakteristischen Funktionen f_1 und f_2 keine Nullstellen in der oberen Halbebene haben, dann sind F_1 und F_2 auch normal. Linniks Theorem läßt sich analog verallgemeinern. Auch allgemeinere Varianten werden erörtert.

Возбуждены проблемой Круглова и результатом Титова мы доказываем элементарное предложение о продолжении функций распределения. Как следствие мы получаем следующее обобщение теоремы Крамера. Пусть F_1 и F_2 две невырожденные функции распределения, которые удовлетворяют условию

$$F_1 * F_2(x) = \Phi_{a,\sigma}(x), \qquad x \leqq x_0$$

где $x_0 \in \mathbf{R}_1$, а $\Phi_{a,\sigma}$ означает нормальную функцию распределения $N(a, \sigma^2)$. Если соответствующие характеристические функции f_1 и f_2 не имеют нулей в верхной полуплоскости, то F_1 и F_2 также являются нормальными. Подобным образом обобщается теорема Линника. Дискутируются также более общие варианты.

Stimulated by a problem of Kruglov and a result of Titov we derive an elementary continuation theorem for distribution functions. It implies the following generalization of Carmér's theorem. Let F_1 and F_2 be two non-degenerate distribution functions sucht that

$$F_1 * F_2(x) = \Phi_{a,\sigma}(x), \qquad x \leqq x_0,$$

where $x_0 \in \mathbf{R}_1$ and $\Phi_{a,\sigma}$ stands for the normal distribution $N(a, \sigma^2)$; if the corresponding characteristic functions f_1 and f_2 do not vanish in the upper half plane, then F_1 and F_2 are also normal. Linnik's theorem can be analogously generalized. More general variants are also discussed.

1. Introduction

Throughout this paper $\Phi_{a,\sigma}$ stands for the normal distribution function $N(a, \sigma^2)$ and $*$ denotes a convolution. The following theorem was first conjectured by P. Lévy and somewhat later proved by H. Cramér; it is well known in the analytic theory of probability, for details see [5] or [12].

Theorem 1.1: *Let F_1 and F_2 be non-degenerated distribution functions such that*

$$F_1 * F_2 = \Phi_{a,\sigma} \qquad (a \text{ real}, \sigma^2 > 0). \tag{1.1}$$

Then $F_i = \Phi_{a_i,\sigma_i}$ $(i = 1, 2)$ where $a_1 + a_2 = a$, $\sigma_1^2 + \sigma_2^2 = \sigma^2$.

In the usual proof, one switches from (1.1) to the corresponding characteristic functions; so $f_1 f_2 = \varphi_{a,\sigma}$. Then one proceeds in the following three steps.

1. A theorem of D. A. RAJKOV implies that f_1 and f_2 are integral characteristic functions without zeros.

2. Next one proves that the orders of f_1 and f_2 are not greater than 2.

3. We use the fact that such integral functions without zeros can be represented as e^{P_2} where P_2 denotes a polynomial with a degree ≤ 2.

Later it was Y. V. LINNIK who proved the following analogous result. Its proof is by far more complicated and was remarkably simplified by E. LUKACS [5].

Theorem 1.2: *Let F_1 and F_2 be two non-degenerated distribution functions such that*

$$F_1 * F_2 = \Phi_{a,\sigma} * P_\lambda \quad (a \text{ real}, \sigma^2 > 0, \lambda \geq 0)$$

where P_λ stands for the Poisson distribution function with parameter λ. Then $F_i = \Phi_{a_i,\sigma_i} * P_{\lambda_i}$ $(i = 1, 2)$ where $a_1 + a_2 = a$, $\sigma_1^2 + \sigma_2^2 = \sigma^2$, $\lambda_1 + \lambda_2 = \lambda$.

It is the aim of this paper to generalize these results in a unified manner. The essential tool will be firstly the elementary new continuation Theorem 3.1 (and its Corollary 3.1) and secondly deep results of B. JESIAK [4] and N. M. BLANK [1] which are based on the value distribution theory.

2. Remarks on the continuation theory of distribution functions

It was A. N. KOLMOGOROV who conjectured that an infinitely divisible distribution function F coinciding with $\Phi_{a,\sigma}$ on a half line equals $\Phi_{a,\sigma}$. The affirmative answer to this question was published in [8] and can now be considered the starting point of what we call the continuation theory of distribution functions. Its present state is surveyed in [11] and elaborated in [12]. The theory of Phragmén and Lindelöf plays a considerable role in it as is seen also from the present paper. Already at several occasions, we have taken the view that that theory provides powerful tools which are very applicable also in other fields of analytic probability theory, see e.g. [9] and [10].

