Zeitschrift für Analysis
und ihre Anwendungen
Bd. 6 (3) 1987, S. 281–286

A General Random Fixed Point Theorem for Upper Semicontinuous Multivalued 1-Set-Contractions

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Es wird ein zufälliger Fixpunktsatz vom Leray-Schauder-Typ für zufällige mengenwertige nach oben halbstetige 1-verdichtende Operatoren bewiesen. Die Definitionsbereiche werden als zufällig vorausgesetzt. Das Resultat verallgemeinert verschiedene zufällige Fixpunktsätze und impliziert eine stochastische Version eines Fixpunktsatzes von Petryshyn für mengenwertige Abbildungen.

Доказывается теорема о неподвижной точке вида Лере-Шаудера для случайных многозначных полунепрерывных 1-уплотняющих отображений. Рассматриваются случайные области определения. Результат обобщает известные теоремы о случайных неподвижных точках и включает в себя, как частный случай, одна теорема Петришина для многозначных случайных отображений.

A random fixed point theorem of Leray-Schauder type for multivalued upper semicontinuous 1-set-contractions is proved. The domains are allowed to be random. The result generalizes several random fixed point theorems and implies a stochastic version of a fixed point theorem of Petryshyn for multivalued mappings.

1. Introduction

The study of random operator equations was initiated by the Prague school of probabilists around Špaček and Hanš in the 1950's. The survey by BHARUCHA-REID [1] initiated an essential improvement of the theory of random fixed points. Especially the papers by ENGL [5, 6] and NOWAK [14, 15] contain very general fixed point theorems. Theorem 6 in [6] assures the existence of a random fixed point of a random continuous multivalued operator with stochastic domain provided that the corresponding deterministic fixed point-problem is solvable. Many random fixed point theorems (cf. [5, 10, 11]) are contained in this result. However, this general theorem is unknown for the important case of the upper semicontinuous multivalued random operators. Therefore it is useful to prove special random fixed point theorems for such random operators. In this paper we prove such a result by use of an idea of ENGL [5]. Our theorem generalizes results of ENGL [6] for compact and of ITOH [12] for condensing random operators.

2. Definitions and preliminary results

Throughout this paper let E be a real separable Banach space, $(\Omega, \mathfrak{S}, \mu)$ a σ-finite complete measure space and $\mathfrak{B}(E)$ the σ-algebra of Borel sets on E. By $\mathfrak{S} \times \mathfrak{B}(E)$ we denote the smallest σ-algebra containing $\{S \times B : S \in \mathfrak{S}, B \in \mathfrak{B}(E)\}$.

Let $M \subseteqq E$. By $\overline{\text{co}}\, M$, $\overline{M}$, ∂M and int M we denote the closed convex hull, the closed hull, the boundary and the interior of M, respectively. We define $2^E = \{X \subseteqq E:$

$X \neq \emptyset\}$, $\mathrm{Cl}(E) = \{X \in 2^E : X$ is closed$\}$, $\mathrm{C}(E) = \{X \in \mathrm{Cl}(E) : X$ is convex$\}$, $\mathrm{KC}(E) = \{X \in \mathrm{C}(E) : X$ is compact$\}$. We define for $M \subseteqq E, N \subseteqq E$, $a \in E$, $r > 0$ and $t \in \mathbf{R}$: $a + M = \{a + x : x \in M\}$, $tM = \{tx : x \in M\}$, $M + N = \{x + y : x \in M$ and $y \in N\}$ and $K_r(a) = \{z \in E : \|z - a\| < r\}$. Let D be a set and $A : D \to 2^E$ be a (multivalued) mapping. The graph of A will be denoted by $\mathrm{Gr}\, A = \{(x, y) \in D \times E : y \in A(x)\}$ and for $G \subseteqq E$ we define $A^{-1}(G) = \{x \in D : A(x) \cap G \neq \emptyset\}$. The set $A(D) = \bigcup \{A(x) : x \in D\}$ is called the range of $A : D \to 2^E$.

