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Generalized Solutions of the Cauchy Problem for a Nonlinear Functional Partial Differential Equation

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Es wird ein Satz über die Existenz und Eindeutigkeit der verallgemeinerten Lösung (in Sinne „fast überall") des Cauchy-Problems für nichtlineare Funktional-Differential-Gleichungen mit partiellen Ableitungen erster Ordnung bewiesen.

Доказана теорема о существовании и единственности обобщенного решения (в смысле „почти всюду") задачи Коши для нелинейного дифференциально-функционального уравнения с частными производными первого порядка.

An existence and uniqueness theorem for the generalized solution (in the sense "almost everywhere") of the Cauchy problem for a nonlinear functional partial differential equation of first order is proved.

1. Introduction. Let us consider the Cauchy problem

$$D_x u(x, y) = F\big(x, y, u(x, y), (Vu)(x, y), D_y u(x, y)\big)$$

$$\text{(a.e. in } [0, a];\ y \in \mathbf{R}), \tag{1}$$

$$u(0, y) = \varphi(y) \qquad (y \in \mathbf{R}), \tag{2}$$

where $D_x = \partial/\partial x$, $D_y = \partial/\partial y$, and V is an operator of Volterra type.

Equation (1) contains as particular cases $\big((Vu)(x, y) = u(\alpha(x, y), \beta(x, y))\big)$ the differential equations with a retarded argument, the special cases of which arise in the theory of the distribution of wealth [5]. A few kinds of integral-differential equations can be obtained from (1) by specializing the operator V. For instance, problems arising from laser problems in Nonlinear Optics are also particular cases of the problem with $(Vu)(x, y) = \int_{-\infty}^{y} K(y - t)\, u(x, t)\, dt$ [1].

In recent papers P. Brandi and R. Ceppitelli [2], Z. Kamont [6, 7], and A. Salvadori [8] have considered the existence and uniqueness of continuously differentiable solutions of problem (1), (2) under the assumption that F is differentiable. The aim of the present paper is to extend these results to a more general case where the given function F is not necessarily continuous and solutions of problem (1), (2) are *generalized* (or weak) in the sense "almost everywhere" (a.e.). The method applied is of fixed point type. It is based on defining an operator, whose range consists of solutions of suitable equations without functional argument. By applying differential inequalities (see Lemma 1) it is proved that this operator is a contraction, and its fixed point is a solution of problem (1), (2). The solution is local in x and global in y, and it is unique in a class of bounded functions, absolutely continuous in x and possessing Lipschitzian derivatives in y.

2. The auxiliary results. In the sequal we will use the existence theorem for the nonlinear partial differential equation

$$D_x u(x, y) = f\big(x, y, u(x, y), D_y u(x, y)\big) \qquad \text{(a.e. in } [0, a];\ y \in \mathbf{R}). \tag{3}$$

Assumption $\mathbf{H}_1$: Suppose that

1° $f(x, \cdot)\colon \mathbf{R}^3 \to \mathbf{R}$ is continuous, and derivatives $D_y f(x, \cdot)$, $D_z f(x, \cdot)$, $D_q f(x, \cdot)\colon \mathbf{R}^3 \to \mathbf{R}$ exist and are continuous for every $x \in [0, a_0]$, $a_0 > 0$;

2° $f(\cdot, y, z, q)$, $D_y f(\cdot, y, z, q)$, $D_z f(\cdot, y, z, q)$, $D_q f(\cdot, y, z, q)\colon [0, a_0] \to \mathbf{R}$ are measurable for every $(y, z, q) \in \mathbf{R}^3$;

