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Interpretation of Variations of the Earth's Vector of Rotation Using Inverse Solution[1])

H. JOCHMANN

Globale geophysikalische Prozesse verursachen Variationen des Rotationsvektors der Erde. Liegen nur geringe Informationen über diese Prozesse vor, empfiehlt es sich, die Lösung der Euler-Liouville-Gleichungen als inverses Problem zu behandeln. Auf diese Weise konnte erstmalig eine globale Meeresspiegelschwankung zwischen der Nord- und der Südhalbkugel der Erde, mit 4- bis 5jähriger Periode, nachgewiesen werden.

Глобальные геофизические процессы вызывают вариации вектора скорости вращения Земли. Решаются уравнения Эйлера-Лиувилля как обратные задачи относительно параметров описывающих эти процессы. Такой подход особенно рекомендуется при незначительной степени информации об этих параметров. Таким образом удалось впервые доказать глобальные колебания уровня океанов между северным и южным полушарами Земли с периодом в 4–5 лет.

Global geophysical processes cause variations of the Earth's vector of rotation. If only little information on these processes is available, it is suitable to consider the solution of the Euler-Liouville equations as an inverse problem. A global oscillation of the sea level (with a 4–5 years period) between the north and the south hemisphere of the Earth could be proved in this way for the first time.

1. Fundamental relations

As is known from theoretical mechanics, the rotation of a rigid body is described by Euler's equation

$$\frac{d\mathsf{H}}{dt} + (\boldsymbol{\omega} \times \mathsf{H}) = \mathsf{L} \tag{1}$$

in an earth-fixed coordinate system, where H is the angular momentum, $\boldsymbol{\omega}$ is the vector of rotation and L is an external torque. Since the Earth cannot assumed to be a rigid body in a rigorous physical sense, equation (1) must be slightly modified considering a small deviation from the rigid body rotation. This is obtained by introducing the expression

$$\mathsf{H} = \mathsf{I}\boldsymbol{\omega} + \mathsf{h} \tag{2}$$

for the angular momentum in equation (1), where the inertia tensor I and the relative angular momentum h are variable with time. The temporal variations of these quantities are caused by mass motions on Earth. Substituting (2) into (1) we obtain the Euler-Liouville equation

$$\frac{d}{dt}(\mathsf{I}\boldsymbol{\omega} + \mathsf{h}) + \big(\boldsymbol{\omega} \times (\mathsf{I} + \mathsf{h})\big) = \mathsf{L}, \tag{3}$$

[1]) This paper is also registered as *Mitt. Zentralinst. Phys. Erde (Potsdam, GDR)* Nr. 1646.

which governs the rotation of a deformable body. For the description of the variation of the vector of rotation, an earth-fixed coordinate system is chosen with axes oriented along the principal axes of inertia. The x_3-axis of this coordinate system is oriented along the mean position of that axis of inertia which coincides nearly with the vector of rotation. The x_1- and x_2-axes are oriented perpendicular to the x_3-axis and to each other. The x_1- and x_2-axes are situated in the equator plane. In this coordinate system the inertia tensor is given by the matrix

$$\mathsf{I} = \begin{pmatrix} A + c_{11} & c_{12} & c_{13} \\ c_{12} & A + c_{22} & c_{23} \\ c_{13} & c_{23} & C + c_{33} \end{pmatrix}, \tag{4}$$

where A and C are the principal moments of inertia, c_{ii} $(i = 1, 2, 3)$ denote the temporally variable parts of the moments of inertia and c_{ij} $(i, j = 1, 2, 3; i < j)$ are the temporally variable products of inertia. The temporal variability of these quantities depends on variations of the mass geometry caused by geophysical processes or variations of the centrifugal force owing to relative changes of the vector of rotation in the earth-fixed coordinate system. The components of the inertia tensor are calculated according to

$$C_{ij} = \int_V \varrho(x_k x_k \delta_{ij} - x_i x_j)\, dV, \tag{5}$$

where ϱ is the density of a volume element dV of the Earth and δ_{ij} the well-known Kronecker symbol. For the diagonal components the time-dependent parts are obtained by subtracting the principal moments of inertia A or C, respectively, from equation (5). The relative angular momentum is obtained according to