Definition 1: Let $A : \Omega \to \mathrm{Cl}(E)$. A is called *measurable* if for all open $G \subseteqq E$ we have $A^{-1}(G) \in \mathfrak{S}$.

Remark 1 [9: Th. 3.5 (iii)]: The mapping $A : \Omega \to \mathrm{Cl}(E)$ is measurable iff $\mathrm{Gr}\, A \in \mathfrak{S} \times \mathfrak{B}(E)$ or iff $A^{-1}(B) \in \mathfrak{S}$ for all $B \in \mathfrak{B}(E)$.

Definition 2 [4]: Let $A : \Omega \to 2^E$. A is called *separable* if it is measurable and there exists a countable set $Z \subseteqq E$ such that for all $w \in \Omega$ we have $A(w) = \overline{Z \cap A(w)}$.

Remark 2: **1.** If $A : \Omega \to 2^E$ is separable, then $A(w) \in \mathrm{Cl}(E)$ $(w \in \Omega)$. **2.** If $A(w) = A_0 \in \mathrm{Cl}(E)$ for all $w \in \Omega$, then A is separable. **3.** If $A(w) = \overline{\mathrm{int}\, A(w)}$ and $A : \Omega \to 2^E$ is measurable, then A is separable. Especially: If $A(w)$ is convex and closed with $\mathrm{int}\, A(w) \neq \emptyset$ $(w \in \Omega)$ and $A : \Omega \to 2^E$ is measurable, then A is separable [6: p. 70].

Definition 3 [4]: Let $A : \Omega \to 2^E$ and $F : \mathrm{Gr}\, A \to \mathrm{Cl}(E)$ are mappings. F is called (multivalued) *random operator with stochastic domain* A if A is measurable and if for all $x \in E$ and open $G \subseteqq E$ we have $\{w \in \Omega : x \in A(w)$ and $F(w, x) \cap G \neq \emptyset\} \in \mathfrak{S}$.

For $A_0 \in \mathrm{Cl}(E)$, $F : \Omega \times A_0 \to 2^E$, F is especially a random operator (but with 'deterministic' domain) iff $F(\cdot, x)$ is measurable for all $x \in E$.

Definition 4: Let $A : \Omega \to 2^E$ and $F : \mathrm{Gr}\, A \to \mathrm{Cl}(E)$ be a random operator with stochastic domain A. A measurable function $x : \Omega \to E$ is called *random fixed point* of F if for all $w \in \Omega$ we have $x(w) \in A(w)$ and $x(w) \in F\big(w, x(w)\big)$.

The following result is a fundamental lemma for the proof of random fixed point theorems.

Lemma 1 [13]: *Let $P : \Omega \to \mathrm{Cl}(E)$ be a measurable multivalued mapping. Then there exists a measurable function $x : \Omega \to E$ such that $x(w) \in P(w)$ for all $w \in \Omega$.*

Definition 5: Let $M \subseteqq E$ and $F : M \to 2^E$. F is called *upper semicontinuous* if for all $x \in M$ and all open $G \subseteqq E$ with $G \supseteqq F(x)$ there exists a neighborhood U of x such that for all $z \in U \cap M$ we have $F(z) \subseteqq G$. F is called *closed* if $\mathrm{Gr}\, F$ is closed in the product topology. F is called *compact* if F is closed and $\overline{F(M)}$ is compact.

Remark 3 (cf. [2]): Let $M \subseteqq E$ and $F : M \to 2^E$. **1.** If F is upper semicontinuous and $F(x)$ is compact for all $x \in M$, then F is closed. **2.** If F is closed and $\overline{F(M)}$ is compact, then F is upper semicontinuous. **3.** F is upper semicontinuous iff for all closed $A \subseteqq E$, $F^{-1}(A)$ is closed.