3° there are measurable and integrable functions $M_i, L_i\colon [0, a_0] \to \mathbf{R}_+ = [0, +\infty)$, $(i = 0, 1, 2, 3)$ such that, a.e. in $[0, a_0]$,

$$|f(x, y, z, q)| \leqq M_0(x), \qquad |D_y f(x, y, z, q)| \leqq M_1(x),$$

$$|D_z f(x, y, z, q)| \leqq M_2(x), \qquad |D_q f(x, y, z, q)| \leqq M_3(x)$$

and

$$|f(x, y, z, q) - f(x, \bar{y}, \bar{z}, \bar{q})| \leqq L_0(x)\,(|y - \bar{y}| + |z - \bar{z}| + |q - \bar{q}|),$$

$$|D_y f(x, y, z, q) - D_y f(x, \bar{y}, \bar{z}, \bar{q})| \leqq L_1(x)\,(|y - \bar{y}| + |z - \bar{z}| + |q - \bar{q}|),$$

$$|D_z f(x, y, z, q) - D_z f(x, \bar{y}, \bar{z}, \bar{q})| \leqq L_2(x)\,(|y - \bar{y}| + |z - \bar{z}| + |q - \bar{q}|),$$

$$|D_q f(x, y, z, q) - D_q f(x, \bar{y}, \bar{z}, \bar{q})| \leqq L_3(x)\,(|y - \bar{y}| + |z - \bar{z}| + |q - \bar{q}|)$$

for all $(y, z, q), (\bar{y}, \bar{z}, \bar{q}) \in \mathbf{R}^3$;

4° the initial function φ in (2) belongs to $C^1(\mathbf{R}, \mathbf{R})$ ($C^1(\mathbf{R}, \mathbf{R})$ denotes the set of all continuously differentiable functions on $\mathbf{R}$ into $\mathbf{R}$), and there exist constants $k_1, k_2 > 0$ such that $|\varphi'(y)| \leqq k_1$ and $|\varphi'(y) - \varphi'(\bar{y})| \leqq k_2\,|y - \bar{y}|$ $(y, \bar{y} \in \mathbf{R})$.

Let us define the constants

$$K_1 = k_1 + (1 + k_1 + k_2) \int_0^a \left(\left(L_0 + \Omega(a_0)\, L_3 + M_3\right) \exp \int_0^x G\, dt \right) dx,$$

$$K_2 = k_2 + (1 + k_1 + k_2) \int_0^a \left(\left(L_1 + \Omega(a_0)\, L_2 + M_2\right) \exp \int_0^x G\, dt \right) dx,$$

$$g_0 = 1 - (1 + k_1 + k_2) \int_0^a \left(L_3 \exp \int_0^x G\, dt \right) dx,$$

where

$$G = L_0 + L_1 + L_3 + \Omega(a_0)\,(L_2 + L_3) + M_2 + M_3,$$

$$\Omega(a_0) = \exp \int_0^{a_0} M_2\, dt \left\{ k_1 + \int_0^{a_0} \left(M_1 \exp\left(-\int_0^t M_2\, ds \right) \right) dt \right\}.$$

Theorem 1 [3, 4]: *If Assumption H_1 is satisfied, then there are a constant $a \in (0, a_0]$ and a function $u\colon E_a = [0, a] \times \mathbf{R} \to \mathbf{R}$ satisfying equation (3) and condition (2). This solution is unique in that class of functions $u\colon E_a \to \mathbf{R}$ for which $u(\cdot, y)$, $D_y u(\cdot, y)\colon [0, a] \to \mathbf{R}$ are absolutely continuous for every $y \in \mathbf{R}$, and*

$$|D_y u(x, y)| \leqq \Omega(a_0),$$

$$|u(x, y) - u(x, \bar{y})| \leqq \frac{K_1}{g_0}\,|y - \bar{y}|, \qquad |D_y u(x, y) - D_y u(x, \bar{y})| \leqq \frac{K_2}{g_0}\,|y - \bar{y}|;$$

for every $(x, y), (x, \bar{y}) \in E_a$.

Remark: In the above theorem a is chosen sufficiently small such that $g_0 > 0$.

