On Strong Unboundedness of Symmetric Operators

J. Friedrich

Es wird gezeigt, daß es zu jeder ungeraden natürlichen Zahl n einen symmetrischen Operator $\mathcal T$ im separablen Hilbertraum $\mathscr X$ gibt, so daß $\mathcal T$; $\mathcal T^3$, ..., $\mathcal T^n$ von unten unbeschränkt sind und $\mathcal{J}^k \geq 0$ für $k > n$ ist.

Доказывается, что для любого нечётного натурального числа и существует симметрический оператор $\mathcal T$ в сепарабельном гильбертовом пространстве $\mathscr X$, такой, что $\mathscr T,\mathscr T^3,\ldots,$ \mathcal{F}^n неограничены снизу и $\mathcal{F}^k \geq 0$ для всех $k > n$.

It will be shown that for each positive odd integer n'there is a symmetric operator $\mathcal F$ in a. separable Hilbert space H such that $\mathcal{J}, \mathcal{J}^s, ..., \mathcal{J}^n$ are unbounded from below and $\mathcal{J}^k \geq 0$ for $k > n$.

A symmetric operator $\mathcal T$ with dense invariant domain $\mathcal D\subseteq D(\mathcal T)$ is said to be strongly unbounded from above (below), if

 $\sup_{\varphi \in B_k(\mathcal{D})} \langle \mathcal{J}^k \varphi, \varphi \rangle = +\infty \quad \left(\inf_{\varphi \in B_k(\mathcal{D})} \langle \mathcal{J}^k \varphi, \varphi \rangle = -\infty \right)$

 \vec{f} or $k = 1, 3, 5, ...$, where

$$
B_k(\mathcal{D}) = \{ \varphi \in \mathcal{D} : |\langle \mathcal{J}^j \varphi, \varphi \rangle| \leq 1, j = 1, 2, ..., k-1 \}.
$$

It was shown in [1] that two unbounded symmetric operators \mathcal{J}_1 , \mathcal{J}_2 with dense invariant domains $\mathcal{D}_j = \tilde{\cap} D(\overline{\mathcal{F}_j}^t)$, $j = 1, 2$, possess dense invariant domains \mathcal{D}_{j_0} $\subseteq \mathcal{D}_i$, $j=1,2$, such that $\mathcal{J}_1 \upharpoonright \mathcal{D}_{10}$ and $\mathcal{J}_2 \upharpoonright \mathcal{D}_{20}$ are unitarily equivalent if and only if they are both strongly unbounded from below or both strongly unbounded from above. The latter result has applications to representations of unbounded operator algebras (see [3]). Clearly, a self-adjoint operator A is strongly unbounded from below (above), if and only if it is unbounded from below (above) in the usual sense. The following theorem shows that the word "strongly" can not be omitted in general. But the theorem seems to be of interest in itself.

Theorem: Let n be a given positive odd integer. There is a closed symmetric operator $\mathcal T$ in a separable Hilbert space $\mathcal H$ such that

The idea of the proof is to construct T as a restriction of a suitable self-adjoint operator. We start with an auxiliary construction of symmetric operators T_t , $t > 0$.

 $\label{eq:2} \begin{array}{l} \mathcal{L}_{\text{max}}(\mathcal{L}_{\text{max}}) \geq 0 \\ \mathcal{L}_{\text{max}}(\mathcal{L}_{\text{max}}) \geq 0 \end{array}$

From now on, let *n* be fixed. Consider a positive bounded operator B in a separable Hilbert space *H* with the properties that ker $B = \{0\}$ and $BH + H$. Choose some vector $x \in H \setminus BH$. We define self-adjoint operators A_t , $t > 0$, in the Hilbert space $C \bigoplus H =: \mathbf{H}$ by $D(A_t) := C \bigoplus BH$ and $A_t(\lambda, B) = (-t^2 \lambda, f)$ for $(\lambda, B) \in D(A_t)$. The operators T_t are defined by 86 J. FRIEDRICH

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Hilbert space *H* with the properties there

vector $x \in H \setminus BH$. We define self-adjo
 $C \bigoplus H =: H$ by $D(A_t) := C \bigoplus BH$ and

