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On Capillary Free Surfaces without Gravity

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Dedicated to Professor Herbert Beckert on his sixty fifth birthday

Das nichtparametrische Kapillaritätsproblem ohne Schwerkraft läßt sich mittels einer Stromfunktion in ein Dirichlet-Problem für eine quasilineare Gleichung vom gemischten Typ überführen. Wir beweisen ein Maximumprinzip für eine dem Gradienten der Stromfunktion zugeordnete Größe und Regularitätseigenschaften von $C^1(\bar{\Omega})$-Lösungen in Gebieten Ω mit Ecken.

Непараметрическая задача капиллярности без действия силы тяжести с помощью функции тока может быть сведена к задаче Дирихле для квазилинейного уравнения смешанного типа. Доказывается принцип максимума для величины, сопоставленной градиенту функции тока, и свойства регулярности для $C^1(\bar{\Omega})$-решений в областях Ω с угловыми точками.

The non-parametric capillary problem in the absence of gravity can be replaced by a Dirichlet problem for a quasilinear equation of mixed type by introducing a stream function. We prove a maximum principle for an expression depending on the gradient of the stream function and, furthermore, regularity properties of $C^1(\bar{\Omega})$-solutions in domains Ω with corners.

1. Introduction

We consider the non-parametric capillary problem in the absence of gravity. One seeks a surface S: $u = u(x)$, of constant mean curvature H, defined over a simply connected and bounded base domain $\Omega \subset \mathbf{R}^2$, such that S meets vertical cylinder walls over the boundary $\partial\Omega$ in a prescribed constant angle γ, where $0 \leqq \gamma \leqq \frac{\pi}{2}$. This problem leads to the equation

$$\sum_{i=1}^{2} \frac{\partial}{\partial x_i} \frac{u_{x_i}}{\sqrt{1 + |u_x|^2}} = 2H \quad \text{in} \quad \Omega \tag{1.1}$$

with the boundary condition

$$\frac{\dfrac{\partial u}{\partial n}}{\sqrt{1 + |u_x|^2}} = \cos \gamma \quad \text{on} \quad \partial\Omega, \tag{1.2}$$

see Finn and Concus [2] and Finn [3]. Here n is the exterior unit normal on $\partial\Omega$, $|u_x|^2 = u_{x_1}^2 + u_{x_2}^2$ and $2H = \frac{|\partial\Omega|}{|\Omega|} \cos \gamma$, where $|\Omega|$ is the area of Ω and $|\partial\Omega|$ the length of $\partial\Omega$.

The problem to find explicit geometric criteria for the existence of solutions of (1.1), (1.2) has been met with only partial success up to the present time, see Finn [3]. It can be replaced by a Dirichlet problem for a quasilinear equation of mixed type by introducing a stream function. We show that the problem of existence of solutions for this problem can be reduced to a certain boundary gradient estimate. In the last part of this paper we study the behavior of $C^1(\bar{\Omega})$-solutions of (1.1), (1.2) over domains with corners. We prove, in particular, that $C^1(\bar{\Omega})$-solutions belong to the class $C^{1,\lambda}(\bar{\Omega})$, $0 < \lambda < 1$, if $0 < \alpha < \pi$ is satisfied for the interior angles α at the corners.

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2. The associated Dirichlet problem

Define

$$\nu_i = \frac{u_{x_i}}{\sqrt{1 + |u_x|^2}} \qquad (i = 1, 2).$$

Since $(\nu_1 - Hx_1)_{x_1} + (\nu_2 - Hx_2)_{x_2} = 0$, it is possible to introduce a stream function ψ by setting

$$\begin{aligned} \psi_{x_1} &= -\nu_2 + Hx_2, \\ \psi_{x_2} &= \nu_1 - Hx_1. \end{aligned} \tag{2.1}$$

Suppose the boundary $\partial\Omega$ is given by $x(s) = \big(x_1(s), x_2(s)\big)$, where s is the arc length. We assume that the boundary $\partial\Omega$ is piecewise smooth. On its smooth parts we have

$$\begin{aligned} \dot{\psi} = \psi_{x_1}\dot{x}_1 + \psi_{x_2}\dot{x}_2 &= (-\nu_2 + Hx_2)\,\dot{x}_1 + (\nu_1 - Hx_1)\,\dot{x}_2 \\ &= \cos\gamma + H(x_2\dot{x}_1 - \dot{x}_2 x_1). \end{aligned}$$

By integrating from 0 to s we get

$$\psi(s) = \left(s - |\partial\Omega|\,\frac{|\Omega(s)|}{|\Omega|}\right)\cos\gamma \quad \text{on} \quad \partial\Omega \tag{2.2}$$

where $\Omega(s)$ is sketched in Figure 1. We mention that $\psi(|\partial\Omega|) = 0$.

