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Perturbation of Temperature Fields by a Small Inclusion

D. Gönde

In einem Gebiet, das ein kleines Teilgebiet Ω , enthält, wird ein Randwertproblem zweiter Ordnung mit Kopplungsbedingungen auf dem Rand von Ω_{ϵ} gestellt. Für seine Lösung wird eine asymptotische Entwicklung bezüglich des nach 0 gehenden Durchmessers von Ω_i konstruiert.

В области содержащей малую подобласть Ω_t ставится краевая задача второго порядка с переходным условием на границе Ω_{ϵ} . Для её решения строится асимптотика относительно диаметра Ω , стремящегося к нулю.

In a domain which contains a small subdomain Ω_t a boundary value problem of second order with transition conditions on the boundary of Ω_k is posed. For its solution an asymptotic expansion is constructed with respect to the diameter of Ω , tending to zero.

For the last decades, several phenomena in physical science and engineering have given raise to models of mathematical analysis which are concerned with domains with irregular boundaries (e.g., porous media or perforated walls) and therefore, especially since about ten years, to a literature referring to this. Within this topic, it seems A.M. Il'in had been the first to investigate, to some extent, a fundamental type of problems, namely, boundary value problems in domains with one small hole: From a bounded domain G, a small subset Ω , of diameter ε is removed, and the solution of a boundary value problem in G is perturbed by imposing additional boundary conditions on the boundary Γ_{ϵ} of Ω_{ϵ} . A. M. Illin constructed representations of such perturbed solutions u_t in $G_t = G \overline{\Omega}_t$, which are asymptotic with respect to ϵ tending to zero (e.g. $[4-6]$. A simplified method for simpler but most important cases in applications was proposed by the author in [2]. Also numerical solutions have been given for special cases $(e.g. [8])$.

Up to now, however, concerning the local asymptotic behaviour, apparently only fixed boundary conditions have been considered at the boundary of the hole, though transition (coupling) conditions could also be significant. We imagine, e.g., temperature field in a domain occupied by a body with thermal conductivity λ^+ which contains but a little inclusion Ω , filled with material of another conductibility λ ⁻. Therefore we shall describe, by means of the first stages of approximation, a method to construct an asymptotic expansion, above all, for a three-dimensional model $problem - Poisson's equation$; further it is shown that the method will also work in the two-dimensional case and, on principle, for more general equations. Finally, in the special case of a spherical inclusion, a more direct procedure can be used, taking its pattern from $[2, 3]$. $-$ For proving the asymptotic character of the expansions obtained a variant of the maximum principle is applied which admits jumps of the first derivatives.

It should be remarked that there are investigations of related problems for, e.g., bodies including thin sheets with material constants growing to infinity while their

thickness tends to zero (e.g., [1, 8]); the very interesting results consist in global rather weak convergence statements, but without precise description of the local behaviour, and, of course, the methods used are quite different from ours. 204 D. Göune

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1. Formulation of the model problem

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Let $G \subseteq \mathbb{R}^3$ be a bounded domain containing the origin 0, and h a smooth function defined in \overline{G} . The boundary $S = \partial G$ of G should be smooth enough to permit classi-
cal solutions of the Dirichlet problem $G: -\Delta w = h(x)$, $S: w = g(x)$ (1) cal solutions of the Dirichlet problem.

$$
G: -\Delta w = h(x), \quad S: w = g(x) \tag{1}
$$

for continuous boundary values g. Let, further, Ω be another bounded domain in \mathbb{R}^3 containing 0, with smooth boundary $\Gamma = \partial \Omega$. Set $\Omega_{\epsilon} = \epsilon \Omega = \{x : x/\epsilon \in \Omega\}$, $\Gamma_{\epsilon} = \epsilon \Gamma$ $=\partial\Omega_t$, and $G_t = G \setminus \bar{\Omega}_t$, which is a domain with a small hole (provided ε is sufficiently \setminus small). (1) can be regarded as a system governing a stationary temperature distribution w in a body with constant thermal conductivity $\lambda^+ = 1$. Now, within the subset i Ω_{ϵ} , the material is displaced by another one with different conductibility $\lambda^{-} = \lambda > 0$. In that way the temperature will vary, at least locally; the perturbed temperature **1. Formulation of the model problem**

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 $\Omega d G_t = \varepsilon \sqrt{\Omega$ for continuous boundary values g. Let, further, Ω be another bounded domain in \mathbb{R}^n containing 0, with smooth boundary $T = \partial \Omega$. Set $\Omega_t = \varepsilon \Omega = \{x : x | \varepsilon \in \Omega\}$, $T_{\epsilon} = \varepsilon I = \partial \Omega$, and $G_{\epsilon} = G \setminus \overline{\Omega}_{\epsilon}$, w trinuous boundary values g. Let, further, Ω be anothing 0, with smooth boundary $\Gamma = \partial \Omega$. Set $\Omega_{\epsilon} = \epsilon$, and $G_{\epsilon} = G \setminus \overline{\Omega}_{\epsilon}$, which is a domain with a small hot (1) can be regarded as a system governing a stat 204 D. Genox

thickness tends to zero $(x_k, [1, 8])$; the very interesting results consist in global,

rather veck convergence statements, but without precise description of the local

behaviour, and, of course, the methods

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G_{\epsilon} : -\Delta u = h(x), \quad S: u = g(x),
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\begin{array}{ccc}\n&\mathcal{Q}_{\epsilon} : -\lambda \Delta u = h(x), & \mathcal{Q}_{\epsilon} : -\lambda \Delta u = h(x), & \mathcal{Q}_{\epsilon} : -\lambda \Delta u = h(x), & \mathcal{Q}_{\epsilon} : \mathcal{Q}_{\epsilon} : \mathcal{Q}_{\epsilon} & \mathcal{Q
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 $_{\rm where}$

$$
u = u_{\epsilon}(x) = \begin{cases} u^+(x) & \text{for } x \in G_{\epsilon} \\ u^-(x) & \text{for } x \in \Omega_{\epsilon} \end{cases}, \quad l(u) = \lambda \frac{\partial u^-}{\partial n} - \frac{\partial u^+}{\partial n};
$$

n denotes the outer normal with respect to Ω . (For existence confer, e.g., [9].)