Definition 6: Let $A : \Omega \to 2^E$ and $F : \mathrm{Gr}\, A \to 2^E$ be a random operator with stochastic domain A. F is called *upper semicontinuous* (*closed, compact*) if for all $w \in \Omega$ the mapping $F(w, \cdot)$ is upper semicontinuous (closed, compact).

Let B be a bounded subset of E. We define $\gamma(B)$, the *set-measure of noncompactness* of B, by $\gamma(B) = \inf\{d > 0 : B$ can be covered by a finite number of sets of diameter $\leqq d\}$.

Remark 4: Let B and C be bounded subsets. Then the following result is well known: **1.** $\gamma(B) = 0$ iff $\overline{B}$ is compact. **2.** $\gamma(\overline{\mathrm{co}}\, B) = \gamma(B)$. **3.** $\gamma(B \cap \{a\}) = \gamma(B)$ $(a \in E)$. **4.** $B \subseteqq C$ implies $\gamma(B) \leqq \gamma(C)$. **5.** $\gamma(B + C) \leqq \gamma(B) + \gamma(C)$. **6.** $\gamma(tB) = |t|\, \gamma(B)$ $(t \in \mathbf{R})$.

Definition 7: Let $k \geqq 0$, $A \subseteqq E$ and $F : A \to 2^E$ be an upper semicontinuous mapping with bounded range. F is called *k-set-contraction* (*1-set-contraction*) if for all bounded $B \subseteqq A$ we have $\gamma(F(B)) \leqq k\gamma(B)$ $(\gamma(F(B)) \leqq \gamma(B))$. F is called *condensing* if for any bounded $B \subseteqq A$ with $\gamma(B) > 0$ we have $\gamma(F(B)) < \gamma(B)$.

Each k-set-contraction has compact values, $F : A \to 2^E$ is a (closed) 0-set-contraction iff F is compact and F is condensing if F is a k-set-contraction with $k < 1$.

Definition 8: Let $A : \Omega \to 2^E$ and $F : \operatorname{Gr} A \to 2^E$ be a random operator with stochastic domain A. F is called a *random 1-set-contraction* (a random condensing operator) if for all $w \in \Omega$ the mappings $F(w, \cdot)$ are 1-set-contractions (condensing operators).

3. The main result

Let $A : \Omega \to 2^E$ be separable, Z a countable set such as appears in Definition 2 and $F : \operatorname{Gr} A \to 2^E$ a random operator with stochastic domain. Following Engl [5, 6], we define

$$H(w, x) = \bigcap \{F_n(w, x) : n \in \mathbf{N}\} \qquad \big((w, x) \in \operatorname{Gr} A\big)$$

with

$$F_n(w, x) = \overline{\operatorname{co}} \bigcup \{F(w, z) : z \in Z \cap A(w) \text{ and } \|z - x\| < 1/n\}.$$

Lemma 2: *Let $F : \operatorname{Gr} A \to \operatorname{KC}(E)$ be a random upper semicontinuous operator. Then we have for $H : \operatorname{Gr} A \to \operatorname{C}(E)$ the following properties:*

1. *$F(w, x) \supseteqq H(w, x) \neq \emptyset$ for all $(w, x) \in \operatorname{Gr} A$.*
2. *Let $k \geqq 0$ and let F be a random k-set-contraction. Then H is a random k-set-contraction and $H(w, x) \in \operatorname{KC}(E)$ for all $(w, x) \in \operatorname{Gr} A$.*
3. *Let $T(w, x) := x - H(w, x)$ $\big((w, x) \in \operatorname{Gr} A\big)$. Then $T^{-1}(D) \in \mathfrak{S} \times \mathfrak{B}(E)$ for all compact $D \subseteqq E$.*