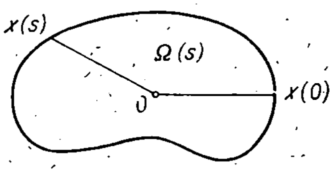

Fig. 1

Put $F(x, \psi_x) = (\psi_{x_1} - Hx_2)^2 + (\psi_{x_2} + Hx_1)^2$. Since

$$\begin{aligned} u_{x_1} &= \frac{1}{\sqrt{1 - F}}\,(\psi_{x_2} + Hx_1), \\ u_{x_2} &= \frac{1}{\sqrt{1 - F}}\,(-\psi_{x_1} + Hx_2), \end{aligned} \tag{2.3}$$

the relation $u_{x_1 x_2} = u_{x_2 x_1}$ gives us the differential equation (we are using the summation convention)

$$a_{ij}(p)\,\psi_{x_i x_j} = 0 \quad \text{in} \quad \Omega \tag{2.4}$$

with the boundary condition (2.2) on $\partial\Omega$, where $p_1 = \psi_{x_1} - Hx_2$, $p_2 = \psi_{x_2} + Hx_1$ and $a_{11} = (1 - p_2^2)$, $a_{22} = (1 - p_1^2)$, $a_{12} = a_{21} = p_1 p_2$. If $\psi \in C^1(\bar{\Omega})$ and $\max\limits_{\bar{\Omega}} F < 1$, then the equation (2.4) is of elliptic type since

$$(1 - F)\,|\xi|^2 \leqq a_{ij}\xi_i\xi_j \leqq |\xi|^2 \quad \text{for all} \quad \xi \in \mathbf{R}^2 .$$

Let $\Omega = B_R(0)$ be a disk with radius R and the center at the coordinate origin. Then we have $\psi(s) = 0$ on $\partial\Omega$. If $\max\limits_{\bar{\Omega}} F < 1$ for a solution $\psi \in C^1(\bar{\Omega})$ of the equation (2.4) with the boundary condition $\psi = 0$ on $\partial\Omega$, then the maximum principle implies $\psi = 0$ in Ω. From (2.3) we obtain immediately

$$u = -\frac{1}{H}\sqrt{1 - H^2\,|x|^2} + \text{const}.$$

If $< \gamma \leqq \frac{\pi}{2}$ is satisfied, then the assumption $\max\limits_{\bar{\Omega}} F < 1$ is fulfilled.

Now we ask for domains with $\psi(s) = 0$ on $\partial\Omega$. The equation (2.2) implies that

$$x_1\dot{x}_2 - x_2\dot{x}_1 = \frac{2\,|\Omega|}{|\partial\Omega|} \tag{2.5}$$

on the smooth parts of $\partial\Omega$. By differentiating we have $x_1\ddot{x}_2 - x_2\ddot{x}_1 = 0$ and, since $x = -\varkappa n$ where $\varkappa$ is the curvature of $\partial\Omega$ at $x(s)$, we can infer that

$$\varkappa \cdot \frac{d\,|x|^2}{ds} = 0 \quad \text{by using} \quad n = (\dot{x}_2, -\dot{x}_1).$$

Hence, since (2.5) must be satisfied on the smooth parts of $\partial\Omega$, we obtain regular m-gons and domains which we get from these polygons by rounding off one or some corners by the incircle with coordinate origin at the point of symmetry. The inequality $\max\limits_{\bar{\Omega}} F < 1$ is in these cases equivalent to $H \max\limits_{\partial\Omega} |x| < 1$. This means that $\gamma > \frac{\pi}{m}$ must be fulfilled. In fact, this is exactly the corner condition, see FINN [3], $\frac{\alpha}{2} + \gamma > \frac{\pi}{2}$, where α is the interior angle at the corners.

3. A maximum principle for F

Now we prove a maximum principle for F by using a method of Bernstein, see GILBARG and TRUDINGER [4: Chapter 14.1].

Theorem: *Let $\psi \in C^1(\bar{\Omega}) \cap C^3(\Omega)$ be a solution of* (2.4) *and assume that* $\max\limits_{\bar{\Omega}} F\big(x, \psi_x(x)\big) < 1$. *Then* $\max\limits_{\bar{\Omega}} F = \max\limits_{\partial\Omega} F$.