$$\mathsf{h} = \int_V \varrho(\mathsf{r} \times \mathsf{v})\, dV, \tag{6}$$

where v is the velocity of mass motion. Subsequently the quantities c_{ii}, c_{ij} and h can be considered as small values, because the rotation of the Earth deviates only little from the rotation of a rigid body. Therefore the products and square of these quantities will be omitted in the following considerations. In the subsequent discussions we will notice that the excursions from a uniform rotation of the Earth are small. Therefore we can write for the vector of rotation

$$\boldsymbol{\omega} = \omega_0(m_1, m_2, 1 + m_3)^{\mathrm{T}}, \tag{7}$$

where ω_0 is the mean value of the rotational velocity, m_1 and m_2 are the temporally variable pole coordinates and $m_3 = -(\Delta\,\text{l.o.d.})/\text{l.o.d.}$ (l.o.d. = length of day) is the negative value of the relative length of day (m_1, m_2, and m_3 are obtained from astronomical observations and are published by the *International Polar Motion Service* (Mizusawa, Japan) and the *Bureau International de l'Heuré* (Paris)). To get a more compact form of the differential equations, usually the complex quantities

$$m = m_1 + \mathrm{i}m_2, \qquad c = c_{13} + \mathrm{i}c_{23}, \qquad h = h_1 + \mathrm{i}h_2, \qquad L = L_1 + \mathrm{i}L_2$$

are introduced in the evaluation of the differential equations. Here h_1, h_2 and L_1, L_2 are the components of the relative angular momentum and the external torques along the coordinate axes x_1 and x_2. Substituting (4), (6), and (7) into (3), we obtain two differential equations representing the relations between the quantities c_{13}, c_{23},

h_1, h_2 and polar motion and between c_{33}, h_3 and the relative length of day:

$$\frac{dm}{dt} + \alpha m = \mathrm{i}\sigma_{\mathrm{CH}}(m - \psi), \qquad m_3 = \psi_3 + c_0. \tag{8}$$

The complex differential equation in (8) describes polar motion and the second equation, which follows from the solution of a differential equation in a straightforward way, describes the variation of the length of day. In (8) ψ and ψ_3,

$$\psi = \chi - \frac{\mathrm{i}}{\omega_0}\frac{d\chi}{dt}, \qquad \chi = \frac{c}{C - A} - \frac{h}{C - A}\frac{1}{\omega_0}$$

$$\text{and} \quad \psi_3 = -\frac{c_{33}}{C} - \frac{h_3}{C}\frac{1}{\omega_0}, \tag{9}$$

are the excitation functions of the variations of polar motion and the length of day. In (9) the external torques were deleted, because they are out of the scope of the following discussions. Equations (8) describe polar motion and the variation of the length of day in linear approximation. The deformation caused by back reactions to polar motion is contained in the values of the parameters of the differential equation in (8)

$$\sigma_{\mathrm{CH}} = 2\pi f_0 := 2\pi/1.19, \qquad \alpha = 0.05.$$

σ_{CH} is the circular frequency (given in cycles per year) of the free polar motion, the so-called Chandler wobble, and α is the damping parameter (given in units per year). They are obtained from the homogeneous solution of the first equation in (8), considering an excitation function depending linearly on the polar motion m. In this excitation function the influence of the visco-elasticity of the Earth's mantle, the influence of the fluid core, and the pole tides of the ocean must be taken into account. We shall not discuss these problems in detail and refer to the monographs of MUNK and MACDONALD [8] and LAMBECK [6].

2. Solutions of the equations of rotational motion

The equations (8) are the basic relations for the investigation of geophysical processes which influence the Earth's rotation by mass motion. It is obvious that there exist a large number of geophysical processes which excite variations of polar motion and the length of day. Mass motions in the atmosphere, the hydrosphere, the Earth's crust and mantle, and the fluid core should be mentioned. A first hint on the source of a certain component of the excitation function is its time scale. Processes containing short-period (periods essentially shorter than 10 years) mass motions are located in the atmosphere or hydrosphere, while longperiod motions (up to thousands of years) must be attributed to processes in the more solid layers of the Earth. This situation induces us to investigate our problem in the frequency domain.