Set, for $a, b > 0$, $M \geqq 0$, and $Ma \leqq b$,

$$E_{ab} = \{(x, y): 0 \leqq x \leqq a, |y| \leqq b - Mx\},$$

$$E_{xb} = \{(s, y) \in E_{ab}: s \leqq x\}, \qquad S_x = \{y: (x, y) \in E_{ab}\}.$$

We shall need the following

Lemma 1: *Suppose that*

1° $u: E_{ab} \to \mathbf{R}$ *is continuous, and* $D_x u$ *exists for a.e.* $x \in [0, a]$ *and every* $y \in [-b + Mx, b - Mx]$;

2° *for every* $x \in [0, a]$, $u(x, \cdot)$ *fulfils a Lipschitz condition;*

3° *there are constants* $c_1, c_2 \geqq 0$ *such that*

$$|D_x u(x, y)| \leqq c_1 |u(x, y)| + c_2 + M |D_y u(x, y)| \qquad (\text{a.e. in } [0, a]; \ |y| \leqq b - Mx).$$

Then the derivative γ' *of the function* γ,

$$\gamma(x) = \max \{|u(s, t)|: (s, t) \in E_{xb}\} \qquad (x \in [0, a]),$$

exists a.e. and $\gamma'(x) \leqq c_1 \gamma(x) + c_2$ *a.e. in* $[0, a]$.

Proof: Since $\delta(x) = \max \{|u(x, y)|: y \in S_x\}$ is continuous on $[0, a]$ (see [9]) and $\gamma(x) = \max \{\delta(s): s \in [0, x]\}$, γ is continuous on $[0, a]$ and γ' exists a.e. in $[0, a]$. Suppose that $(\bar{x}, \bar{y}) \in E_{xb}$ is such that

$$\gamma(x) = |u(\bar{x}, \bar{y})|, \tag{4}$$

and $u_x(\bar{x}, \bar{y})$, $u_y(\bar{x}, \bar{y})$, $\gamma'(x)$ exist. Thus we have $0 \leqq \bar{x} \leqq x$. First, let $\bar{x} \neq 0$. From the definitions of γ and δ it follows that $\gamma(x) = \delta(\bar{x})$. For $h < 0$, we get

$$\gamma(x + h) \geqq \delta(x + h) \qquad \text{and} \qquad \frac{\gamma(x + h) - \gamma(x)}{h} \leqq \frac{\delta(\bar{x} + h) - \delta(\bar{x})}{h}.$$

Hence, by $h \to 0^-$, we obtain

$$\gamma'(x) \leqq D_-\delta(x) \tag{5}$$

(D_- is the left-hand lower Dini derivative). For $\bar{x} = 0$ inequality (5) is certainly satisfied, since γ is constant in $[0, x]$. If $(\bar{x}, \bar{y})$ is an interior point of E_{ab}, then we have [9] $D_-\delta(\bar{x}) \leqq |D_x u(\bar{x}, \bar{y})|$, and also $|D_y u(\bar{x}, \bar{y})| = 0$. Hence, by (5) and assumption 3° it follows that

$$\gamma'(x) \leqq D_-\delta(\bar{x}) \leqq |D_x u(x, y)| \leqq c_1 |u(\bar{x}, \bar{y})| + c_2 = c_1 \gamma(x) + c_2.$$

Now, suppose that $(\bar{x}, \bar{y})$ is not an interior point of E_{ab}. Then $\bar{y} = b - M\bar{x}$ or $\bar{y} = -b + M\bar{x}$. We consider only the first case. Assume also, that in (4) we have $\gamma(x) = u(\bar{x}, \bar{y})$ (for $\gamma(x) = -u(\bar{x}, \bar{y})$ the proof is quite similar). Let us consider the function $\bar{m}$ defined by $\bar{m}(x) = u(x, b - Mx)$. Since $\bar{m}(x) \leqq \delta(x)$, $x \in [0, \bar{x}]$, and $\bar{m}(\bar{x}) = \delta(\bar{x})$,

$$\frac{\bar{m}(\bar{x} + h) - \bar{m}(\bar{x})}{h} \geqq \frac{\delta(\bar{x} + h) - \delta(\bar{x})}{h} \quad \text{for } h < 0.$$