Proof: We derive a differential equation of second order for F. The assertion then follows from the classical maximum principle. Set $f_1 = -Hx_2$, $f_2 = Hx_1$ and $F = p_1^2$

$+ p_2^2$, where $p_1 = \psi_{x_1} + f_1$, $p_2 = \psi_{x_2} + f_2$. We have

$$F_{x_i} = 2p_r p_{r,i}, \tag{3.1}$$

$$F_{x_i x_j} = 2p_{r,j} p_{r,i} + 2p_r \psi_{,rij}, \tag{3.2}$$

$$p_{l,k} = p_{k,l} + (f_{l,k} - f_{k,l}). \tag{3.3}$$

(By writing $v_{,i}$ we mean $\frac{\partial v}{\partial x_i}$.) By differentiating the equation (2.4) with respect to x_k one obtains

$$a_{ij}(p)\,\psi_{,kij} + D_{p_l} a_{ij}(p)\,p_{l,k}\psi_{,ij} = 0$$

where $D_{p_l} = \frac{\partial}{\partial p_l}$. Multiplying by p_k and summing over k, we thus have

$$a_{ij}(p)\,p_k\psi_{,kij} + D_{p_l} a_{ij}(p)\,p_{l,k}p_k\psi_{,ij} = 0$$

or, with (3.2), the equation

$$\frac{1}{2}\,a_{ij}(p)\,F_{,ij} - a_{ij}(p)\,p_{r,i}p_{r,j} + D_{p_l} a_{ij}(p)\,p_{l,k}p_k\psi_{,ij} = 0.$$

By using (3.1) and (3.3) we get

$$\frac{1}{2}\,a_{ij}(p)\,F_{,ij} + \frac{1}{2}\,D_{p_l} a_{ij}(p)\,\psi_{,ij}F_{,l}$$
$$= a_{ij}(p)\psi_{,ri}\psi_{,rj} + a_{ij}(p)\,f_{r,i}f_{r,j} + b_{ij}\psi_{,ij}$$

where $b_{11} = 2Hp_1p_2$, $b_{22} = -2Hp_1p_2$, $b_{12} = b_{21} = H(p_2^2 - p_1^2)$. Since

$$\begin{aligned} \frac{1}{2}\,F_{x_1} &= p_1\psi_{,11} + p_2(\psi_{,21} + H), \\ \frac{1}{2}\,F_{x_2} &= p_1(\psi_{,12} - H) + p_2\psi_{,22}, \end{aligned} \tag{3.4}$$

cf. (3.1), we see that $b_{ij}\psi_{,ij} = H(p_2F_{,1} - p_1F_{,2}) - 2H^2F$. It is easy to check that $a_{ij}(p)\,f_{r,i}f_{r,j} = 2H^2 - FH^2$. According to our hypothesis with respect to F the $\psi_{,ri}$ can be calculated directly from (3.4) and the equation (2.4). This calculation yields

$$\psi_{,11} = g_i^{11}(p)\,F_{,i} - \frac{2H}{F}\,p_1p_2,$$

$$\psi_{,22} = g_i^{22}(p)\,F_{,i} + \frac{2H}{F}\,p_1p_2,$$

$$\psi_{,12} = g_i^{12}(p)\,F_{,i} + \frac{H}{F}\,(p_1^2 - p_2^2)$$

where the g_i^{lk} are certain regular functions depending on p. Thus we get from the above that F satisfies the equation

$$\frac{1}{2}\,a_{ij}(p)\,F_{,ij} + b_lF_{,l} = 4H^2(1 - F)$$

with known functions b_l belonging to $C^1(\Omega)$. The classical maximum principle implies $\max_{\bar{\Omega}} F = \max_{\partial\Omega} F$, provided $\psi \in C^1(\bar{\Omega}) \cap C^3(\Omega)$ and $\max_{\bar{\Omega}} F < 1$ ∎

Corollary: *There exists a solution of* (1.1), (1.2), *provided* $0 < \gamma \leqq \frac{\pi}{2}$ *and* $\partial\Omega$ *is near a circle with respect to the* C^2*-norm.*

For the proof we observe that a bounded slope condition, cf. GILBARG and TRUDINGER [4: p. 225], is fulfilled for such domains. Using this condition it is possible to derive the inequality $\max_{\bar{\Omega}} F(x, \psi_x(x)) = C < 1$ for a $C^1(\bar{\Omega})$-solution of (2.4), (2.2) if $\partial\Omega$ is sufficiently near a circle. The constant C' does not depend on ψ. Then, by a standard argument, from the above maximum principle it follows the existence of a solution of (2.4), (2.2) ∎

Remark: We may weaken the assumption of the above theorem to $\psi \in C^1(\bar{\Omega})$ only and $\max_{\bar{\Omega}} F < 1$. This follows from the weak formulation of the equation (2.4), see Part 4, since $\psi \in C^\infty(\Omega)$ is a consequence of the regularity theory for quasilinear elliptic equations of second order.