The operators T_t are defined by
 $D(T_t) := \$ pace *H* with the properties that ker $B = \{0\}$ and $BH + H$.
 $E \in H \setminus BH$. We define self-adjoint operators $A_t, t > 0$, in the
 $E \in H$ by $D(A_t) := C \oplus BH$ and $A_t(\lambda, Bt) = (-t^2 \lambda, t)$ for $(\lambda \text{ators } T_t \text{ are defined by}$
 $D(T_t) := \{(\lambda, t) \in D(A_t) : A_t(\lambda,$

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$$
D(T_t) := \{ (\lambda, f) \in D(A_t) : A_t(\lambda, f) \perp (t^{n-1}, x) \},
$$

\n
$$
T_t := A_t \upharpoonright D(T_t).
$$

Since T_t is a densely defined closed symmetric operator and has deficiency indices

Lemma 1: $D_t := \bigcap_{i=1}^{\infty} D(T_t^i)$ is dense in **H** and.

$$
\overline{T_t^k \restriction D_t} = T_t^k, \qquad k = 1, 2, \ldots.
$$

In the following we will need a suitable description of $D(T_t^k)$, which is given by Lemma 2: Let k be a positive odd integer. $(\lambda, f) \in H$ belongs to $D(T_t^k)$ if and only if $(\lambda, f) = \lambda(1, B^k h_t^k) + (0, B^k g)$,
where $g \perp B^l x$, $l = 0, 1, ..., k - 1$, and

$$
(\lambda, f) = \lambda(1, B^k h_t^k) + (0, B^k g),
$$

$$
T_t
$$
 is a densely defined closed symmetric operator and has def
\nTheorem 1.9] yields
\n
$$
1: D_t := \bigcap_{j=1}^{\infty} D(T_t^j)
$$
 is dense in H and
\n
$$
\overline{T_t^k \mid D_t} = T_t^k, \qquad k = 1, 2,
$$

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\n2: Let k be a positive odd integer. $(\lambda, f) \in H$ belongs to $D(T_t^k)$ i
\n $(\lambda, f) = \lambda(1, B^k h_t^k) + (0, B^k g),$
\n
$$
B^l x, l = 0, 1, ..., k - 1, and
$$

\n
$$
B^l x, l = 0, 1, ..., k - 1, and
$$

\n
$$
h_t^k = \begin{vmatrix} 0 & x & Bx & ... & B^{k-1}x \\ -t^{2k+n-1} & b_0 & b_1 & ... & b_{k-1} \\ +t^{2k+n-3} & b_1 & b_2 & ... & b_k \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -t^{n+1} & b_{k-1} & b_k & ... & b_{2k-2} \end{vmatrix}
$$

\n $b_j := \langle B^j x, x \rangle, \qquad j = 0, 1,$
\n \therefore The definition of h k is correct, since

Proof: The definition of h_t ^{k} is correct, since

$$
\begin{bmatrix}\n1 & 0 & \dots & 0 \\
\vdots & \vdots & \vdots \\
b_{k-1} & \dots & b_{2k-2}\n\end{bmatrix}
$$

is the Gramian determinant of the vectors $x, Bx, ..., B^{k-1}x$, which are linearly independent since x 4 *BH.*

the Gramian determinant of the vectors $x, Bx, ..., B^{k-1}x$, which are linearly in-
pendent since $x \notin BH$.
Consider $(\lambda, f) \in D(T_t^k)$. Since $D(T_t^k) \subseteq D(A_t^k)$, $(\lambda, f) = \lambda(1, B^k h_t^k) + (0, B^k g)$ for
me $g \in H$. By definition of $D(T_t^k)$ some $g \in H$. By definition of $D(T_t^k)$, **The ALA ISS ANDER AND APPLE ASSAUTE ASSAUTE ASSAUTE ASSAUTE As** *A***^{***i***}(***2***,** *f***)** \in *D***(***A***_{***i***}** *H***). By definition of** *D***(***T***_{***i***}^{***k***}),
A_{***i***}^{***i***}(***2***,** *f***)** \perp **(***t***ⁿ⁻¹,** *x***),** *j* **= 1, 2, ...,** *k***,** is the Greender
Consider Consider
Some $g \in$
i.e.