We set $u = w + v$ and ask for an asymptotic expansion, for $\varepsilon \to +0$, of the difference

where
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$$
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\n2. Construction of an asymptotic expansion
\nWe set $u = w + v$ and ask for an asymptotic expansion, for $\varepsilon \to +0$, of the difference
\n*v* which is solution of the following problem:
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$$
G_t: -\Delta v = 0, \quad S: v = 0,
$$
\n
$$
\Omega_t: -\lambda \Delta v = (\lambda - 1) \Delta w = (1 - \lambda) \hbar,
$$
\n
$$
\Gamma_t: v^- - v^+ = 0, \quad l(v) = (1 - \lambda) \partial w/\partial n.
$$
\nNow the boundary condition on *S* is omitted, we only demand the (extended) solution
\nto be regular at infinity. So the exact difference *v* is changed to an approximation \overline{v} .
\nNext the two inhomogenities in (3) are removed for a jump of \overline{v} along Γ_t by
\n
$$
\overline{v} = v_0 + \hat{w}, \quad \hat{w}(x) = \begin{cases} 0 & \text{for } x \in G_t, \\ (1 - \lambda) \lambda^{-1} w(x) & \text{for } x \in \Omega_t, \end{cases}
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\n(4)
\nafter which
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$$
G_t: -\Delta v_0 = 0, \quad \Omega_t: -\Delta v_0 = 0
$$
\n
$$
\Gamma_t: v_0^- - v_0^+ = (\lambda - 1) \lambda^{-1} w, \quad l(v_0) = 0
$$
\n(5)

to be regular at infinity. So the exact difference v is changed to an approximation \bar{v} . Next the two inhomogenities in (3) are removed for a jump of \bar{v} along \bar{T}_{e} by to be regular at infinity. So the exact difference v is changed to
Next the two inhomogenities in (3) are removed for a jump of \overline{v}
 $\overline{v} = v_0 + \hat{w}, \quad \hat{w}(x) = \begin{cases} 0 & \text{if } x \in G, \\ (1, 0, 1) \in \text{Im}(x) & \text{for } x \in G. \end{cases}$ We set $u = w + v$ and ask for an asymptotic expansion, for $\varepsilon \to +0$, of the diffe
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\overline{v} = v_0 + \hat{w}, \quad \hat{w}(x) = \begin{cases} 0 & \text{for } x \in G_{\varepsilon}, \\ (1 - \lambda) \lambda^{-1} w(x) & \text{for } x \in \Omega_{\varepsilon}, \end{cases}
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G_{\varepsilon}: -\Delta v_0 = 0, \quad \Omega_{\varepsilon}: -\Delta v_0 = 0
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$$
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$$
G_{\epsilon} : -\Delta v_0 = 0, \quad \Omega_{\epsilon} : -\Delta v_0 = 0
$$

$$
\Gamma_{\epsilon} : v_0^- - v_0^+ = (\lambda - 1) \lambda^{-1} w, \quad l(v_0) = 0
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(5)

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and $v_0(\infty) = 0$. As third step we set

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y(\infty) = 0.
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 As third step we set

$$
v_0(x) = \begin{cases} W[\psi](x) & \text{for } x \in G_{\epsilon}, \\ \lambda^{-1}W[\psi](x) & \text{for } x \in \Omega_{\epsilon}, \end{cases}
$$

where $W[\psi]$ denotes the double layer potential with density ψ on Γ_e , so that all con $v_0(x) = \begin{cases} \n\lambda^{-1}W[\psi](x) & \text{for } x \in \Omega_\epsilon, \\
\lambda^{-1}W[\psi](x) & \text{for } x \in \Omega_\epsilon,\n\end{cases}$
where $W[\psi]$ denotes the double layer potential with density ψ on Γ_ϵ , so that all conditions of (5), except the third one, will be satisfied and $v_0(\infty) = 0$. As third step we set
 $v_0(x) = \begin{cases} W[\psi](x) & \text{for } x \in G_\epsilon, \\ \lambda^{-1}W[\psi](x) & \text{for } x \in \Omega_\epsilon, \end{cases}$