Proof: Itoh [12: Lemma 1.1] proved statement 1 and that $H(w, \cdot)$ is upper semicontinuous for all $w \in \Omega$. In the proof of statement 2 we choose $w \in \Omega$ fixed (but arbitrary). Therefore we do not write the argument w. Let B be a bounded subset of A. Then

$$\begin{aligned} H(B) &= \bigcup \{H(x) : x \in B\} \subseteqq \bigcup \{F_n(x) : x \in B\} \\ &\subseteqq \bigcup \overline{\operatorname{co}} \big\{F\big(K_{1/n}(x)\big) : x \in B\big\} \subseteqq \overline{\operatorname{co}}\, F\big(B + K_{1/n}(0)\big) \quad (n \in \mathbf{N}). \end{aligned}$$

Because F is a k-set-contraction we obtain with Remark 4

$$\gamma\big(H(B)\big) \leqq \gamma\big(F(B + K_{1/n}(0))\big) \leqq k\gamma\big(B + K_{1/n}(0)\big) \leqq k\gamma(B) + k\gamma\big(K_{1/n}(0)\big)$$

for all $n \in \mathbf{N}$. With $n \to \infty$ we have $\gamma\big(H(B)\big) \leqq k\gamma(B)$ and therefore $H(w, \cdot)$ is a k-set-contraction for all $w \in \Omega$. This implies $H(w, x) \in \operatorname{KC}(E)$ for all $(w, x) \in \operatorname{Gr} A$.

Now we prove statement 3. Let $G \subseteqq E$ be open and $n \in \mathbf{N}$. Then we have (cf. Rem. 1 and Def. 3)

$$\begin{aligned} &\left\{(w, x) \in \Omega \times E : x \in A(w), \left(\bigcup \left\{F(w, z) : z \in Z \cap A(w), \|z - x\| < \frac{1}{n}\right\}\right) \cap G \neq \emptyset\right\} \\ &= \bigcup_{z \in Z} \left[\Omega \times \left\{x \in E : \|x - z\| < \frac{1}{n}\right\}\right] \cap \operatorname{Gr} A \\ &\qquad \cap \left[\{w \in \Omega : z \in A(w), F(w, z) \cap G \neq \emptyset\} \times E\right] \in \mathfrak{S} \times \mathfrak{B}(E). \end{aligned}$$

Now we apply [9: Prop. 2.6 and Th. 9.1] and get that F_n is measurable on $(\mathrm{Gr}\, A, (\mathfrak{S} \times \mathfrak{B}(E)) \cap \mathrm{Gr}\, A)$. Let $T_n(w, x) = x - F_n(w, x)$ $((w, x) \in \mathrm{Gr}\, A)$. Then T_n is measurable on $\mathrm{Gr}\, A$, too. With Remark 1 we have, especially,

$$\{(w, x) \in \mathrm{Gr}\, A : T_n(w, x) \cap D \neq \emptyset\} \in \mathfrak{S} \times \mathfrak{B}(E)$$

for all compact $D \subseteq E$. Now we have

$$T(w, x) = x - \bigcap \{F_n(w, x) : n \in \mathbb{N}\} = \bigcap \{T_n(w, x) : n \in \mathbb{N}\} \ \big((w, x) \in \mathrm{Gr}\, A\big)$$

and we apply [9: Cor. 4.3]. Therefore $\{(w, x) \in \mathrm{Gr}\, A : T(w, x) \cap D \neq \emptyset\} \in \mathfrak{S} \times \mathfrak{B}(E)$ for all compact $D \subseteq E$ ∎

For the proof of our main result we need the following deterministic fixed point theorem, which is a corollary from [8: Th. 6.1.6].

Lemma 3: *Let K be closed, convex and U an open subset of E with $U \cap K \neq \emptyset$. Let $A = \overline{U} \cap K$ and $H : A \to \mathrm{C}(E)$ be a mapping with $H(A) \subseteq K$. We suppose:*

1. *H is a 1-set-contraction.*
2. *If $(x_n) \subset A$ and (z_n) with $z_n \in H(z_n)$ are sequences with $x_n - z_n \to 0$, then there exists an $x' \in A$ with $x' \in H(x')$.*
3. *There exists an $a \in U \cap K$ such that $\beta x + (1 - \beta)\, a \notin H(x)$ $(x \in \partial U \cap K, \beta > 1)$.*

Then there exists an $x_0 \in A$ with $x_0 \in H(x_0)$ (we can find a similar result for instance in [16], though only for $K = E$ and point-valued mappings).