$$
A_i^j(\lambda, f) \perp (t^{n-1}, x), \qquad j = 1, 2, ..., k,
$$

$$
f: \text{The definition of } h_t^k \text{ is correct, since}
$$
\n
$$
\begin{vmatrix}\nb_0 & \dots b_{k-1} \\
\vdots & \vdots \\
b_{k-1} & \dots b_{2k-2}\n\end{vmatrix}
$$
\n
$$
\text{tramian determinant of the vectors } x, Bx, \dots, B^{k-1}x, \text{ which are line:}
$$
\n
$$
\text{at a linear function of } h_t^k \text{ is correct, since}
$$
\n
$$
f: Bx = \sum_{k=1}^{k-1} f_k^k \text{ is the identity of } \sum_{k=1}^{k-1} f_k^k \text
$$

for $j = 1, 2, ..., k$. By definition of h_t^k , $\langle h_t^k, B^{k-j}x \rangle = (-1)^{k-j} t^{2j+n-1}$, which implies. that $\langle g, B^{k-j}x \rangle = 0, j = 1, 2, ..., k$.

On the other hand, each vector of the described form belongs to $D(T_i^k)$

Since we want to study the operators T_t for $t \to 0$, the following statement will be useful.

Corollary 3: $h_i^k = t^{n+1}(h^k + O(t))$, where h^k is a fixed non-zero vector and $O(t)$ *tends to zero i/I tends to zero.*

Next we express the positivity of T_i^k in terms of h_i^k .

Lemma 4: Let k be a positive odd integer. $\langle T_t^k \varphi, \varphi \rangle \geq 0$ for all $\varphi \in D_t$ if and only if

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\nSince we want to study the operators
$$
T_t
$$
 for $t \to 0$, the following statement will be
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\nCorollary 3: $h_t^* = t^{n+1}(h^k + O(t))$, where h^k is a fixed non-zero vector and $O(t)$
\ntends to zero if *t* tends to zero.
\nNext we express the positivity of T_t^k in terms of h_t^k .
\nLemma 4: Let *k* be a positive odd integer. $\langle T_t^k \varphi, \varphi \rangle \ge 0$ for all $\varphi \in D_t$ if and only if
\n
$$
1 - \sup_{\substack{0 \neq f \perp B^t x \\ l = 0, 1, \ldots, k-1}} \frac{|\langle B^k h_t^k, f \rangle|^2}{\langle B^k f, f \rangle \langle B^k h_t^k, h_t^k \rangle} \ge i^{2k} \langle B^k h_t^k, h_t^k \rangle^{-1}.
$$
\nProof: $\langle T_t^k \varphi, \varphi \rangle \ge 0$ for all $\varphi \in D_t$ if $\langle T_t^k \varphi, \varphi \rangle \ge 0$ for all $\varphi \in D(T_t^k)$, since D_t is a
\ncore for T_t^k by Lemma 1. Obviously $\langle T_t^k (0, B^k g), (0, B^k g) \rangle \ge 0$. Thus we conclude by

 $\lim_{l=0,1,\ldots,k-1} \frac{0+f_1B^l\mathbf{z}}{l=0,1,\ldots,k-1}$
Proof: $\langle T_l^k\varphi,\varphi\rangle \geq 0$
core for T_l^k by Lemma
Lemma 2 that $\langle T_l^k\varphi,\varphi\rangle$ *dy* the operators T_t for $t \to 0$, the following stat
 $t^{n+1}[h^k + O(t)]$, where h^k is a fixed non-zero ve

positivity of T_t^k in terms of h_t^k .