A remarkable deep result of continuation theory is due to I. A. IBRAGIMOV [3] who generalized the above mentioned continuation theorem in the following way.

Theorem 2.1: *Let F be an infinitely divisible distribution function which is positive on the whole line, and assume that its characteristic function f is continuable to the upper half plane*

$$\mathfrak{H}_+ = \{z = t + iy : y \geq 0\}.$$

Let G be an infinitely divisible distribution function satisfying

$$F(x) = G(x), \quad x \leq x_0, \tag{2.1}$$

for some real x_0. Then $F = G$.

For the proof Ibragimov used Nevanlinna's value distribution theory. Later on, it was B. JESIAK [4] who recognized that part of this result has little to do with infinite divisibility. Namely, using Ibragimov's method he proved that the problem concerns mainly the zeros of the characteristic function under consideration. For a brief formulation of his result, we write $\mathfrak{T}$ for the class of all distribution functions F having a characteristic function f which is continuable to $\mathfrak{H}_+$ and satisfies the conditions

$$\lim_{y\to\infty} \frac{\log f(iy)}{y \log y} = \infty, \quad n(r; f) = O(\log r), r \to \infty, \tag{2.2}$$

where n stands for the number of zeros of f in the semi-disk $(z: |z| \leqq r, y > 0)$.

Theorem 2.2: *Assume that $F \in \mathfrak{T}$ and that for the distribution function G we have* (2.1). *If, moreover, $n(r; g) = O(\log r)$, then $F = G$.*

The Harkov school of analytic probability theory also dealt with continuation theory and used results of the value distribution theory. It was I. V. Ostrovskij [6] who introduced the following class of functions.

Definition 2.1: Let $H: \mathbf{R}_1 \to \mathbf{R}_1$ be a function of bounded variation. We say that it belongs to the *class* $\mathfrak{B}$ if it possesses the following properties (where lext stands for "left extremity"):

(i) $H(-\infty) = 0, H(\infty) = 1, H(x - 0) = H(x)$;

(ii) lext $H = -\infty$;

(iii) the "characteristic function" $h(t) = \int_{-\infty}^{\infty} e^{itx} \, dH(x)$ is continuable to $\mathfrak{H}_+$ and has no zeros here.

I. V. Ostrovskij obtained the following result.

Theorem 2.3: *If H_1 and H_2 belong to $\mathfrak{B}$ and if*

$$H_1(x) = H_2(x), \quad x \leqq x_0, \tag{2.3}$$

for some $x_0 \in \mathbf{R}_1$, then $H_1 = H_2$.

In the particular case that $H_2 = \Phi_{a,\sigma}$ we need considerably weaker assumptions than the coincidence (2.3) on a half lime. The first to notice this was M. Riedel [7]. Generalizing his result N. M. Blank and I. V. Ostrovskij [2] proved the following.

Theorem 2.4: *If H belongs to $\mathfrak{B}$ and*

$$|H(-x) - \Phi_{0,1}(-x)| < \exp\left(-\frac{x^2}{2} - x\Omega(x)\right), \qquad x > 0,$$

where $\Omega: (0, \infty) \to \mathbf{R}_1$ satisfies $\lim_{x\to\infty} \Omega(x) = \infty$, then $H = \Phi_{0,1}$.

Finally, N. M. Blank [1] considered the subclass $\mathfrak{B}_1 \subset \mathfrak{B}$ of functions H whose "characteristic function" h satisfy

$$h(iy) \geqq \exp(p^{-1} y^p)$$

for some $p > 1$ and sufficiently large $y > 0$; since $\Phi_{0,1} \in \mathfrak{B}_1$ her result stated as Theorem 2.5 generalizes Theorem 2.4.

Theorem 2.5: *Let Ω be as in Theorem 2.4. If $H_1 \in \mathfrak{B}$ and $H_2 \in \mathfrak{B}_1$ and*

$$|H_1(-x) - H_2(-x)| \leqq \exp\left(-\frac{1}{k} x^k - x^{k-1}\, \Omega(x)\right), \qquad x \geqq 0,$$

where $k = \frac{p}{p-1}$, then $H_1 = H_2$.