Applying the Fourier transformation on equations (8) we obtain

$$m(t) = \sum_{f=-\infty}^{\infty} I(f)\,\psi(f, t), \qquad m_3(t) = \sum_{f=-\infty}^{\infty} \psi_3(f, t),$$

where $\psi(f, \cdot)$ and $\psi_3(f, \cdot)$ are the periodic constituents of the excitation functions and $I(f) = \left((1 - f/f_0)^2 + 1/4Q^2\right)^{-1}(1 - f/f_0 - \mathrm{i}/2Q)$ is the frequency-dependent transfer function. Here $f_0 = \sigma_{\mathrm{CH}}/2\pi = 0.84$ is the Chandler frequency, $Q = \pi f_0/\alpha \approx 50$ is the quality factor and f the frequency of a periodic constituent of the excitation function. The frequencies in previous formulae and in the subsequent discussions are given in cycles per year. In case of geophysical interpretation of the Earth's rotation generally certain periodic constituents of the excitation function are investigated, thus in the following the reduced formulae

$$m(f, t) = I(f)\,\psi(f, t) \quad \text{and} \quad m_3(f, t) = \psi_3(f, t)$$

are applied. These equations govern the direct solution. The inverse solution is given by

$$\psi(f, t) = I(f)^{-1}\, m(f, t), \qquad \psi_3(f, t) = m_3(f, t). \tag{10}$$

Further we mention input-output analysis, which can be considered as a particular inverse solution. This is governed by the equation

$$I(f) = m(f, t)\,\psi^{-1}(f, t) \quad \text{or} \quad I(f)^{-1} = \psi(f, t)\, m^{-1}(f, t).$$

The direct solution has been successfully applied to the investigation of atmospheric excitations (see e.g. [1, 3, 4]). The possibility of the nearly exact calculation of an annual-periodic excitation function, using meteorological data, allows the determination of the transfer function by input-output analysis. The inverse solution seems to be a useful tool for the investigation of global geophysical processes on which little information is available. From equations (10) it follows that it is easy to derive the periodic constituents of the excitation function from the corresponding constituents of the variations of polar motion and the length of day, provided that the transfer function is known. But, we must consider that the excitation function consists of integrals to be taken through the whole Earth, which contain unknown solution functions depending on the geophysical process to be investigated.

2.1 The excitation function. The excitation function consists of integrals which represent the geometric and dynamic effects of mass motions. In all that follows, long-period variations will be discussed. Hence it is allowed to neglect the dynamical effect of mass motion. Therefore the relative angular momentum and the temporal derivatives of the products of inertia in equations (9) can be deleted and the excitation functions reduce to $\psi = c/(C - A)$ and $\psi_3 = -c_{33}/C$. Introducing spherical coordinates (φ, λ, r) in (5), we obtain these equations in the following notation:

$$\psi(t) = -\frac{a^4}{C - A} \int\limits_{\varphi=-\pi/2}^{\pi/2} \int\limits_{\lambda=0}^{2\pi} D(\varphi, \lambda, t) \sin\varphi \cos^2\varphi \exp(i\lambda)\, d\varphi\, d\lambda \tag{11}$$

and

$$\psi_3(t) = -\frac{a^4}{C} \int\limits_{\varphi=-\pi/2}^{\pi/2} \int\limits_{\lambda=0}^{2\pi} D(\varphi, \lambda, t) \cos^3\varphi\, d\varphi\, d\lambda, \tag{12}$$

where

$$D(\varphi, \lambda, t) = \frac{1}{a^4} \int\limits_{r=R}^{R+h} r^4\big(\varrho(\varphi, \lambda, r, t) - \varrho_0(\varphi, \lambda, r)\big)\, dr. \tag{13}$$