4. Domains with corners

L. SIMON [7] has shown that if $\frac{\alpha}{2} + \gamma > \frac{\pi}{2}$ and $\alpha < \pi$, then a solution of (1.1), (1.2) is differentiable up to the corner. Here α denotes the interior angle at this corner. From Part 2 it follows that (1.1), (1.2) can be replaced by the Dirichlet problem

$$\begin{aligned} &\int_\Omega a_i(x, \psi_x)\, v_{x_i}\, dx = 0 \quad \text{for all } v \in C_0^\infty(\Omega), \\ &\psi = \psi(s) \quad \text{on } \partial\Omega \end{aligned} \tag{4.1}$$

where

$$a_1(x, \psi_x) = \frac{1}{\sqrt{1-F}} (\psi_{x_1} - Hx_2),$$

$$a_2(x, \psi_x) = \frac{1}{\sqrt{1-F}} (\psi_{x_2} + Hx_1)$$

and ψ is given by (2.2).

Let the origin be a corner of Ω. We assume that $\overline{\partial_1\Omega}$, $\overline{\partial_2\Omega}$ are in C^2 and that $\partial_1\Omega$ has positive and $\partial_2\Omega$ negative slope at the origin, see Figure 2.

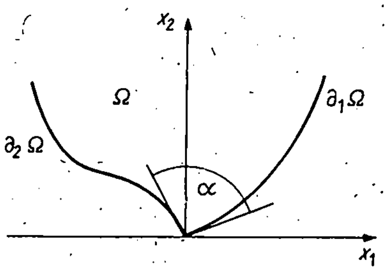

Fig. 2

Suppose $u \in C^1(\bar{\Omega})$ is a solution of (1.1), (1.2) and $\alpha < \pi$. This implies that $\frac{\alpha}{2} + \gamma > \frac{\pi}{2}$ must be satisfied. The inequality can be verified by an easy calculation by using (1.2) and the fact that (1.1) is elliptic at the corner. The assumption $u \in C^1(\bar{\Omega})$

also implies that (4.1) is an elliptic problem since the associated ψ satisfies $\psi \in C^1(\bar{\Omega})$, $F < 1$ in $\bar{\Omega}$ and the equation (4.1).

Theorem: *Let $u \in C^1(\bar{\Omega}) \cap C^2(\Omega)$ be a solution of* (1.1), (1.2). *Under the condition $0 < \alpha < \pi$ for all interior angles at the corners we have $u \in C^{1,\lambda}(\bar{\Omega}) \cap H^2(\Omega)$, $0 < \lambda < 1$.*

Proof: Results of this type were proved by MIERSEMANN [5, 6]. Some details differ from [5, 6]. The first distinction is that the vector field $(a_1(x, q), a_2(x, q))$, $q = \psi_x$, is strongly monotone with respect to q for $|p| < 1$ only where again $p_1 = \psi_{x_1} - Hx_2$, $p_2 = \psi_{x_2} + Hx_1$. The second one is the dependence on x. It is not hard to prove that the results of [5] stay true for strongly monotone C^1-vector fields depending on x. We omit the proof of this fact.

It is sufficient to prove the regularity properties near the corners. Set $\Omega_\varrho = \Omega \cap B_\varrho(0)$ where $B_\varrho(0)$ is a disk with radius ϱ and the center at the origin. Assuming $\overline{\partial_i \Omega} \in C^2$, there exists a continuation $\Phi \in C^2(\overline{\Omega_\varrho})$ of ψ, provided $0 < \alpha < \pi$ and $\varrho > 0$ is sufficiently small. The proof of the Theorem is almost the same as in [5] ∎

Since (2.4) is of mixed type, the barrier construction of [5: p. 61] must be modified. Let E be the set of all functions $\varphi \in C^1(\overline{\Omega_\varrho})$ such that $F(x, \varphi_x) < 1$ in $\overline{\Omega_\varrho}$. Clearly, E is a convex set.