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In view of these deep results, the following continuation theorem of A. N. Titov [14] deserves particular attention for two reasons. Namely, neither infinite divisibility nor zeros of characteristic functions are mentioned; moreover, the corresponding proof is nothing but an elegant application of elementary facts which are well known from the theory of Phagmén and Lindelöf. In Remark 3.1 we indicate how this result can be obtained in our context though our approach differs from that of Titov. In [12] we also present further results of Titov which are relevant in this connection.

Theorem 2.6: *Assume that for a distribution function H and some natural number $n \geqq 2$ we have*

$$H^{n*}(x) = \Phi_{a,\sigma}(x), \qquad x \leqq x_0,$$

for some real x_0; then H is a normal distribution.

This is obviously related to Cramér's theorem, and in view of this result it is natural to ask if even the assumption

$$F_1 * F_2(x) = \Phi_{a,\sigma}(x), \qquad x \leqq x_0, \tag{2.3}$$

implies that the distribution functions F_1 and F_2 are also normal. Unfortunately such a desirable statement is wrong.

Example (oral com. of A. N. Titov): Put

$$F_1(x) = \begin{cases} 2\sum\limits_0^\infty \left(\Phi_{0,1}(x - 2n) - \Phi_{0,1}(x - 2n - 1)\right), & x \leqq 0 \\ v(x), & x \geqq 0 \end{cases}$$

where v stands for a monotone function on $[0, \infty)$ such that $F_1(0) = v(0)$, $v(\infty) = 1$. Let then F_2 denote the binomial distribution function attributing mass 1/2 to the atoms 0 and 1. Then (2.3) holds for $x \leqq 0$, but $F_1 * F_2 \not\equiv \Phi_{0,1}$. ○

3. A problem of V. M. Kruglov and the main result

The results of A. N. Titov described above made V. M. Kruglov pose the following problem:

Assume that in (2.3) we have $F_i(x) = F(x/\sigma_i)$ $(i = 1, 2)$ for $0 < \sigma_1 < \sigma_2$. Does it then follow that F is normal?

This question seems to be easy since Theorem 2.6 can be proved easily and with elementary means. But in the present paper we will give only a partial answer to this problem. Nevertheless, it proved stimulating since it led us to introduce the root which plays an essential rôle in our proof of Theorem 3.1.

First of all we have to quote two lemmas the second of which is due to I. V. Ostrovskij. For the proofs see e.g. [12]. For the sake of brevity, we write $\mathfrak{C}$ for the class of all functions w analytic in the interior of $\mathfrak{H}_+$ and continuous in $\mathfrak{H}_+$ which are bounded on the real axis $\mathbf{R}_1$. Further, for $w \in \mathfrak{C}$, $w \not\equiv 0$, we let

$$k_w = \overline{\lim_{y\to\infty}}\, y^{-1} \log |w(iy)|.$$

Lemma 3.1: *For a function $w \in \mathfrak{C}$, $w \not\equiv 0$, which is bounded in $\mathfrak{H}_+$ we have $k_w > -\infty$.*

Lemma 3.2: *Suppose that $w \in \mathfrak{C}$, $w \not\equiv 0$, satisfies $-\infty \leqq k_w \leqq 0$ and*

$$|w(z)| \leqq K \exp(dy^2), \qquad z \in \mathfrak{H}_+,$$

for some positive numbers K and d. Then w is bounded in $\mathfrak{H}_+$.

We are now in a position to prove the following continuation theorem.

Theorem 3.1: *Let F be a distribution function which is positive on the line. Assume that its characteristic function is continuable to $\mathfrak{H}_+$, has no zeros in $\mathfrak{H}_+$, and satisfies*

$$f(iy) \leqq L \exp(dy^2), \qquad y \geqq 0, \tag{3.1}$$

for some $L > 0$, $d > 0$. If the distribution function G fulfills

$$G(x) = F(x), \qquad x \leqq x_0, \tag{3.2}$$

for some real x_0 and if its characteristic function has also no zeros in $\mathfrak{H}_+$, then $G = F$.