Here the parameter a is the mean radius of the Earth and R is the radius of a layer of the Earth in which the supposed geophysical process takes place; in (13) mass motion is described by temporal density variations with respect to a mean density ϱ_0. In equations (11), (12) and (13) f was not introduced, because in these formulae and all that follows we discuss excitation functions consisting of one constituent only with a constant frequency. The calculation of (13) requires that the mass conservation law is satisfied. Hence we must add a correction $\Delta\varrho(t)$ to the density variations in

(13) which must fulfil the condition

$$\int_{\varphi=-\pi/2}^{\pi/2} \int_{\lambda=0}^{2\pi} \int_{r=R}^{R+h} \big(\varrho(\varphi, \lambda, r, t) - \varrho_0(\varphi, \lambda, r)\big)\, r^2 \cos\varphi \, d\varphi \, d\lambda \, dr$$

$$+ \Delta\varrho(t) \int_{\pi=-\pi/2}^{\pi/2} \int_{\lambda=0}^{2\pi} \int_{r=R}^{R+h} r^2 \cos\varphi \, d\varphi \, d\lambda \, dr = 0.$$

If we substitute the equations (10) into the left-hand sides of (11) and (12), we obtain the integral equations governing the inverse solution; $D(\varphi, \lambda, t)$ is the solution function to be determined.

2.2 A geometrical property of the excitation function of polar motion. In this subsection we shall consider a property of $\psi(f, t)$ which can be used to find a suitable ansatz for the solution function $D(\varphi, \lambda, t)$. If the axes of the earth-fixed coordinate system are oriented along the axes of the principal moments of inertia, the inertia tensor becomes

$$\mathsf{I}' = \begin{pmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & C \end{pmatrix}. \tag{14}$$

If the axes of principal inertia deviate from the axes of the earth-fixed coordinate system, we obtain the corresponding inertia tensor by appplying the tensor transformation

$$\mathsf{I} = \mathsf{R}\mathsf{I}'\mathsf{R}^{-1}. \tag{15}$$

The structure of the inertia tensor will be, after the transformation, the same as the structure of the inertia tensor according to (4). Because the deviation between the principal axes of inertia and the axes of the coordinate system will be small, we can use a simplified version of the transformation matrix

$$\mathsf{R} = \begin{pmatrix} 1 & 0 & m_1' \\ 0 & 1 & m_2' \\ -m_1' & -m_2' & 1 \end{pmatrix}, \tag{16}$$

where the components m_1' and m_2' can be considered as motions of the pole of inertia. Substituting (14), (16) and (4) into (15), the following relations between the components of the excitation function and the components of the transformation matrix are obtained: $\psi_1 = \dfrac{c_{13}}{C - A} = m_1'$ and $\psi_2 = \dfrac{c_{23}}{C - A} = m_2'$. From this it follows that the excitation function of polar motion can be interpreted as a motion of the pole of inertia (figure axis) of the Earth.

A periodic constituent of the excitation function of polar motion can be written in the form

$$\begin{aligned} \psi(t) &= A_{11} \cos 2\pi f t + A_{12} \sin 2\pi f t + \mathrm{i}(A_{21} \cos 2\pi f t + A_{22} \sin 2\pi f t) \\ &= \psi_{10} \sin(2\pi f t + \gamma_1) + \mathrm{i}\psi_{20} \sin(2\pi f t + \gamma_2), \end{aligned}$$

which corresponds to an elliptical motion of the pole of inertia. The semi-axes of this ellipse are obtained according to

$$\left.\begin{matrix} a \\ b \end{matrix}\right\} = \frac{1}{2}\left(A_{11}^2 + A_{12}^2 + A_{21}^2 + A_{22}^2 + 2(A_{11}A_{22} - A_{21}A_{12})\right)^{1/2}$$

$$\left\{\pm \frac{1}{2}\left(A_{11}^2 + A_{12}^2 + A_{21}^2 + A_{22}^2 - 2(A_{11}A_{22} - A_{21}A_{12})\right)^{1/2}. \right. \quad (17)$$

For many types of excitation functions the elliptical motion reduces to a linear motion. From (17) it follows that this requires the condition

$$A_{11}A_{22} - A_{21}A_{12} = 0, \quad (18)$$

which is equivalent to

$$\gamma_1 = \gamma_2 \quad \text{or} \quad \gamma_1 = \gamma_2 + \pi. \quad (19)$$

From (11) it follows that the solution function $D(\varphi, \lambda, t)$ must have invariable phase angles γ to fulfil the conditions (18) and (19) for an excitation function. The quantities A_{ij} in the above-mentioned formulae are constant coefficients that are obtained by harmonic analyses of the time series of pole coordinates and length of day, respectively.