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except the third one, will be satisfied automaticall where $W[\psi]$ denotes the double layer pote
ditions of (5), except the third one, will be
favourable for the following, we introduce
 $\xi = x/\varepsilon$
and, corresponding, $|\xi| = \varrho = r/\varepsilon = |x|/\varepsilon$,
the fixed boundary Γ of the pr *q,(r)* $\alpha \in G_i$,
 $[\psi](x)$ for $x \in G_i$,
 q,(r) $[\psi](x)$ for $x \in \Omega_i$,
 a double layer potential with density ψ on Γ_i , so that all continues third one, will be astisfied automatically. As it will be more
 $\alpha \in \mathfr$

$$
\xi = x/\varepsilon \qquad (1)
$$

so that the potential *W* is now given on the fixed boundary Γ of the prototype domain Ω by

$$
\xi = x/\varepsilon
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\xi = \frac{z}{\varepsilon}
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$$
\xi = \frac{z}{\varepsilon} = \frac{z}{\varepsilon}, \quad \text{so that the potential } W \text{ is now given on}
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\text{boundary } \Gamma \text{ of the prototype domain } \Omega \text{ by}
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\n
$$
W[\varphi](\xi) = \iint_{\Gamma} \varphi(\tau) \frac{\partial \varphi^{-1}}{\partial n_{\varepsilon}} d\sigma_{\varepsilon}, \qquad (7)
$$

where $\varphi(\tau) = \psi(\varepsilon\tau)$. Adequately, we also rewrite the function v_0 as a function of ξ , too: $v_0(\xi) = v_0(x/\varepsilon)$. Using the jump property of W at Γ

$$
W^+(\xi) = 2\pi \varphi(\xi) + W(\xi), \quad W^-(\xi) = -2\pi \varphi(\xi) + W(\xi),
$$

where W^+ , W^- are the limits from the outside and from the inside, respectively, while $W(\xi)$ denotes the value on Γ itself, we obtain the integral equation for the density *q',* ing, $|\xi| = \varrho = r/\varepsilon = |x|/\varepsilon$, so that the potential *W* is now given on
 ary T of the prototype domain Ω by
 $\int \int \varrho(r) \partial \varrho^{-1} / \partial n_r d\sigma_r$, (7)
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 $\int \int \frac{r}{r}$, $\int \int \frac{dr}{r}$ and $\$

$$
2\pi\varphi(\xi) - \frac{1-\lambda}{1+\lambda}\dot{W}[\varphi](\xi) = \frac{1-\lambda}{1+\lambda}w(\varepsilon\xi)
$$
 (8)

on the surface Γ , with weakly singular kernel. Because the homogeneous adjoint The same of (5) with *w* replaced by 0, regular at infinity, from the lemma below (maximum principle) it will follow that this solution is necessarily the trivial one. Therefore (8) will always be solvable; the solution w of (5) with *w* replaced by 0, regular at infinity, from the lemma below (maximum principle) it will follow that this solution is necessarily the trivial 'one. Therefore (8) will always be solvable; the solution will depend continuously on the right-hand side. • obtain for limits from-the outside and from the inglue on Γ itself, we obtain the integral equality all the integral equality of $\hat{W}[\varphi](\xi) = \frac{1 - \lambda}{1 + \lambda} w(\varepsilon \xi)$
weakly singular kernel. Because the hom out to provide, but as integral equation turns

of (5) with w replaced b

principle) it will follow

will always be solvable;

Since a double layer po

obtain for
 $v_0(\xi) = v_0(x/\varepsilon)$

the estimate
 $|v_0(x/\varepsilon)| \leq C\varepsilon^2/$ on the surface Γ , with weakly singular kernel. Because the
integral equation turns out to provide, but as a single layer p
of (5) with w replaced by 0, regular at infinity, from the lem
principle) it will follow that ntial V, a solution
below (maximum
one. Therefore (8)
he right-hand side.
cording to ϱ^{-2} , we
(9)
(9)
uniform boundedprinciple) it will follow that this solution is necessarily the trivial one. Therefore (8)
will always be solvable; the solution will depend continuously on the right-hand side.
Since a double layer potential will decreas

Since a double layer potential will decrease for
$$
\rho = |\xi| \to \infty
$$
, according to ρ^{-2} , we obtain for
\n
$$
v_0(\xi) = v_0(x/\varepsilon) = \begin{cases} W[\varphi](x/\varepsilon) & \text{for } x \in G_{\varepsilon}, \\ \lambda^{-1}W[\varphi](x/\varepsilon) & \text{for } x \in \Omega_{\varepsilon} \end{cases}
$$
\n(b) (9)

with a constant C independent of ε , for φ will be bounded from the uniform boundedness of the right-hand side of (8) . With the function v_0 just constructed the approximate difference \bar{v} in (4) satifies the conditions (3) exactly except that on S: there will be $\bar{v} = v_0 = O(\epsilon^2)$. By the lemma already mentioned it will turn out that $v_0(\xi) = v_0(x/\varepsilon) = \begin{cases} W[\varphi](x/\varepsilon) & \text{for } x \\ \lambda^{-1}W[\varphi](x/\varepsilon) & \text{for } x \end{cases}$
the estimate
 $|v_0(x/\varepsilon)| \leq C\varepsilon^2/r^2$
with a constant *C* independent of ε , for φ will be bo
ness of the right-hand side of (8). With the fun

$$
u(x) = \overline{\tilde{w}}(x) + \hat{w}(x) + v_0(x/\varepsilon) + \varepsilon^2 z_1(x) \qquad (11)
$$

with z_1 bounded in G , independently of ε .

The further procedure in order to get

function z_1 will be expanded
 $z_1 = w_1 + \hat{w}_1 + v_1 + \varepsilon^2 z_2$,

where w_1 is the solution of (1) with $h = 0$ The further procedure in order to get higher approximations is apparent: The function z_1 will be expanded will be $\bar{v} = v_0 = O(\varepsilon^2)$. By the lemma already mentioned it will turn out that
 $u(x) = \tilde{w}(x) + \hat{w}(x) + v_0(x/\varepsilon) + \varepsilon^2 z_1(x)$ (11)

with z_1 bounded in *G*, independently of ε .