Proof: Without loss of generality we set $x_0 = 0$ since $F_0(x) = F(x + x_0)$, too, satisfies the assumptions of the theorem. We introduce the function

$$k(t) = g(t) - f(t) = \int_0^\infty e^{itx} d\big(G(x) - F(x)\big)$$

where the latter equation is obvious by assumption. The continuation

$$k(z) = g(z) - f(z) = \int_0^\infty e^{izx} d\big(G(x) - F(x)\big), \qquad z \in \mathfrak{H}_+,$$

belongs to $\mathfrak{C}$ and is bounded in $\mathfrak{H}_+$, $|k(z)| \leqq 2$. Thus we have by the ridge property of characteristic functions $|g(z)| \leqq |k(z)| + |f(z)| \leqq 2 + f(iy)$, $z \in \mathfrak{H}_+$, and from (3.1) it is seen that there exists $K > 0$ such that

$$|g(z)| \leqq K \exp(dy^2), \qquad z \in \mathfrak{H}_+. \tag{3.3}$$

The essential idea of the proof consists in introducing

$$\Delta(z) = \sqrt{g(z)} - \sqrt{f(z)}, \qquad z \in \mathfrak{H}_+, \tag{3.4}$$

where $\sqrt{g(0)} = \sqrt{f(0)} = 1$. Then $k(z) = \Delta(z)\left(\sqrt{g(z)} + \sqrt{f(z)}\right)$ and, moreover, we have the estimate

$$2 \geqq |k(iy)| = |\Delta(iy)| \left(\sqrt{g(iy)} + \sqrt{f(iy)}\right) \geqq |\Delta(iy)| \sqrt{f(iy)}. \tag{3.5}$$

Combining (3.1), (3.3), and (3.4) we obtain from the ridge property $|\Delta(z)| \leqq \left(\sqrt{L} + \sqrt{K}\right) \times \exp(dy^2/2)$, $y \geqq 0$. On the other hand, the assumption $F > 0$ implies

$$\lim_{y \to \infty} y^{-1} \log f(iy) = \infty. \tag{3.6}$$

Thus it is clear from (3.5) that $|\Delta(iy)|$ is bounded for $y > 0$. Hence Δ satisfies all assumptions of Lemma 3.2, and applying it we see that Δ is bounded. Assume now $\Delta \not\equiv 0$. Then our result contradicts Lemma 3.1 since from (3.5) and (3.6) it follows that $k_\Delta = -\infty$. Thus $\Delta = 0$ and $f = g$ ∎

Remark: A function B which has bounded variation on the lime can be represented as $B = c_1 F_1 - c_2 F_2$ where F_1 and F_2 are distribution functions and $c_1 \geqq 0$, $c_2 \geqq 0$. This is the reason why Theorem 3.1 is true also for functions $G, F \in \mathfrak{B}$ with a minor change in the proof.

Now we turn to the particular case in which

$$F = \Phi_{a,\sigma} * P_\lambda .$$

The condition (3.1) is trivially satisfied for $d = \sigma^2/2$, and (3.2) can be replaced by an asymptotic relation. Note that (3.7) implies $G(-x) = O(e^{-rx})$, $x \to +\infty$, for all $r > 0$, so that g is continuable to $\mathfrak{H}_+$.

Corollary 3.1: *Let G be an arbitrary distribution function and $a \in \mathbf{R}_1$, $\sigma^2 > 0$, $\lambda \geqq 0$. Assume that for some $c > \sigma^{-2}$*

$$|G(-x) - \Phi_{a,\sigma} * P_\lambda(-x)| = O(e^{-cx^2}), \quad x \to \infty; \tag{3.7}$$

if the characteristic function g has no zeros in $\mathfrak{H}_+$, then $G = \Phi_{a,\sigma} * P_\lambda$.

The proof (R. SCHARM [13]) essentially proceeds like above. Namely, in this case we have

$$|k(z)| \leqq 3 + |z| \frac{K}{2} \sqrt{\frac{\pi}{c}} \exp\left(\frac{y^2}{4c}\right), \quad y \geqq 0,$$

so that we get a similar estimate for g. Thus, instead of (3.5), we may write

$$O\left(y \exp\left(\frac{y^2}{4c}\right)\right) = |k(iy)| \geqq |\Delta(iy)| \exp\left(\frac{y^2\sigma^2}{4} - \frac{ya}{2} - \frac{\lambda}{2}\right).$$

Hence $\Delta(iy)$, $y \geqq 0$, proves to be bounded. Moreover, $\Delta(z)/(3 + iz)$, $y \geqq 0$, satisfies all assumptions of Lemma 3.2. Thus we get the desired result with similar conclusions as before ∎

Corollary 3.1 obviously implies the following theorem which generalizes Linnik's Theorem 1.2 and, at the same time (for $\lambda = 0$) Cramér's Theorem 1.1. Since

$$F_1\left(\frac{x}{2}\right) F_2\left(\frac{x}{2}\right) \leqq F_1 * F_2(x)$$

it is clear from (3.8) that $F_j(-x) = O(e^{-rx})$, $x \to \infty$, for all $r > 0$, so that the characteristic function f_j $(j = 1, 2)$ are continuable to $\mathfrak{H}_+$.