Hence, we get

$$\bar{m}'(x) \geqq D_-\delta(\bar{x}). \tag{6}$$

From assumption 2° it follows that $\tilde{m}'$ exists a.e. in $[0, a]$ and $\tilde{m}'(x) = D_x u(x, b - Mx) - MD_y u(x, b - Mx)$. In particular, we have $\tilde{m}'(\bar{x}) = D_x u(\bar{x}, \bar{y}) - MD_y u(\bar{x}, \bar{y})$. Hence, and by (6), we obtain $D_-\delta(\bar{x}) \leqq D_x u(\bar{x}, \bar{y}) - MD_y u(\bar{x}, \bar{y})$. The inequality together with assumption 3° yields

$$D_-\delta(\bar{x}) \leqq D_x u(\bar{x}, \bar{y}) - MD_y u(\bar{x}, \bar{y}) \leqq c_1 |u(\bar{x}, \bar{y})| + c_2 = c_1\gamma(x) + c_2.$$

Hence and by (5) we get the assertion of the lemma ∎

3. The existence theorem. We denote by $K(a, P, Q)$ the class of all continuous and bounded functions $u: E_a \to \mathbf{R}$ satisfying the following conditions:

(i) $u(\cdot, y), D_y u(\cdot, y): [0, a] \to \mathbf{R}$ are absolutely continuous for every $y \in \mathbf{R}$;

(ii) there are constants $P, Q \geqq 0$ such that

$$|u(x, y) - u(x, \bar{y})| \leqq P |y - \bar{y}|, \qquad |D_y u(x, y) - D_y u(x, \bar{y})| \leqq Q |y - \bar{y}|,$$

for all $(x, y), (x, \bar{y}) \in E_a$.

Assumption $\mathbf{H}_2$: Suppose that

1° $F(x, \cdot): \mathbf{R}^4 \to \mathbf{R}$ is continuous, and the derivatives $D_y F(x, \cdot)$, $D_z F(x, \cdot)$, $D_p F(x, \cdot)$, $D_q F(x, \cdot): \mathbf{R}^4 \to \mathbf{R}$ exist and are continuous for every $x \in [0, a_0]$;

2° $F(\cdot, y, z, p, q)$, $D_y F(\cdot, y, z, p, q)$, $D_z F(\cdot, y, z, p, q)$, $D_p F(\cdot, y, z, p, q)$, $D_q F(\cdot, y, z, p, q): [0, a_0] \to \mathbf{R}$ are measurable for every $(y, z, p, q) \in \mathbf{R}^4$;

3° there are a constant $l_0 \geqq 0$, and measurable and integrable functions $m_i, l_j: [0, a_0] \to \mathbf{R}_+$ $(i = 0, 1, 2, 3, 4;\ j = 1, 2, 3, 4)$ such that a.e. in $[0, a_0]$

$$|F(x, y, z, p, q)| \leqq m_0(x),$$

$$|D_y F(x, y, z, p, q)| \leqq m_1(x), \qquad |D_z F(x, y, z, p, q)| \leqq m_2(x),$$

$$|D_p F(x, y, z, p, q)| \leqq m_3(x), \qquad |D_q F(x, y, z, p, q)| \leqq m_4(x),$$

and

$$|F(x, y, z, p, q) - F(x, \bar{y}, \bar{z}, \bar{p}, \bar{q})| \leqq l_0(|y - \bar{y}| + |z - \bar{z}| + |p - \bar{p}| + |q - \bar{q}|);$$

$$|D_y F(x, y, z, p, q) - D_y F(x, \bar{y}, \bar{z}, \bar{p}, \bar{q})| \leqq l_1(x)\,(|y - \bar{y}| + |z - \bar{z}| + |p - \bar{p}| + |q - \bar{q}|),$$