The further procedure in order to

$$
z_1 = w_1 + \hat{w}_1 + v_1 + \varepsilon^2 z_2,
$$

deviation of the boundary values on S induced by v_0 , \hat{w}_1 is defined analogously by

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(4) with w_1 instead of w, and v_1 is obtained by (5), (8), and (9) with w replaced by w_1 just as done for v_0 , and so on. Summing up we have

Theorem 1: The solution u of the perturbed problem (2) admits an asymptotic expan-
 $u(x) = \sum_{k=0}^{m} \epsilon^{2k} (w_k(x) + \hat{w}_k(x) + v_k(x/\epsilon))' + \epsilon^{2(m+1)} z_{m+1}(x)$, (13) *sion*

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\n
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$$

\n $= w$ is the solution of the unperturbed problem (1), w_k ($k \geq 1$) the solution of

\n $v = 0$, $q = -v_{k-1}/\varepsilon^2$.

'where $w_0 = w$ *is the solution of the unperturbed problem (1),* w_k ($k \ge 1$) *the solution of*

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\n(4) with
$$
w_1
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 instead of w , and v_1 is obtained by (5)
\njust as done for v_0 , and so on. Summing up we have
\n
$$
\text{Theorem 1: The solution } u \text{ of the perturbed problem}
$$
\n
$$
u(x) = \sum_{k=0}^{m} \varepsilon^{2k} (w_k(x) + \hat{w}_k(x) + v_k(x/\varepsilon))' +
$$
\n
$$
\text{where } w_0 = w \text{ is the solution of the unperturbed problem}
$$
\n(1) with $h = 0$, $g = -v_{k-1}/\varepsilon^2$,
\n
$$
\hat{w}_k(x) = \begin{cases} 0 & \text{for } x \in G_\varepsilon, \\ (1 - \lambda) \lambda^{-1} w_k(x) & \text{for } x \in \Omega_\varepsilon, \\ \lambda^{-1} W(\xi) & \text{for } \varepsilon \xi \in G_\varepsilon, \end{cases}
$$

 $W_k(x) = \begin{cases} W(\xi) & \text{for } x \in \Omega_\epsilon, \\ \lambda^{-1}W(\xi) & \text{for } \epsilon\xi \in G_\epsilon, \end{cases}$
 $W = W[\varphi_k]$ the double layer potential on Γ with density φ_k which is a solution of the integral equation *integral equation* $v_k(\xi) = \begin{cases} W(\xi) & \text{for } \varepsilon \xi \in G_\varepsilon, \\ \lambda^{-1} W(\xi) & \text{for } \varepsilon \xi \in \Omega_\varepsilon, \end{cases}$
 $W = W[\varphi_k]$ the double layer potential on Γ with ι
 and z_{m+1} *is bounded in all of G, independently of* ε .

$$
v_k(\xi) = \begin{cases} \lambda^{-1}W(\xi) & \text{for } \varepsilon \xi \in \Omega_{\varepsilon}, \\ v_k \end{cases}
$$

the double layer potential on Γ with density φ_k will
quation

$$
2\pi \varphi_k(\xi) - \frac{1-\lambda}{1+\lambda} W[\varphi_k](\xi) = \frac{1-\lambda}{1+\lambda} w_k(\varepsilon \xi), \quad \xi \in \Gamma,
$$

Remark: The functions v_k depend continuously on ε because the corresponding solutions φ_k of the integral equation (8) will do. Of course one could expand the right-hand side of (8) and obtain, for each v_k , an expansion with powers of ε , the coefficient functions depending on $\xi = x/\varepsilon$ only. $\mathcal{U}(1 - \lambda) \lambda^{-1} w_k(x)$ for $x \in \Omega_c$,
 $v_k(\xi) = \begin{cases} W(\xi) & \text{for } \varepsilon \in G_c, \\ \lambda^{-1} W(\xi) & \text{for } \varepsilon \in \Omega_c, \end{cases}$
 $W = W[\varphi_k]$ the double layer potential on Γ with a

integral equation
 $\begin{cases} 2\pi \varphi_k(\xi) - \frac{1-\lambda}{1+\lambda} W[\varphi_k](\xi) = \frac{1-\lambda$ $W = W[\varphi_k]$ the double layer potential on Γ with density φ_k which is
integral equation
 $2\pi\varphi_k(\xi) - \frac{1-\lambda}{1+\lambda}W[\varphi_k](\xi) = \frac{1-\lambda}{1+\lambda}w_k(\varepsilon\xi)$, $\xi \in \Gamma$,
and z_{m+1} is bounded in all of G , independently of ε . and z_{m+1} is bounded in all of G, independently of ε .