3. The inverse solution

According to (10) we obtain from the periodic constituents of polar motion and the length of day the corresponding excitation functions in the notation

$$\psi(t) = A_{11}\cos 2\pi f t + A_{12}\sin 2\pi f t + \mathrm{i}(A_{21}\cos 2\pi f t + A_{22}\sin 2\pi f t),$$

$$\psi_3(t) = A_{31}\cos 2\pi f t + A_{32}\sin 2\pi f t.$$

If these expressions are substituted into (11) and (12), a system of six integral equations is got from which the solution function $D(\varphi, \lambda, t)$ must be derived. Generally, the solution function becomes

$$D(\varphi, \lambda, t) = D_0(\varphi, \lambda)\sin\left(2\pi f t + \gamma(\varphi, \lambda)\right).$$

From the system of integral equations we obtain a number of equivalent solutions and it is impossible to find a correct solution without additional physical information. The solution of the system of integral equations can be simplified for excitation functions represented by a linear motion of the pole of inertia. This restriction is justified, because by numerical investigations it is found that a large number of periodic constituents of polar motion are caused by linear motions of the pole of inertia. It can be supposed that only this type of excitation is caused by merely one global geophysical process. Excitation functions corresponding to an elliptical motion of the pole of inertia are caused by a combination of several global geophysical processes. The linear motion of the pole of inertia corresponds to the excitation function

$$\psi(t) = (\psi_{10} + \mathrm{i}\psi_{20})\sin(2\pi f t + \gamma), \qquad \psi_3(t) = \psi_{30}\sin(2\pi f t + \gamma).$$

The solution function becomes $D(\varphi, \lambda, t) = D_0(\varphi, \lambda)\sin(2\pi f t + \gamma)$ and the system of integral equations reduces to

$$\psi_{10} + i\psi_{20} = -\frac{a^4}{C - A} \int_{\varphi=-\pi/2}^{\pi/2} \int_{\lambda=0}^{2\pi} D_0(\varphi, \lambda) \sin\varphi \cos^2\varphi \exp(i\lambda)\, d\varphi\, d\lambda,$$

$$\psi_{30} = -\frac{a^4}{C} \int_{\varphi=-\pi/2}^{\pi/2} \int_{\lambda=0}^{2\pi} D_0(\varphi, \lambda) \cos^3\varphi\, d\varphi\, d\lambda. \tag{20}$$

This is a system of three integral equations determining the spatially variable amplitude of the solution function. The phase angle of the solution function is the same as the phase angle of the excitation function.

An ansatz for $D_0(\varphi, \lambda)$. Since $D_0(\varphi, \lambda)$ describes the distribution of variable amplitudes of time-dependent densities on a spherical surface, it is obvious to approximate it by a series of spherical harmonics

$$F(\varphi, \lambda) = \sum_{n=0}^{\infty} \sum_{m=0}^{n} (a_{nm} \cos m\lambda + b_{nm} \sin m\lambda) P_{mn}(\sin\varphi). \tag{21}$$

The coefficients of this series should be estimated according to presumed distributions of spatially variable amplitudes. For instance, the surface of the Earth could be divided into areas where the density varies ($F(\varphi, \lambda) = 1$ or $F(\varphi, \lambda) = -1$) and others without density variations $\left(F(\varphi, \lambda) = 0\right)$. Using (21), we introduce in (20) the ansatz $D_0(\varphi, \lambda) = \varkappa F(\varphi, \lambda)$, where $\varkappa$ is a constant. By solving the integrals (20), three values $\varkappa_1$, $\varkappa_2$, and $\varkappa_3$ are obtained which must be equal if the coefficients of the function $F(\varphi, \lambda)$ are correctly determined. From $\varkappa_1 \neq \varkappa_2$ it follows that the tesseral spherical harmonics are incorrect, $\varkappa_1 = \varkappa_2 \neq \varkappa_3$ means that coefficients of the zonal spherical harmonics are incorrectly estimated. If the condition $\varkappa_1 = \varkappa_2 = \varkappa_3$ is not fulfilled, the distribution of values at the surface of the Earth must be changed until equality between the three values $\varkappa$ is obtained.