$$|D_z F(x, y, z, p, q) - D_z F(x, \bar{y}, \bar{z}, \bar{p}, \bar{q})| \leqq l_2(x)\,(|y - \bar{y}| + |z - \bar{z}| + |p - \bar{p}| + |q - \bar{q}|),$$

$$|D_p F(x, y, z, p, q) - D_p F(x, \bar{y}, \bar{z}, \bar{p}, \bar{q})| \leqq l_3(x)\,(|y - \bar{y}| + |z - \bar{z}| + |p - \bar{p}| + |q - \bar{q}|),$$

$$|D_q F(x, y, z, p, q) - D_q F(x, \bar{y}, \bar{z}, \bar{p}, \bar{q})| \leqq l_4(x)\,(|y - \bar{y}| + |z - \bar{z}| + |p - \bar{p}| + |q - \bar{q}|)$$

for all $(y, z, p, q), (\bar{y}, \bar{z}, \bar{p}, \bar{q}) \in \mathbf{R}^4$;

4° $(Vu)(\cdot, y): [0, a_0] \to \mathbf{R}$ is measurable for $y \in \mathbf{R}$, $u \in K(a_0, P, Q)$, there exists $D_y(Vu) \in C(E_{a_0}, \mathbf{R})$ for each $u \in K(a_0, P, Q)$, and there are measurable and integrable functions $p, r_i\colon [0, a_0] \to \mathbf{R}_+$ ($i = 0, 1$) such that a.e. in $[0, a_0]$

$$|(Vu)(x, y) - (Vu)(x, \bar{y})| \leqq r_0(x)\,|y - \bar{y}|;$$

$$|D_y(Vu)(x, y)| \leqq p(x), \qquad |D_y(Vu)(x, y) - D_y(Vu)(x, \bar{y})| \leqq r_1(x)\,|y - \bar{y}|$$

for each $u \in K(a_0, P, Q)$;

5° there is a constant $s \geqq 0$ such that $\|Vu - V\bar{u}\|_x \leqq s\,\|u - \bar{u}\|_x$ for any u, $\bar{u} \in K(a_0, P, Q)$, where $\|u\|_x = \sup\{|u(s, y)|\colon (s, y) \in E_x\}$, $E_x = [0, x] \times \mathbf{R}$;

6° the initial function φ in (2) belongs to $C^1(\mathbf{R}, \mathbf{R})$, and there exist constants $k_0, k_1 > 0$ and $k_2 \geqq 0$ such that $|\varphi(y)| \leqq k_0$, $|\varphi'(y)| \leqq k_1$ and $|\varphi'(y) - \varphi'(\bar{y})| \leqq k_2\,|y - \bar{y}|$ ($y, \bar{y} \in \mathbf{R}$).

Lemma 2: *If Assumption H_2 is satisfied, then there exist constants $a \in (0, a_0]$ and $P, Q \geqq 0$, such that for every $w \in K(a, P, Q)$ there is a unique solution $u[w] \in K(a, P, Q)$ of the equation*

$$D_x u(x, y) = F\big(x, y, u(x, y); (Vw)(x, y), D_y u(x, y)\big) \quad (\text{a.e. in } [0, a];\ y \in \mathbf{R}) \tag{7}$$

satisfying condition (2).

Proof: In order to prove this lemma, we show that all assumptions of Theorem 1 are satisfied with $f(x, y, z, q) = F\big(x, y, z, (Vw)(x, y), q\big)$, where $w \in K(a, P, Q)$. From $H_2/1°$, 2° and 4° it follows that $H_1/1°$, 2° are satisfied, $H_1/3°$ is satisfied with

$$M_0 = m_0, \qquad M_1 = m_1 + m_3 p, \qquad M_2 = m_2, \qquad M_3 = m_4,$$
$$L_0 = l_0(1 + r_0), \qquad L_1 = (1 + r_0)(l_1 + pl_3) + m_3 r_1,$$
$$L_2 = l_2(1 + r_0), \qquad L_3 = l_4(1 + r_0).$$