a positive odd integer. $\langle T_t^k \varphi, \varphi \rangle \ge 0$ for all $\varphi \in D_t$
 $\frac{$ $\lim_{h\to 0+} \frac{|\langle B^kh_t, f\rangle|^2}{\langle B^k h_t, f\rangle\langle B^kh_t^k, h_t^k\rangle} \geq t^{2k} \langle B^{k-1}, \ldots, k-1 \rangle$
 $\text{Proof: } \langle T_t^k \varphi, \varphi \rangle \geq 0 \text{ for all } \varphi \in D_t \text{ iff } \langle T_t^k \varphi, \varphi \rangle$
 $\text{core for } T_t^k \text{ by Lemma 1. Obviously } \langle T_t^k(0, B^k g),$

Lemma 2 that $\langle T_t^k \varphi, \varphi \rangle \geq 0 \text{ for$

$$
\left\langle T_t^{\ k}(1,B^k(h_t{}^k+g)),\big(1,B^k(h_t{}^k+g)\big)\right\rangle=-t^{2k}+\left\langle B^k(h_t{}^k+g),\,h_t{}^k+g\right\rangle\geqq 0
$$

Next we express the positivity of
$$
T_t^k
$$
 in terms of h_t^k .
\nLemma 4: Let k be a positive odd integer. $\langle T_t^k \varphi, \varphi \rangle \ge 0$ for all $\varphi \in D_t$ if and only
\n
$$
1 - \sup_{\begin{subarray}{l}0 \ne f_1 \ne h_1,\ k_2 \ne 0\end{subarray}} \frac{|\langle B^kh,t,f\rangle|^2}{\langle B^kh,t,h\rangle^k|} \ge i^{2k}\langle B^kh_t^k,h_t^k\rangle^{-1}.
$$
\nProof: $\langle T_t^k \varphi, \varphi \rangle \ge 0$ for all $\varphi \in D_t$ iff $\langle T_t^k \varphi, \varphi \rangle \ge 0$ for all $\varphi \in D(T_t^k)$, since D_t
\ncore for T_t^k by Lemma 1. Obviously $\langle T_t^k(0, B^kg), (0, B^kg) \rangle \ge 0$. Thus we conclude
\nLemma 2 that $\langle T_t^k \varphi, \varphi \rangle \ge 0$ for all $\varphi \in D_t$ iff
\n
$$
\langle T_t^k(1, B^k(h_t^k + g)), (1, B^k(h_t^k + g)) \rangle = -t^{2k} + \langle B^k(h_t^k + g), h_t^k + g \rangle \ge
$$

\nfor all $g \perp B^lx, l = 0, 1, ..., k - 1$. But
\n
$$
\min_{k \in \mathbb{C}} \langle B^kh_t^k, h_t^k \rangle (1 - |\langle B^kh_t^k, g \rangle|^2 \langle \langle B^kh_t^k, h_t^k \rangle \langle B^kg, g \rangle \rangle^{-1})
$$

\nfor $g \ne 0$. Using this, (1) follows from the last inequality **I**
\nNow we consider the left-hand side of inequality (1) for small t.
\n
$$
\text{Lemma 5: } It \text{ is}
$$

\n
$$
\liminf_{t \to +0} \left(1 - \sup_{0 \ne f_1, B^k} \frac{|\langle B^k g_t^k, f \rangle|^2}{\langle B^kh_t^k, h_t^k \rangle \langle B^k f, f \rangle} \right) > 0
$$

\nfor $k = 3, 5, ...$

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Now we consider the left-hand side of inequality (1) f
\n
$$
\liminf_{t\to+0} \left(1 - \sup_{0+f\perp B^i x} \frac{|\langle B^k g_t^k, f \rangle|^2}{\langle B^k h_t^k, h_t^k \rangle \langle B^k f, f \rangle} \right) > 0
$$
\nfor $k = 3, 5, ...$
\nProof: Let H_* denote the completion of H with r ,
\n $\langle \cdot, \cdot \rangle_* := \langle B^k, \cdot \rangle$. With some abuse of notation we denote
\n B to H_* again by B . Since $k \geq 3$,
\n $|\langle f, B^{k-1} x \rangle| \leq \langle B^k f, f \rangle \langle B^{k-2} x, x \rangle$
\nfor all $f \in H$, i.e., the linear functional $f \mapsto \langle f, B^{k-1} x \rangle$ has a
\nThus, there is a vector $x_* \in H_*$ such that $\langle f, B^{k-1} x \rangle = \langle f, \rangle$