4. The determination of global sea level changes by inverse solution

From pole coordinates published in [9] an amplitude spectrum of polar motion was calculated. The corresponding spectrum of the excitation function was calculated from the spectrum of polar motion according to (10). This was transformed into spectra of both semi-axes of the elliptical path of the moving pole of inertia (Fig. 1). From these spectra it was found that there exists a constituent of the excitation function with a four years period which is represented by a linear motion of the pole of inertia. We supposed that this excitation function could be caused by global sea level changes and calculated the solution function $D_0(\varphi, \lambda)$ according to the suppositions

$$F(\varphi, \lambda) = \begin{cases} 0 & \text{at the continents,} \\ \operatorname{sign}(\varphi)\, 1 & \text{at the oceans.} \end{cases}$$

By this supposition the mass conservation law is sufficiently satisfied. From the complex integral equation of (20) we obtained two values $\varkappa$. In case of equality of these values we confirmed the result by calculating the integral equation for variations of the length of day. This result must be in accordance with the corresponding

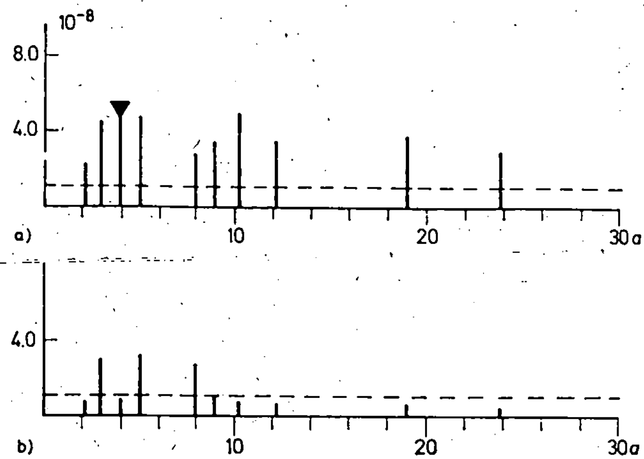

Fig. 1 Amplitude spectra of the semi-axes (a and b) of the excitation functions derived from polar motion. The dashed line denotes the standard deviation of the amplitudes.

periodic constituent of the amplitude spectrum of the variations of the length of day. A relation between sea level changes and the solution function is obtained from (13).

$$D(\varphi, \lambda, t) = \frac{\varrho_w}{a^4} \cdot \int_{r=a}^{a+\Delta h(\varphi,\lambda,t)} r^4 \, dr \approx \varrho_w \Delta h(\varphi, \lambda, t), \tag{22}$$

where ϱ_w is the density of sea water.

4.1 Numerical results. From the calculation of the amplitude spectrum of the semi-major axes of the excitation function of polar motion (Fig. 1) the following quantities for a four-years period were obtained:

$$\psi_{10} = -3.020 \cdot 10^{-8} \text{ rad} \quad \text{and} \quad \psi_{20} = -3.040 \cdot 10^{-8} \text{ rad}.$$

The corresponding quantities in Fig. 1—3 are denoted by arrow heads. Introducing these quantities into the first equation of (20), we got by numerical integration for the real and the imaginary part

$$\frac{\psi_{10}}{\varkappa_1} = -0.181 \cdot 10^{-8} \text{ rad} \quad \text{and} \quad \frac{\psi_{20}}{\varkappa_2} = -0.154 \cdot 10^{-8} \text{ rad}.$$