Theorem 3.2: *Let $a \in \mathbf{R}_1$ and $\sigma^2 > 0$, $\lambda \geqq 0$. Let, moreover, F_1, F_2 be two distribution functions satisfying*

$$|F_1 * F_2(x) - \Phi_{a,\sigma} * P_\lambda(x)| = O(e^{-cx^2}) \tag{3.8}$$

for some $c > \sigma^{-2}$; if, moreover, their characteristic functions f_1, f_2 have no zeros in $\mathfrak{H}_+$, then the assertions of Theorem 1.1 and Theorem 1.2, resp., are true.

Proof: Putting $G = F_1 * F_2$ we can apply Corollary 3.1 and obtain the assertion ∎

Remark 3.1: Though in Titov's Theorem 2.6 zeros of f are not mentioned, our method enables us to prove it. Namely, let us put $F = \Phi_{a,\sigma}$ in Theorem 3.1. Then the assumptions made on the zeros of g can be replaced by "$\sqrt{g} \in \mathfrak{E}$". Further, a slight modification of the proof shows that the more general assumption "there exists $n \geqq 2$ such that $\sqrt[n]{g} \in \mathfrak{E}$" is also sufficient. Then Theorem 2.6 follows from this new version of Theorem 3.1, since we have then to replace g by h^n.

Corollary 3.2: *Kruglov's question can be answered in the affirmative provided that f has no zeros or only zeros of an even order in $\mathfrak{H}_+$.*

Clearly, we can now also generalize Theorem 1.2. Namely, putting

$$G = F_1 * F_2 \quad \text{and} \quad f(t) = \exp\left(iat - \sigma^2 t^2/2 + \lambda(e^{it} - 1)\right)$$

we can apply Theorem 3.1 to obtain the following result.

Theorem 3.3: *If*

$$F_1 * F_2(x) = \Phi_{a,\sigma} * P_\lambda(x), \qquad x \leqq x_0$$

and the characteristic functions f_j of F_j ($j = 1, 2$) have no zeros in $\mathfrak{H}_+$, then the assertion of Theorem 1.2 *is true.*

4. Conclusions from the value distribution theory

The results of § 3 are, amongst others, of interest since they can be obtained with very elementary analytic methods and can be understood without any special probabilistic education. Now we state briefly what directly follows from Theorem 2.2 and Theorem 2.5, resp.

Theorem 4.1: *Let F_1 and F_2 be two distribution functions satisfying*

$$F_1 * F_2(x) = \Phi_{a,\sigma} * P_\lambda(x), \qquad x \leqq x_0, \tag{4.1}$$

for some $\lambda \geqq 0$; if, moreover, their characteristic functions f_1 and f_2 have only "few zeros" in $\mathfrak{H}_+$ in the sense that for $j = 1, 2$

$$n(r, f_j) = O(\log r),$$

as $r \to \infty$, then the assertions of the Theorem 1.1 *and* 1.2, *resp., are true.*

Theorem 4.2: *Let F_1 and F_2 be two distribution functions satisfying*

$$|F_1 * F_2(-x) - \Phi_{a,\sigma} * P_\lambda(-x)| \leqq \exp\left(-\frac{x^2}{2} - x\Omega(x)\right), \qquad x \geqq 0,$$

for some $\lambda \geqq 0$, where Ω: $(0, \infty) \to \mathbf{R}_1$ is a function such that $\lim_{x\to\infty} \Omega(+x) = \infty$; if, moreover, their characteristic functions f_1 and f_2 have no zeros in $\mathfrak{H}_+$, then the assertions of the Theorems 1.1 *and* 1.2, *resp., are true.*

Acknowledgement. This paper owes much to discussions with J. Focke, B. Jesiak, M. Riedel, and R. Scharm, and I am very grateful to them.

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Manuskripteingang: 30. 12. 1982; revidierter Fassung: 10. 02. 1984

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