Remark: The functions v_k depend continuously on ε because the corresponding solu-

tions φ_k of the integral equation (8) will do. Of course one could Remark: The functions v_k
tions φ_k of the integral equations
side of (8) and obtain, for expression of $\xi = 2$
functions depending on $\xi = 2$
3. The maximum principle
In order to estimate the rem
solvability of the i . Of course one could expansion with powers of
xpansion with powers of
f the expansion (13), and
use the following variant
 \overline{G}_t) $\cap C^2(\Omega_t) \cap C^1(\overline{\Omega}_t)$,
 ∂u^- **Example 1.** The functions v_k depend continuously on ε because the correspondions φ_k of the integral equation (8) will do. Of course one could expand the ri side of (8) and obtain, for each v_k , an expansion wi

In order to estimate the remainder z_{m+1} of the expansion (13), and for the proof of principle. tions φ_k of the integral equation (8) will do. Of course one could
side of (8) and obtain, for each v_k , an expansion with power
functions depending on $\xi = x/\varepsilon$ only.
3. The maximum principle
In order to estimate t solvability of the integral equation (8), we use the following variant of the maximum **(i)** α i) α is α only β is α is α) in the parameter of the transmit probability of the integral equation (8), we use the following variancy of the integral equation (8), we use the following variancy o In order to estimate the remainder z_{m+1} of the expansion (13), and for the proof of
solvability of the integral equation (8), we use the following variant of the maximum
principle.
Lemma: Let $z, Z \in C(\overline{G}) \cap C^2(G_i) \cap C^$

$$
\text{Lemma: Let } z, Z \in C(\overline{G}) \cap C^2(G_\epsilon) \cap C^1(\overline{G}_\epsilon) \cap C^2(\Omega_\epsilon) \cap C^1(\overline{\Omega}_\epsilon),
$$

$$
\text{ma:} \text{ Let } z, Z \in C(\overline{G}) \cap C^2(G_\epsilon) \cap C^1(\overline{G}_\epsilon) \cap C^2(\Omega_\epsilon) \cap C^1(\overline{\Omega}_\epsilon),
$$
\n
$$
L^{\pm}u = -\lambda^{\pm} \Delta u + c(x) u, \quad l u = \lambda^{-} \frac{\partial u^{-}}{\partial n} - \lambda^{+} \frac{\partial u^{+}}{\partial n}
$$

where $c \in C(\overline{G})$ is nonnegative, λ^- , λ^+ are positive constants, n is the outside normal (with respect to Ω). If $Q(G)$ is nonnegative, λ , λ , λ
 *Q*_c: $|L^+z| \leq L^+Z$, $S: |z| \leq Z$
 $Q_c: |L^-z| \leq L^-Z$, $\Gamma_c: |z| \leq \lambda$ l^{\prime} $l^{\prime}(G_{\epsilon}) \cap C^{2}(\Omega_{\epsilon}) \cap C^{1}(\Omega_{\epsilon}),$
 $l^{\prime} = \lambda^{-} \frac{\partial u^{-}}{\partial n} - \lambda^{+} \frac{\partial u^{+}}{\partial n}$
 ire positive constants, n is the outside normal
 lZ,

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$$
G_{\epsilon}: |L^+z| \leqq L^+Z, \quad S: |z| \leqq Z,
$$

$$
\Omega_{\epsilon}: |L^-z| \leq L^-Z, \quad \Gamma_{\epsilon}: |z| \leq \mathcal{U}Z,
$$

The proof consists in the usual application of the following *proposition* to the differences $v = \pm z - Z$, with apparently corresponding notations: where $c \in C(\overline{G})$ is nonnegative, λ^- , λ^+ are positive constants, n is the outside not

(with respect to Ω_i). If
 $G_i : |L^+z| \leq L^+Z$, $S : |z| \leq Z$,
 $\Omega_i : |L^-z| \leq L^-Z$, $\Gamma_i : |z| \leq |Z$,

then $|z| \leq Z$ in all of G

 $L^{\pm}u = -\lambda^{\pm} \Delta u + c(x) u$, $lu = \lambda^{-} \frac{\partial u^{-}}{\partial n} - \lambda^{+} \frac{\partial u^{+}}{\partial n}$

here $c \in C(\overline{G})$ is nonnegative, λ^{-} , λ^{+} are positive constants, n is the outside normal

ith respect to Ω ,). If
 $G_i : |L^{\pm}z| \leq L^{\pm}Z$, $S : |z|$ Let $G_1 \subseteq G_2 \subseteq \cdots \subseteq G_m = G$ be bounded domains in \mathbb{R}^n with smooth boundaries $S_i = \partial G_i$ which do not intersect each other. In the closure of each $D_i = G_i \setminus \overline{G}_{i-1}$ $G_i: |L^+z| \leq L^+Z$, $S: |z| \leq Z$,
 $\Omega_i: |L^-z| \leq L^-Z$, $I_i: |z| \leq Z$,
 $\Omega_i \geq Z$ in all of G .

The proof consists in the usual application of the following proposition to the

differences $v = \pm z - Z$, with apparently corresp $(i = 1, ..., m; G₀$ empty set) a linear uniformly elliptic positive operator L_i of second *nonnegative,* Λ^{-} , Λ^{+} are positive.
 If
 $\leq L^{+}Z$, $S: |z| \leq Z$,
 $\leq L^{-}Z$, $\Gamma_{\epsilon}: |z| \leq |Z|$,

of G .

sists in the usual application c
 $z - Z$, with apparently corresp
 $\cdots \subseteq G_m = G$ be bounded doms

onto int order, with bounded coefficients, is defined for which the strong maximum principle

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is valid. Let *v* be continuous in \overline{G} , with $v \leq 0$ on $S = S_m = \partial G$, $v(x) = v_i(x)$ for $x \in \overline{D}_i$, and, in each D_i , be a classical solution of $L_i v_i \leq 0$ with first derivatives con-Perturbation of Temperature Fields 207

is valid. Let *v* be continuous in \overline{G} , with $v \le 0$ on $S = S_m = \partial G$, $v(x) = v_i(x)$ for
 $x \in \overline{D}_i$, and, in each D_i , be'a classical solution of $L_i v_i \le 0$ with first derivative tinuous up to the boundaries. In any point of a boundary S_i separating D_i and D_{i+1} , for the outer normal derivatives the transition condition shall be fulfilled: is valid.
 $x \in \overline{D}_i$, and

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 $(*)$

Then v can

If *av1 ,1 Jan* < 0, then a*vi lOn* < 0.