Lemma: *Assume that for $\psi, \varphi \in E$ we have $\psi \leqq \varphi$ on $\partial\Omega_\varrho$ and*

$$\int_\Omega a_i(x, \psi_x)\, v_{x_i}\, dx \leqq \int_\Omega a_i(x, \varphi_x)\, v_{x_i}\, dx$$

for all $v \in C_0^1(\Omega_\varrho)$ with $v \geqq 0$ in Ω_ϱ. Then one has $\psi \leqq \varphi$ in Ω_ϱ.

Proof: This comparison principle follows immediately by using the convexity of E and the fact that for all $\xi \in \mathbf{R}^2$ we have

$$(1 - F)^{-1/2}\, |\xi|^2 \leqq a_{ij}\xi_i\xi_j \leqq (1 - F)^{-3/2}\, |\xi|^2,$$

where in this part we put $a_{ij} = \dfrac{\partial a_i(x, q)}{\partial q_j}$ with $q_i = \varphi_{x_i}$.
Here φ is a function belonging to E ∎

From the Lemma it follows that there exists at most one solution $\psi \in C^1(\bar{\Omega})$ of (4.1) such that $F(x, \psi_x) < 1$.

A function $\varphi \in C^1(\overline{\Omega_\varrho}) \cap C^2(\overline{\Omega_\varrho} \setminus \{0\})$ is said to be a *barrier* of ψ at $x = 0$ if the following conditions are fulfilled:

(i) $\varphi \in E$,
(ii) $\varphi \geqq \psi$ on $\partial\Omega_\varrho$,
(iii) $a_{ij}(x, \varphi_x)\, \varphi_{x_i x_j} \leqq 0$ in Ω_ϱ.

From the Lemma we obtain $\varphi \geqq \psi$ in $\bar{\Omega}_\varrho$ for a barrier φ. Put

$$\varphi = \Phi(0) + \Phi_{x_i}(0)\, x_i + A r^{1+\mu} \sin\,(1 + \tau)\, \Theta$$

where (r, Θ) denote polar coordinates in the plane and A, μ and τ positive constants. We will show that one can choose these constants in such a way that φ satisfies (i) to (iii), provided $\varrho > 0$ is small enough. Since $\Phi \in C^2(\overline{\Omega_\varrho})$ we have

$$\Phi(x) = \Phi(0) + \Phi_{x_i}(0)\, x_i + h(x) \quad \text{where} \quad |h(x)| \leqq c_1 |x|^2.$$

According to our assumption one has

$$\psi(x) = \Phi(0) + \Phi_{x_i}(0)\, x_i + R(x) \quad \text{where} \quad |R(x)| \leqq c(\varrho)\, \varrho \quad \text{in } \Omega_\varrho$$

with a function $c(\varrho)$ such that $c(\varrho) \to 0$ as $\varrho \to 0$.

We shall prove that

$$\varphi \geqq \psi \quad \text{on} \quad \Omega \cap \partial B_\varrho(0) \tag{4.2}$$

is satisfied. If $A\varrho^{1+\mu} \sin (1 + \tau)\,\Theta \geqq c(\varrho)\,\varrho$ is fulfilled, then we have the inequality (4.2). Set

$$c_2 = \min_{(r,\theta)\in\bar{\Omega}_\varrho} \sin (1 + \tau)\,\Theta .$$

Provided $\tau_0 > 0$ and $\varrho > 0$ are sufficiently small, we see that $c_2 > 0$ for all $0 < \tau \leqq \tau_0$. Put $c_0(\varrho) = \max\{c(\varrho), \varrho^\delta\}$ for a δ with $0 < \delta < 1$. If we choose $A = \frac{c_0(\varrho)}{c_2} \times \varrho^{-\mu}$ then the inequality (4.2) is satisfied.

Now we will prove $\varphi \geqq \psi$ on $\partial\Omega \cap B_\varrho$. Again, this inequality is true if we have

$$c_0(\varrho)\,\varrho^{-\mu} r^{1+\mu} \geqq c_1 r^2 \; (0 < r \leqq \varrho) \quad \text{or} \quad c_0(\varrho) \geqq c_1\varrho .$$

From the definition of $c_0(\varrho)$ it follows that the previous inequality is satisfied for all $0 < \varrho \leqq \varrho_0$, provided ϱ_0 is small enough.