These quantities were compared and the values $\varkappa_1 = 16.68$ and $\varkappa_2 = 19.74$ obtained. They agree sufficiently and we introduced into the following calculations the mean value $\varkappa = 18.2$. Inserting this value together with $F(\varphi, \lambda)$ into the integral equation for the excitation function of the length of day, we got $\psi_{30} = 3.5 \cdot 10^{-10}$. From the amplitude spectrum of the length of day results $\psi_{30} = 7.0 \cdot 10^{-10}$ for the amplitude of the four-years term. These quantities agree sufficiently if we take into account that the value derived from astronomical observations has a standard deviation $\sigma_{u30} = 4.5 \cdot 10^{-10}$. These results show that the model of a north to south oscillating mass motion on ocean areas works well. Inserting the density of sea water $\varrho_w = 1025 \text{ kg/m}^3$, we find the amplitude $\Delta h = 1.8$ cm of sea level variations according to (22). Considering the different surfaces covered by oceans on the north and the south hemisphere of the Earth, we obtain

$\Delta h_n = 2.0$ cm on the north hemisphere,

$\Delta h_s = 1.6$ cm on the south hemisphere.

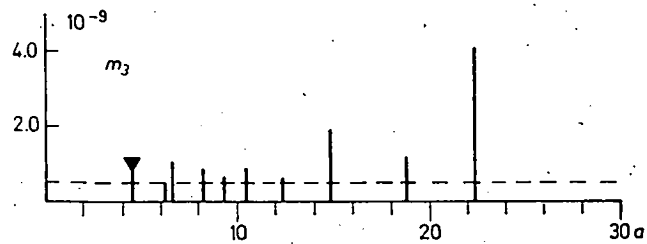

Fig. 2 Amplitude spectrum of the relative length of day. The dashed line denotes the standard deviation of the amplitudes.

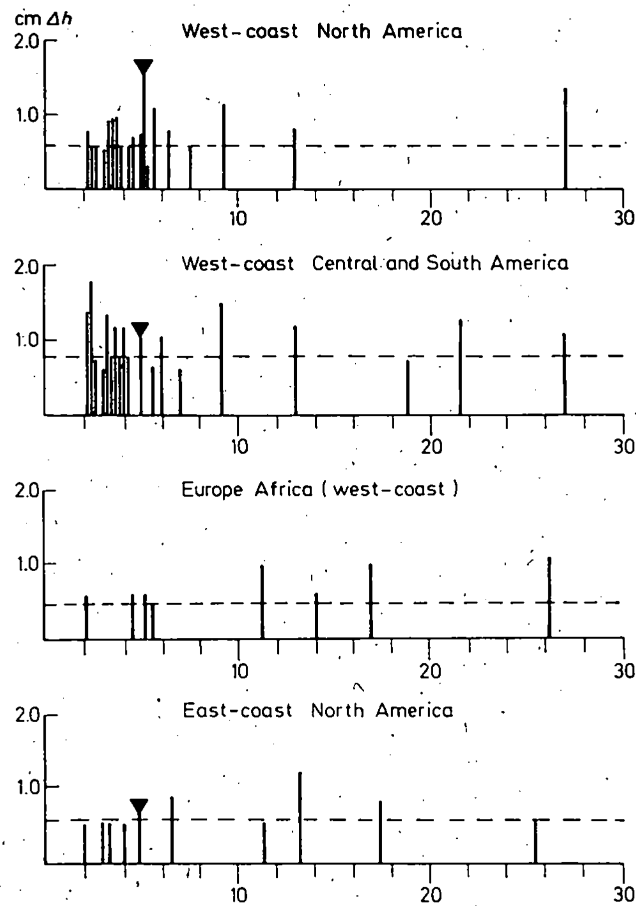

Fig. 3 Amplitude spectra of sea level changes. The dashed lines denote the standard deviations of the amplitudes.

Another suggestion is that only the ocean areas bounded by the Asian east coast and the American west coast and by the American east coast and the west coasts of Europe and Africa take part in sea level oscillations. From this follows an equal amplitude $\Delta h = 1.8$ cm on both hemispheres.