At last, $H_1/4°$ is covered by $H_2/6°$. By Theorem 1 it follows that there is a unique function $u[w]$ satisfying equation (7) and condition (2). Moreover, this soluton satisfies the conditions

$$|u[w](x, y) - u[w](x, \bar{y})| \leqq \frac{\tilde{K}_1}{\tilde{g}_0}\,|y - \bar{y}|,$$

$$|D_y u[w](x, y) - D_y u[w](x, \bar{y})| \leqq \frac{\tilde{K}_2}{\tilde{g}_0}\,|y - \bar{y}|,$$

where

$$\tilde{K}_1 = k_1 + (1 + k_1 + k_2)\int_0^a \Big(\big\{(1 + r_0)\big(l_0 + \tilde{\Omega}(a_0)\,l_4\big) + m_4\big\}\exp\int_0^x \tilde{G}\,dt\Big)\,dx,$$

$$\tilde{K}_2 = k_2 + (1 + k_1 + k_2)\int_0^a \Big(\big\{(1 + r_0)\big(l_1 + pl_3 + \tilde{\Omega}(a_0)\,l_2\big) + Pm_3 r_1 + m_2\big\}$$
$$\times \exp\int_0^x \tilde{G}\,dt\Big)\,dx,$$

$$\tilde{g}_0 = 1 - (1 + k_1 + k_2)\int_0^a \Big(l_4(1 + r_0)\exp\int_0^x \tilde{G}\,dt\Big)\,dx,$$

$$\tilde{\Omega}(a_0) = \exp\int_0^{a_0} m_2\,dt\,\Big\{k_1 + \int_0^{a_0}\Big((m_1 + m_3 p)\exp\Big(-\int_0^t m_2\,ds\Big)\Big)\,dt\Big\},$$

$$\tilde{G} = (1 + r_0)\big(l_0 + l_1 + pl_3 + l_4 + \tilde{\Omega}(a_0)(l_2 + l_4)\big) + m_3 r_1 + m_2 + m_4.$$

Let P and Q in the definition of $K(a, P, Q)$ be constants satisfying the inequalities $\tilde{K}_1 \leqq P\tilde{g}_0$, $\tilde{K}_2 \leqq Q\tilde{g}_0$ which are certainly satisfied for a sufficiently small (for $a \to 0$

these inequalities reduce to $k_1 \leqq P$, $k_2 \leqq Q$). Thus we have

$$|u[w](x, y) - u[w](x, \bar{y})| \leqq P\,|y - \bar{y}|,$$

$$|D_y u[w](x, y) - D_y u[w](x, \bar{y})| \leqq Q\,|y - \bar{y}|.$$

Since $u[w]$ is generated by characteristics [3], then writing the characteristic system for (7) it is easy to show that $u[w]$ is bounded. Hence, we have $u[w] \in K(a, P, Q)$ for every $w \in K(a, P, Q)$ ∎

Theorem 2: *If Assumption* H_2 *is satisfied, then there are constants* $a, P, Q, 0 < a \leqq a_0$, *and a function* $u: E_a \to \mathbf{R}$, *satisfying equation* (1) *and condition* (2). *Furthermore, u is unique in the class* $K(a, P, Q)$.

Proof: Let us define the operator T on $K(a, P, Q)$ by $(Tw)(x, y) = u[w](x, y)$, where a, P, Q are given in Lemma 2. It follows by Lemma 2 that T maps $K(a, P, Q)$ into itself. Let us introduce the norm

$$\|z\| = \sup_{(x,y)\in E_a} |z(x, y)| \exp(-\lambda x), \qquad \text{where } \lambda > l_0(1 + s).$$

By $H_2/5°$ we have

$$|(Vu)(x, y) - (V\bar{u})(x, y)| \exp(-\lambda x) \leqq s \sup_{(s,y)\in E_x} |u(s, y) - \bar{u}(s, y)| \exp(-\lambda x)$$

$$\leqq s \sup_{(s,y)\in E_x} |u(s, y) - \bar{u}(s, y)| \exp(-\lambda s) \leqq s\,\|u - \bar{u}\|.$$