Proof: Let H_* denote the completion of H with respect to the inner product $\langle \cdot \rangle_* := \langle B^k \cdot, \cdot \rangle$. With some abuse of notation we denote the continuous extension of for *g* \neq 0. Using this, (1) follows from the last inequality ■

Now we consider the left-hand side of inequality (1) for sma
 \cdot Lemma 5: *It is*
 $\liminf_{t\to+0} \left(1 - \sup_{0 \neq f_1, B^t x} \frac{|\langle B^k g_i^k, f \rangle|^2}{\langle B^k h_i^k, h_i^k \rangle$ $k = 3, 5, \ldots$

Proof: Let H_* den
 $* := \langle B^k, \cdot \rangle$. With s
 H_* again by B. Si
 $|\langle f, B^{k-1}x \rangle| \le$

all $f \in H$, i.e., the line

$$
|\langle f, B^{k-1}x \rangle| \leq \langle B^k f, f \rangle \langle B^{k-2}x, x \rangle
$$

for all $f \in H$, i.e., the linear functional $f \mapsto \langle f, B^{k-1}x \rangle$ has a continuous extension to H_+ . Thus, there is a vector $x_* \in H_*$ such that $\langle f, B^{k-1}x \rangle = \langle f, x_* \rangle_*$ for all $f \in H$. Obviously, $Bx'_* = x$. $|(-1 x)| \leq \langle B^k f, f \rangle \langle B^{k-2} x, x \rangle$
 \therefore the linear functional $f \mapsto \langle f, B^k \rangle$

vector $x_* \in H_*$ such that $\langle f, B^k \rangle$
 $\sup_{(f,x_*)=0} \frac{|\langle h_t^k, f \rangle_*|^2}{\langle f, f \rangle_* \langle h_t^k, h_t^k \rangle_*}$
 $\lim_{0 \to f \in H_*} \langle B(x_*, x_*, \cdot) \rangle_*$

Consider

$$
a_t := \sup_{\substack{\langle f, x_\bullet\rangle = 0\\0 \neq f \in H_\bullet}} \frac{|\langle h_t^k, f\rangle_*|^2}{\langle f, f\rangle_* \langle h_t^k, h_t^k\rangle_*}.
$$

If we denote the angle between h_t ^k and x_* by α_t , then a_t is nothing but sin² α_t . Hence

$$
a_t := \sup_{\substack{(f,x_*)=0\\0+f\in H_*}} \frac{|\langle h_t^*, f\rangle_*|^2}{\langle f, f\rangle_* \langle h_t^k, h_t^k\rangle_*}.
$$

denote the angle between h_t^k and x_* by α_t , then a_t .

$$
a_t = 1 - \cos^2 \alpha_t
$$

$$
= 1 - |\langle h_t^k, x_*\rangle|^2 \langle\langle x_*, x_*\rangle_* \langle h_t^k, h_t^k\rangle_*)^{-1}.
$$

$$
= 1 - |\langle B^{k-1}h_t^k, x\rangle|^2 \langle\langle B^{k-2}x, x\rangle \langle B^k h_t^k, h_t^k\rangle_*)^{-1}.
$$

Since by definition of h_i^k

$$
|\langle B^{k-1}h_t^k, x \rangle| = |\langle h_t^k, B^{k-1}x \rangle| = t^{n+1},
$$

we obtain

$$
a_t = 1 - t^{2n+2} \langle \langle B^{k-2}x, x \rangle \langle B^k h_t^k, h_t^k \rangle \rangle^{-1}.
$$