 $\label{eq:2} \frac{1}{\sqrt{2}}\sum_{i=1}^n\frac{1}{\sqrt{2}}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^n\frac{1}{j!}\sum_{j=1}^$

Then v cannot assume a positive maximum in the interior of G. Apparently it is only • to be shown that *v* cannbt attain a positive maximum at a point x, of a bundary S_i ($i \leq m - 1$). But if this should be the case, and v_{i+1} not a constant, then $\partial v_{i+1}/\partial n$ $<$ 0 in x_i , and (*) would entail still greater values of $v = v_i$ in D_i . If $v_{i+1} =$ const, the argument may be repeated with respect to the boundary S_{i+1} , and so on \blacksquare
Supplement: The assertion of the lemma will be maintained if there is a surface (e.g., *(*)* If $\partial v_{i+1}/\partial n < 0$, then $\partial v_i/\partial n < 0$.

Then v cannot assume a positive maximum in the interior of G. Apparently it is only to be shown that v cannot attain a positive maximum at a point x_i of a boundary S_i $x \in D_i$, and, in each D_i , be a classical solution of $L_i v_i \leq 0$ with first derivatives continuous up to the boundaries. In any point of a boundary S_i separating D_i and D_i , for the outer normal derivatives the tra utter normal derivatives the transition condition

If $\partial v_{i+1}/\partial n < 0$, then $\partial v_i/\partial n < 0$.

annot assume a positive maximum in the interior

annot assume a positive maximum in the interior
 $n-1$). But if this should be t (*) If $\partial v_{i+1}/\partial n < 0$, then $\partial v_i/\partial n < 0$.
Then v cannot assume a positive maximum in the interior of G to be shown that v cannot attain a positive maximum at a S_i ($i \leq m - 1$). But if this should be the case, an

a sphere) in *G,* where *Z* has only continuous derivatives of first order.

In order to apply the lemma to prove that z_1 in (11) is bounded we state the conti-
nuity of z_1 at Γ , and

$$
G_{\epsilon}: -\Delta z_1 = 0, \quad S: |z_1| = |-v_0/\epsilon^2| \leq C,
$$

$$
\Omega_{\epsilon}: -\lambda \Delta z_1 = 0, \quad \Gamma_{\epsilon}: |z_1| = 0.
$$
 (14)

Apparently $Z = C$ = const will be an upper bound for $|z_1|$. Preceding a step $(m = 1)$ we have to estimate z_2 from (12). Inserting the definitions or properties of w_1, \hat{w}_1, v_1 , and z_1 we see that z_2 is continuous in G and satisfies a system like (14), with v_0 replaced by v_1 . Thus, by induction, the general assertion can be\proved.

• As to the solvability of the integral equation (8); it is to be shown that a solution of (5), with $w = 0$, represented by a single layer potential, must vanish identically. For this sake, the domain is extended to a ball of arbitrary radius; on its surface the potential will be as small as one likes and, according to the lemma, also in the ball and in the included domain G.

4 **The** two-dimensional case

The assertions of Theorem 1 will apparently be maintained, with obvious modifications, for space dimensions greater than three. Also for dimension two one expects, for the present, the formal expansion procedure torun quite similar: In the expansion The assertions of Theorem 1 will apparently be maintained, with obvious modifications, for space dimensions greater than three. Also for dimension two one expects, for the present, the formal expansion procedure torun qui The assertions of Theorem 1 will apparently be maintained, with obvious modifica-
tions, for space dimensions greater than three. Also for dimension two one expects,
for the present, the formal expansion procedure torun q in the integral equation (8) 2π is substituted by π . Further on, the existence of a uniform bound for z_{m+1} in the remainder $\varepsilon^{m+1}z_{m+1}$ can be proved as just done by the lemma. But it cannot be applied immediately, as at the end of Section 3, to the proof of the solvability of (8): The function *v* satisfying (5) (with $w = 0$), since generated by a single layer, cannot be supposed here, a priori, to tend to zero at infinity, which had been essential above. Formula. But it cannot be applied immediately, as at the end of Section 3, to the proof
of the solvability of (8): The function v satisfying (5) (with $w = 0$), since generated by
a single layer, cannot be supposed here, a or x_{m+1}^{m+1} in the condition ε $e^{-x_{m+1}^{m+1} - \varepsilon}$ in the proved as just
mnot be applied immediately, as at the end of Section 3,
r of (8): The function v satisfying (5) (with $w = 0$), since ε
annot be supp *J* Theorem 1 will apparently be maintained, we dimensions greater than three. Also for dimentihe formal expansion procedure to run quite since ϵ^{2k} are to be replaced by ϵ^k – corresponding to layer potentials, wi bace dimensions greater than three. Als

ent, the formal expansion procedure to reres ε^{2k} are to be replaced by ε^{k} – corres

uuble layer potentials, will now decreas

ral equation (8) 2π is substituted by

But we can conclude as follows: If $v(\xi) = V[\tilde{\chi}] (\xi)$ is a single layer potential which $\times \partial V[\chi]/\partial n = 0$, then a single layer, cannot be supposed here, a priori, to tend to zero at infinity, which
had been essential above.
But we can conclude as follows: If $v(\xi) = V[\chi](\xi)$ is a single layer potential which
will fulfil $lv = 0$ at Γ Potential which
 $(1 - \lambda) (1 + \lambda)^{-1}$
 (15)

$$
\vartheta(\xi) = \begin{cases} v(\xi) + W[\varphi](\xi) & \text{for } \xi \in \Omega, \\ \lambda v(\xi) + W[\varphi](\xi) & \text{for } \xi \in \Omega \end{cases}
$$
 (15)