Hence $\|Vu - V\bar{u}\| \leqq s\,\|u - \bar{u}\|$. Now, we prove that T is a contraction. Indeed, for any $w, \bar{w} \in K(a, P, Q)$, we have

$$\begin{aligned}
|D_x[(Tw)(x, y) - (T\bar{w})(x, y)]| &= \left|D_x\big[u[w](x, y) - u[\bar{w}](x, y)\big]\right| \\
&\leqq l_0\,|u[w](x, y) - u[\bar{w}](x, y)| + l_0\,|(Vw)(x, y) - (V\bar{w})(x, y)| \\
&\quad + l_0\left|D_y\big[u[w](x, y) - u[\bar{w}](x, y)\big]\right| \\
&\leqq l_0\,|u[w](x, y) - u[\bar{w}](x, y)| + l_0 s\,\|w - \bar{w}\| \exp(\lambda x) \\
&\quad + l_0\left|D_y\big[u[w](x, y) - u[\bar{w}](x, y)\big]\right|,
\end{aligned}$$

and $u[w](0, y) - u[\bar{w}](0, y) = 0$. Hence, and by Lemma 1, we obtain $\gamma'(x) \leqq l_0\gamma(x) + l_0 s\,\|w - \bar{w}\| \exp(\lambda x)$, where $\gamma(x) = \sup\{|u[w](s, t) - u[\bar{w}](s, t)| : (s, t) \in E_{xb}\}$ and consequently, by the extended Peano inequality [3], $\gamma(x) \leqq l_0 s(\lambda - l_0)^{-1}\,\|w - \bar{w}\| \exp(\lambda x)$. Since this inequality is satisfied for every b, it follows that

$$\|Tw - T\bar{w}\| \leqq l_0 s(\lambda - l_0)^{-1}\,\|w - \bar{w}\|, \qquad l_0 s(\lambda - l_0)^{-1} < 1.$$

Thus, T is a contraction. It is easily seen that the fixed point of the operator T satisfies condition (2) and equation (1) ∎

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REFERENCES

[1] BASSANINI, P., and M. C. SALVATORI: Problemi ai limiti per sistemi iperbolici quasilineari e generazione di armoniche ottiche. Riv. Mat. Univ. Parma 4 (1979), 55–76.

[2] BRANDI, P., and R. CEPPITELLI: Existence, uniqueness and continuous dependence for a hereditary nonlinear functional partial differential equation of the first order. Ann. Polon. Math. (to appear).

[3] CINQUINI-CIBRARIO, M., and S. CINQUINI: Equazioni alle derivate parziali di tipo iperbolico. Roma: Ed. Cremonese 1964.
[4] CINQUINI-CIBRARIO, M., and S. CINQUINI: Sopra una forma più ampia del problema di Cauchy per l'equazione $p = f(x, y, z, q)$. Ann. di Mat. 32 (1951), 121—155.
[5] EICHHORN, W., and W. GLEISSNER: On a functional differential equation arising in the theory of the distribution of wealth. Aequ. Math. 28 (1985), 190—198.
[6] KAMONT, Z.: Existence of solutions of first order partial differential-functional equations. Ann. Soc. Math. Polon., Ser I Comm. Math. 25 (1985), 249—263.
[7] KAMONT, Z.: On the Cauchy problem for non-linear partial differential-functional equations of the first order. Math. Nachr. 88 (1979), 13—29.
[8] SALVADORI, A.: Sul problema di Cauchy per una struttura ereditaria di tipo iperbolico. Esistenza, unicità e dipendenza continua. Atti Sem. Mat. Fis. Univ. Modena 32 (1983), 329—356.
[9] SZARSKI, J.: Differential inequalities. Warsaw: Polish Sci. Publ. 1967.

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