 $Rk, k, 4$

 \rm{Hence}

$$
1 - \sup_{\substack{\vec{0} \neq f_1, B^t x \\ f \in H, i = 0, 1, \dots, k-1}} \frac{\frac{|\Delta h_i f_j|}{\langle B^k h_i^k, h_i^k \rangle \langle B^k f_j, f \rangle}}{\langle B^k h_i^k, h_i^k \rangle \langle B^k f_j, f \rangle}
$$

$$
\geq 1 - \sup_{\substack{\Delta f_1, B^k = 1x \\ f \in H}} \frac{\langle B^k h_i^k, h_i^k \rangle \langle B^k f_j, f \rangle}{\langle B^k h_i^k, h_i^k \rangle \langle B^k f_j, f \rangle}
$$

$$
\geq 1 - a_t = t^{2n+2} \langle \langle B^{k-2}x, x \rangle \langle B^k h_t^k, h_t^k \rangle \rangle^{-1}.
$$

Thus, by Corollary 3,

$$
\liminf_{t\to+0}\left(1-\sup_{\substack{0+f\perp B^t x\\t=0,1,\ldots,k-1}}\frac{|\langle B^kh_t,k,f\rangle|^2}{\langle B^kh_t^k,h_t^k\rangle\langle B^kf,f\rangle}\right)
$$
\n
$$
\geq \lim_{t\to+0} i^{2n+2}\langle\langle B^{k-2}x,x\rangle\langle B^kh_t^k,h_t^k\rangle\rangle^{-1} = (\langle B^{k-2}x,x\rangle\langle B^kh^k,h^k\rangle)^{-1}
$$

which proves our assertion

Proof of the Theorem: Since

$$
\lim_{t\to+0}t^{2n+4}\langle B^{n+2}h_t^{n+2},h_t^{n+2}\rangle^{-1}=\lim_{t\to+0}t^2\langle B^{n+2}h^{n+2},h^{n+2}\rangle^{-1}=0
$$

and by Lemma 4 and 5, there is an $\varepsilon > 0$ such that $\langle T_t^{n+2}\varphi, \varphi \rangle \geq 0$ for all $t \in (0, \varepsilon)$ and all $\varphi \in D_t$.

Since

$$
\lim_{t\to+0}t^{2n}\langle B^{n}h_{t}^{n},h_{t}^{n}\rangle^{-1}=\lim_{t\to+0}t^{-2}\langle B^{n}h^{n},h^{n}\rangle^{-1}=\infty,
$$

there is some $t_0 \in (0, \varepsilon)$ such that $t_0^{2n} \langle B^n h_{t_0}^n, h_{t_0}^n \rangle > 1$. Writing T and D instead of T_{t_0} and D_{t_0} , this means by Lemma 4 that there is some $\varphi_0 \in D$ such that $\langle T^n \varphi_0, \varphi_0 \rangle < 0$. Obviously, $\langle T^{n-2j}T^j\varphi_0, T^j\varphi_0 \rangle < 0$ for $j = 0, 1, ..., (n-1)/2$ and, since $\langle T^{n+2}\varphi, \varphi \rangle \geq 0$ for all $\varphi \in D$, $\langle T^k \varphi, \varphi \rangle \geq 0$ for all $\varphi \in D$ and $k > n$.

Consider the closed symmetric operator $\mathcal{J} = \sum \oplus iT$ in the separable Hilbert space $\mathcal{H} = \sum \bigoplus \mathbf{H}$. \mathcal{T} satisfies the conditions in the theorem:

(i) is satisfied, since $\mathcal{D} = \bigcap_{i=1}^{\infty} D(\mathcal{J}^i) = \sum_{i=1}^{\infty} \bigoplus D$ and since D is a core for each T^k , $k = 1, 2, ...$

(ii) is true, since

$$
\inf_{B_k(\mathcal{D})}\langle \mathcal{J}^k\varphi, \varphi\rangle \leq j \left(\sum_{l=0}^{k-1} |\langle T^l\varphi_0, \varphi_0\rangle|\right)^{-1} \langle T^k\varphi_0, \varphi_0\rangle.
$$

for $j = 1, 2, ...,$ and since $\langle T^k \varphi_0, \varphi_0 \rangle < 0$ for $k = 1, 3, ..., n$.

(iii) follows immediately from the fact that $\langle T^k \varphi, \varphi \rangle \geq 0$ for $k > n$, which completes the proof

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