 $\times \partial V[\chi]/\partial n = 0$, then
 $\hat{v}(\xi) = \begin{cases} v(\xi) + W[\varphi](\xi) & \text{for } \xi \\ \lambda v(\xi) + W[\varphi](\xi) & \text{for } \xi \end{cases}$

with the double layer potential $W[\varphi], \varphi =$

including its first derivatives. Therefore \hat{v} is $-(1/2\pi)$ $(1 - \lambda)$ v, passes Γ continuously $\vartheta(\xi) = \begin{cases} v(\xi) + W[\varphi](\xi) & \text{for } \xi \in \Omega \\ \lambda v(\xi) + W[\varphi](\xi) & \text{for } \xi \in \Omega \end{cases}$
with the double layer potential $W[\varphi], \varphi = -(1/2\pi) (1 - \lambda) v$, passes Γ continuously including its first derivatives. Therefore ϑ is a potential

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 $\label{eq:2} \begin{aligned} \mathcal{L}^{(1)}(x) &= \mathcal{L}^{(1)}(x) \, , \\ \mathcal{L}^{(1)}(x) &= \mathcal{L}^{(1)}(x) \, , \end{aligned}$

with logarithmic growth at infinity, at most, but bounded from above or below $-$ and such a singularity must be a removable one: $\hat{v} = c = \text{const.}$ Now (15) shows that $v(\xi) = c + O(|\xi|^{-1})$ as $|\xi| \to \infty$, and by the lemma $v(\xi) - c \equiv 0$ will follow. Especially, $\partial v/\partial n = 0$ on Γ_t and, therefore, $\chi = 0$ guarantees solvability of the integral equation (8) also in this case.

Theorem 2: *In a plane domain G with inclusion Q, the solution above* or below $-$ and
 $v(\xi) = c + O(|\xi|^{-1})$ as $|\xi| \to \infty$, and by the lemma $v(\xi) - c \equiv 0$ will follow. Especially, $\partial v/\partial n = 0$ *u*^{*x*} α *uz* α *l* α *y*_{*k*} α *y*_{*l*} α *y*_{*k*} α *ik*_{*n*} α *y*_{*k*} α *ik*_{*l*} α *ik*_{*l*} α *j* α *j* α *j* α *j* α *j* β *il* α *j* α *j* β *il* α *j* $\$

problem (2) *admits an asymptotic expansion*

$$
u = \sum_{k=0}^{m} (w_k + w_k + v_k) \epsilon^k + \epsilon^{m+1} z_{m+1}
$$

 $w = \sum_{k=0}^{n} (w_k + \hat{w}_k + v_k) \hat{e}^k + \hat{e}^{m+1} z_{m+1}$ (13')
 with functions w_k , \hat{w}_k , v_k defined like in Theorem 1, but w_k ($k \ge 1$) solution of (1) with $g = -v_{k-1}/\varepsilon$ and v_k, v_k, v_k aetinea like in Theorem 1, out w_k ($k \geq 1$) solution of (1) $g = -v_{k-1}/\varepsilon$ and $v_k(\xi) = v_k(x/\varepsilon) = W[\varphi_k](\xi)$ given by the integral equation on Γ **5. Followith the BLACE AND ASSET ASSESS**
 5. Following the anti-interpretential CONSTANDING ACT ACT ACT ACT AND $u = \sum_{k=0}^{m} (w_k + \hat{w}_k + v_k) e^k + e^{m+1} z_{m+1}$ **

with functions** w_k **,** \hat{w}_k **,** v_k **defined like in Theorem 1** problem (2) admits an asymptotic expansion
 $u = \sum_{k=0}^{m} (w_k + \hat{w}_k + v_k) \epsilon^k + \epsilon^{m+1} z_{m+1}$

with functions w_k , \hat{w}_k , v_k defined like in Theorem 1, but w_k
 $g = -v_{k-1}/\epsilon$ and $v_k(\xi) = v_k(x/\epsilon) = W[\varphi_k]$ (ξ) given by *•* $u = \sum_{k=0}^{m} (w_k + w_k + v_k) e^k + e^{m+1} z_{m+1}$ (13)
 vith functions w_k , \hat{w}_k , v_k defined like in Theorem 1, but w_k ($k \ge 1$) solution of (1) with
 $g = -v_{k-1} | \varepsilon$ and $v_k(\xi) = v_k(x/\varepsilon) = W[\varphi_k] (\xi)$ given by the int

$$
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, and, therefore, $\chi = 0$ guarantees soivability of the integral
(8) also in this case.
em 2: In a plane domain G with inclusion Ω , the solution u of the perturbed
2) admits an asymptotic expansion

$$
u = \sum_{k=0}^{m} (w_k + \hat{w}_k + v_k) \epsilon^k + \epsilon^{m+1} z_{m+1}
$$
(13')
lions w_k , \hat{w}_k , v_k defined like in Theorem 1, but w_k ($k \ge 1$) solution of (1) with
 $1/\varepsilon$ and $v_k(\xi) = v_k(x/\varepsilon) = W[\varphi_k] (\xi)$ given by the integral equation on Γ
 $\pi \varphi_k(\xi) - \frac{1-\lambda}{1+\lambda} W[\varphi_k] (\xi) = \frac{1-\lambda}{1+\lambda} w_k(\varepsilon \xi).$ (8')

$$
G\colon -\Delta w\,+\,c(x)\,w\,=\,h(x),\quad S\colon w\,=\,g(x)
$$

and the perturbed problem is

ations
$$
w_k
$$
, \hat{w}_k , \hat{v}_k defined like in Theorem 1, but w_k ($k \geq 1$) solution of (1) with $-1/\varepsilon$ and $v_k(\xi) = v_k(x|\varepsilon) = W[\varphi_k](\xi)$ given by the integral equation on Γ