Definition 9: Let $A \subseteq E$. We call $H : A \to 2^E$ *demicompact* in 0 if for bounded sequences $(x_n) \subset A$ and (z_n) with $z_n \in H(x_n)$ and $x_n - z_n \to 0$ there exists an $x \in E$ and a subsequence (x_{n_k}) with $x_{n_k} \to x$ for $k \to \infty$.

We can easily see that any condensing (especially, any compact or any k-set-contraction with $k < 1$) mapping is demicompact in 0.

Now we can prove our general fixed point theorem.

Theorem: *Let $A : \Omega \to 2^E$ be separable and $F : \mathrm{Gr}\, A \to \mathrm{KC}\,(E)$ be a random 1-set-contraction with random domain A. We suppose:*

1. *For all $w \in \Omega$ the mappings $F(w, \cdot)$ are demicompact in 0.*
2. *For all $w \in \Omega$ there exist an open subset $U(w) \subseteq E$ and a set $K(w) \in \mathrm{C}(E)$ with $A(w) = \overline{U(w)} \cap K(w)$, $U(w) \cap K(w) \neq \emptyset$ and $F(w, x) \subseteq K(w)$ $\big(x \in A(w)\big)$.*
3. *For all $w \in \Omega$ there exists $a(w) \in U(w) \cap K(w)$ such that the Leray-Schauder condition $\beta x + (1 - \beta)\, a(w) \notin F(w, x)$ $\big(x \in \partial U(w) \cap K(w), \beta > 1\big)$ holds.*

Then F has a random fixed point.

Proof: Let Z be a countable set such as appears in Definition 2. We define $H : \mathrm{Gr}\, A \to \mathrm{C}(E)$ as before Lemma 2. Let $P(w) = \{x \in A(w) : x \in H(w, x)\}$ $(w \in \Omega)$. We apply Lemma 2 with $k = 1$ and get that $H(w, \cdot)$ is a 1-set-contraction for all $w \in \Omega$, $H(w, x) \subseteq K(w)$ for all $w \in \Omega$ and $x \in A(w)$, and $\emptyset \neq H(w, x) \subseteq F(w, x)$ for all $(w, x) \in \mathrm{Gr}\, A$. Because of assumption 3, for all $w \in \Omega$ there is an $a(w) \in U(w) \cap K(w)$ with $\beta x + (1 - \beta)\, a(w) \notin F(w, x)$, and therefore $\beta x + (1 - \beta)\, a(w) \notin H(w, x)$ $\big(x \in \partial U(w) \cap K(w), \beta > 1\big)$. Let $w \in \Omega$ be fixed (but arbitrary). Now we show that condition 2 from Lemma 3 holds for $H(w, \cdot)$. Let $(x_n) \subset A(w)$ and (z_n) are sequences with $z_n \in H(x_n)$ and $x_n - z_n \to 0$. Because $H(x_n) \subseteq F(x_n)$ we have $z_n \in F(x_n)$ $(n \in \mathbb{N})$. F is demicompact in 0 and therefore there exists a subsequence (x_{n_k}) of (x_n) with $x_{n_k} \to x' \in A(w)$. Then we have $z_{n_k} \to x'$. Because $z_{n_k} \in H(x_{n_k})$ $(k \in \mathbb{N})$ and $H(w, \cdot)$ is closed, we obtain $x' \in H(x')$. Therefore we can apply Lemma 3 and for all $w \in \Omega$ there exist an $x(w) \in A(w)$ with $x(w) \in H(w, x(w))$ and $P(w) \neq \emptyset$. $H(w, \cdot)$ is closed, and then $P(w)$ is closed. Therefore $P : \Omega \to \mathrm{Cl}(E)$. We prove that P is measurable and apply Lemma 1. Let $T(w, x) := x - H(w, x)$ $((w, x) \in \mathrm{Gr}\, A)$. Then we obtain

$$\mathrm{Gr}\, P = \{(w, x) \in \mathrm{Gr}\, A : x \in H(w, x)\} = T^{-1}(\{0\}).$$