4.2 Comparison with sea level observations. To confirm the hypothesis of a north to south sea level oscillation, we must compare the results obtained from variations of the vector of rotation with direct sea level observations. In [2] time series of sea level changes at different coast lines were published. The amplitude spectra of these time series are exhibited in Figure 3. The observations at the west coast of North America can be considered as the most accurate ones, because these stations lie at the boundary of the largest free ocean area. But, most of the observations at other coast lines also confirm the hypothesis of 4–5 years sea level oscillations between the north and the south hemisphere of the Earth. Considering the amplitudes and periods in Table 1, no objections to the investigated hypothesis can be found.

Table 1: Sea level changes with a 4–5 years period

Results obtained from	Δh cm	period a
variations of the vector of rotation	$\Delta h_n = 2.0 \pm 0.5$ $\Delta h_s = 1.6 \pm 0.5$	4.0 ± 0.2
west coast of North America	1.6 ± 0.6	5.2 ± 0.5
west coast of Central and South America	1.1 ± 0.8	5.0 ± 0.8
east coast of America	0.7 ± 0.6	4.8 ± 0.9

The obtained results show that the variations of polar motion and the length of day could serve as a means to detect global properties of geophysical processes. Polar motion can be applied to detect meridional mass motions, while the variations of the length of day correspond mainly to zonal mass motions. The demonstrated example shows that the variations of polar motion and the length of day allow to decide whether a locally observed property is a global one, provided that the property influences the variations of the vector of rotation in an earth-fixed coordinate system.

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VERFASSER:

Dr. HORST JOCHMANN
Zentralinstitut für Physik der Erde
der Akademie der Wissenschaften der DDR
Telegrafenberg
DDR-1561 Potsdam

Buchbesprechung

H.-J. SCHMEISSER and H. TRIEBEL: **Topics in Fourier Analysis and Function Spaces** (Math. u. Anw. Phys. u. Techn.: Bd. 42). Leipzig: Akad. Verlagsges. Geest & Portig 1987, 300 S.

Das Buch ist einem sich intensiv entwickelnden Gebiet der Analysis, und zwar der Theorie der Räume differenzierbarer Funktionen, gewidmet. In ihm werden von einem einheitlichen Gesichtspunkt aus, dessen Mitbegründer H. Triebel ist, Funktionenräume betrachtet, die die Sobolev-, Besov- und Bessel-Potential-Klassen, Räume mit dominierender gemischter Ableitung und andere umfassen.

Das Buch enthält sechs Kapitel. Im ersten Kapitel werden die Räume $L^{\Omega}_{p,\mu}(\mathbb{R}^n)$ der in $\mathbb{R}^n$ ganzen analytischen Funktionen vom Exponentialtyp untersucht, deren Fourier-Transformierte einen Träger im Kompakt $\Omega \subset \mathbb{R}^n$ besitzen. Der Raum $L^{\Omega}_{p,\mu}(\mathbb{R}^n)$ wird mit der Norm $\|\cdot\|_{L_{p,\mu}\mathbb{R}^n}$ ausgestattet, wobei μ ein Borelsches Maß in $\mathbb{R}^n$ ist. Es werden sowohl der quasinormierte Fall für $0 < p < 1$ als auch der Fall gemischter Quasinormen mit $p = (p_1, \ldots, p_n)$, $0 < p_1, \ldots, p_n \leqq \infty$, untersucht. Die Klassen $L^{\Omega}_{p,\mu}(\mathbb{R}^n)$ und einige ihrer Verallgemeinerungen spielen im Buch eine besondere Rolle. Mit ihrer Hilfe werden die in den folgenden Kapiteln betrachteten Funktionenräume definiert und untersucht. Dadurch erklärt sich wahrscheinlich die recht ausführliche Analyse der Klassen $L^{\Omega}_{p,\mu}(\mathbb{R}^n)$. Das Hauptinstrument ist die Fourier-Transformierte langsam wachsender Distributionen und der Distributionen, die im Schwarz-Raum mit exponentiellem Gewicht definiert sind. Für die Elemente der Räume $L^{\Omega}_{p,\mu}(\mathbb{R}^n)$ werden insbesondere Ungleichungen für die Hardy-Littlewood-Maximalfunktion und Sätze