Now we check the assumption (i). An easy calculation yields

$$F(x, \varphi_x) = \left(\Phi_{x_1}(0) - Hx_2 + \frac{c_0(\varrho)}{c_2}\left(\frac{r}{\varrho}\right)^\mu g_1\right)^2 + \left(\Phi_{x_2}(0) + Hx_1 + \frac{c_0(\varrho)}{c_2}\left(\frac{r}{\varrho}\right)^\mu g_2\right)^2$$

where

$$g_1 = (1 + \mu)\cos\Theta \sin (1 + \tau)\,\Theta - (1 + \tau)\sin\Theta\cos (1 + \tau)\,\Theta ,$$

$$g_2 = (1 + \mu)\sin\Theta \sin (1 + \tau)\,\Theta + (1 + \tau)\cos\Theta\cos (1 + \tau)\,\Theta .$$

Consequently, the function φ satisfies $F(x, \varphi_x) < 1$ in $\overline{\Omega_{\varrho_0}}$ for a sufficiently small $\varrho_0 > 0$.

The inequality (iii) was proved in [5: Lemma 2.2].

Replacing A by $-A$ we obtain $|\psi - \Phi| \leqq c\,|x|^{1+\mu}$ in $\overline{\Omega_{\varrho_0}}$. From this inequality, we get

$$\psi \in C^{1,\lambda}(\bar{\Omega}_\varrho) \cap H^2(\Omega_\varrho) \quad \text{for a } \lambda \text{ with} \quad 0 < \lambda < 1,$$

see [5]. Since u_x is given by (2.3), the same is true for u.

R e m a r k: If we assume that $u \in C^{1,\beta}(\bar{\Omega})$, then we can control $c(\varrho)$ and we get $\|u\|_{C^{1,\lambda}(\bar{\Omega})} \leqq C$ for $0 < \beta < \lambda < 1$, provided we have chosen $\beta > 0$ sufficiently small. The constant C depends, for fixed Ω and γ, on $\|u\|_{C^{1,\beta}(\bar{\Omega})}$ only.

Once one has obtained $\psi \in C^{1,\lambda}(\overline{\Omega_\varrho})$, it is possible to apply the theory of linear elliptic equations of second order in domains with corners, see Azzam [1]. The linear transformation which transforms the equation $a_{ij}(0, \psi_x(0))\,\psi_{x_i x_j} = 0$ into the Laplace equation transforms γ into a new angle ω. The vector $\psi_x(0)$ can be obtained from (2.1) and (1.2). An easy calculation yields for ω, $0 < \omega < \pi$, the equation

$$\operatorname{tg}\frac{\omega}{2} = \left(1 - \frac{\cos^2\gamma}{\sin^2\frac{\alpha}{2}}\right)\operatorname{tg}\frac{\alpha}{2} .$$

The angle ω is exactly the angle at the corner of the surface $u(x)$ about the origin. Provided $\overline{\partial_1 \Omega}$, $\overline{\partial_2 \Omega}$ are sufficiently regular, then we have

$$u \in C^{m+2,\lambda}(\overline{\Omega_{\varrho_0}}) \qquad (0 < \lambda < 1;\ m = 0, 1, 2, \ldots)$$

if $\omega < \dfrac{\pi}{2 + m}$ is satisfied.

REFERENCES

[1] AZZAM, A.: Behaviour of solutions of Dirichlet problem for elliptic equations at a corner. Indian J. Pure Appl. Math. **10** (1979), 1453–1459.

[2] CONCUS, P., and R. FINN: On capillary free surfaces in the absence of gravity. Acta Math. **132** (1974), 177–198.

[3] FINN, R.: Existence criteria for capillary free surfaces without gravity. Indiana Univ. Math. J. **32** (1983), 439–460.

[4] GILBARG, D., and N. S. TRUDINGER: Elliptic Partial Differential Equations of Second Order. Berlin: Springer-Verlag 1977.

[5] MIERSEMANN, E.: Zur Regularität verallgemeinerter Lösungen quasilinearer elliptischer Differentialgleichungen zweiter Ordnung in Gebieten mit Ecken. Z. Anal. Anw. **1** (1982), 59–71.

[6] MIERSEMANN, E.: Zur Gleichung der Fläche mit gegebener mittlerer Krümmung in zweidimensionalen eckigen Gebieten. Math. Nachr. **110** (1983), 231–241.

[7] SIMON, L.: Regularity of capillary surfaces over domains with corners. Pacific J. Math. **88** (1980), 363–377.

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