Because of Lemma 2 we have $T^{-1}(\{0\}) \in \mathfrak{S} \times \mathfrak{B}(E)$ and P is measurable (Rem. 1). Because of Lemma 1 there exists a measurable function $x_0 : \Omega \to E$ with $x_0(w) \in P(w)$ for all $w \in \Omega$. Then x_0 is a random fixed point for F, since $x_0(w) \in A(w)$ and $x_0(w) \in H(w, x_0(w)) \subseteq F(w, x_0(w))$ ∎

Corollary 1: *Let $U \subseteq E$ be open, $K \subseteq E$ be closed and convex with $U \cap K \neq \emptyset$ and $F : \Omega \times (\overline{U} \cap K) \to C(E)$ be a random, condensing operator with $F(\Omega \times (\overline{U} \cap K)) \subseteq K$. We suppose: For all $w \in \Omega$ there is an $a(w) \in U \cap K$ with $\beta x + (1 - \beta)\, a(w) \notin F(w, x)$ $(x \in \partial U \cap K, \beta > 1)$. Then F has a random fixed point.*

Proof: $F(w, \cdot)$ is demicompact in 0 because F is condensing. Then we apply the Theorem and Remark 2/2 ∎

Corollary 2: *Let $A : \Omega \to C(E)$ be separable and $F : \mathrm{Gr}\, A \to \mathrm{KC}(E)$ be a multivalued random operator with stochastic domain A. We suppose: For all $x \in \partial A(w)$ we have $F(w, x) \subseteq A(w)$ $(w \in \Omega)$ and for all w the mappings $F(w, \cdot)$ are $k(w)$-set-contractions with $k(w) < 1$. Ghen F has a random fiðed point.*

Proof: If $w \in \Omega$ with $\mathrm{int}\, A(w) = \emptyset$, then the conditions 2 and 3 in our Theorem are realized for $U = E$, since $A(w) = \partial A(w)$. If $w \in \Omega$ with $\mathrm{int}\, A(w) \neq \emptyset$, then the conditions 2 and 3 in our Theorem are realized for $K = E$, since $A(w) = \overline{U}(w)$ is convex and then the Rothe condition implies the Leray-Schauder condition. F is a random 1-set-contraction and all $F(w, \cdot)$ are demicompact in 0, because they are condensing. Now we apply the Theorem ∎

Corollary 3: *Let $A : \Omega \to C(E)$ be separable and measurable, $F : \mathrm{Gr}\, A \to \mathrm{KC}(E)$ be a multivalued random compact operator and $G : \mathrm{Gr}\, A \to E$ a random operator. We suppose for all $w \in \Omega$: $F(w, x) + G(w, x) \subseteq A(w)$ $(x \in A(w))$, there exists $k(w) \in [0, 1)$ with $\|G(w, x) - G(w, y)\| \leqq k(w)\, \|x - y\|$ $(x, y \in A(w))$ and $G(w, \cdot)$ has a bounded range. Then $F + G$ has a random fixed point.*

Proof: For all $w \in \Omega$, $F(w, \cdot) + G(w, \cdot)$ is a $k(w)$-set-contraction with $k(w) < 1$ and we apply Corollary 2 ∎

Corollary 1 generalizes for $U = E$ or for $K = E$ the main theorem of the Rothe type for condensing random mappings from [12]. Corollary 2 generalizes the stochastic versions of the fixed point theorem by Kakutani, which was proved by Engl [5, 6] and by Nowak [15] for compact mappings. Corollary 3 is a multivalued version of Corollary 9 in [6].

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Manuskripteingang: 10. 04. 1985; in revidierter Fassung 08. 10. 1985

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