\n $\pi \varphi_k(\xi) - \frac{1 - \lambda}{1 + \lambda} W[\varphi_k](\xi) = \frac{1 - \lambda}{1 + \lambda} w_k(\varepsilon \xi).$

\nThus, Γ is on the case $-\Delta + c$

\nwhen will now read

\n $G: -\Delta w + c(x) w = h(x), \quad S: w = g(x)$

\nperturbed problem is

\n $G_i: -\Delta u + c(x) u = h(x), \quad S: u = g(x),$

\n $\Omega_i: -\lambda \Delta u + c(x) u = h(x), \quad \Gamma_i: u^- - u^+ = 0, \quad l\bar{u} = 0,$

\nnonnegative continuous function c in \overline{G}, G, Q are bounded domains in \mathbb{R}^3 as

with a nonnegative continuous function c in \bar{G} , G, Q are bounded domains in \mathbb{R}^3 as in Section 1, $\Omega_t = \epsilon \Omega$, $\Gamma_t = \epsilon \Gamma = \partial \Omega_t$. In addition, for technical reasons, Ω is to be supposed star-shaped with respect to a ball centered at the origin. As leading terms of an asymptotic approximation, the same functions as defined in (4) , (9) for the simpler model above turn out to be also suited in this case. The difference $v = u - w$ will here satisfy the conditions corresponding to (3) - the differential equations in. G_t and Ω , augmented by $c(x)$ v on the left-hand side. But as approximating difference \bar{v} we will again choose the solution, regular at infinity, of \cdot . *G_c*: $-\Delta u + c(x) u = h(x)$, $S: u = g(x)$,
 $Q_t: -\lambda \Delta u + c(x) u = h(x)$, $\Gamma_t: u^- - u^+ = 0$, $lu = 0$,

nonnegative continuous function *c* in \overline{G} , G , Ω are bounded domains in \mathbb{R}^3

ion 1, $\Omega_t = \epsilon \Omega$, $\Gamma_t = \epsilon \Gamma = \partial \Omega_t$. In add Q_{ϵ} : $-\Delta u + c(x) u = h(x)$, $\sum u - g(x)$,
 Q_{ϵ} : $-\lambda \Delta u + c(x) u = h(x)$, Γ_{ϵ} : $u^{-} - u^{+}$

with a nonnegative continuous function c in \overline{G} , G ,

in Section 1, $Q_{\epsilon} = \epsilon \Omega$, $\Gamma_{\epsilon} = \epsilon \Gamma = \partial Q_{\epsilon}$. In addition

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 $\therefore -\lambda \Delta \bar{v} = (\lambda + 2\lambda \bar{v})$

nd (9),
 $\begin{cases} (1 - \lambda) \lambda^{-1} \\ (1 - \lambda) \lambda^{-1} \end{cases$ *mptotic approximation, the sa*
 lel above turn out to be also
 zaugmented by $c(x) v$ on the lagain choose the solution, reg
 $\mathbb{R}^3 \setminus \overline{\Omega}_\epsilon$: $-\Delta \overline{v} = 0$, Ω_ϵ : $-\lambda$
 Γ_ϵ : $\overline{v}^- - \overline{v}^+ = 0$, $l\overline{v} =$ he same functions as defined in (4), (9) for the also suited in this case. The difference $v = r$ rresponding to (3) – the differential equation the left-hand side. But as approximating differential equation, regular at inf of an asymptotic approximation, the same functions as defined in (4), (9) for the sim-
pler model above turn out to be also suited in this case. The difference $v = u - u$
will here satisfy the conditions corresponding to (3)

$$
\mathbb{R}^3 \setminus \overline{\Omega}_{\epsilon} : -\Delta \overline{v} = 0, \quad \Omega_{\epsilon} : -\lambda \Delta \overline{v} = (\lambda - 1) \Delta w
$$
\n
$$
\Gamma_{\epsilon} : \overline{v}^- - \overline{v}^+ = 0, \quad l\overline{v} = (1 - \lambda) \partial w/\partial n.
$$
\n(18)

Therefore we set, combining (4) and (9),

$$
\overline{v}(x) = \hat{w}(x) + v_0(x/\varepsilon) = \begin{cases} \n\overline{v}(x) + \overline{v}(x/\varepsilon) & \text{if } x \neq 0 \\
(1 - \lambda) \lambda^{-1} w(x) + \lambda^{-1} W[\varphi](x/\varepsilon) & \text{if } x \neq 0\n\end{cases}
$$
\n(19)

for $x \in G_{\epsilon}$ or $x \in \Omega_{\epsilon}$, respectively, where φ is the solution of the integral equation (8).
The remainder z_0 in $u = w + \hat{w} + v_0 + z_0$ will now be submitted to

Let above turn out to be also suited in this case. The difference
$$
v = u - w
$$

\n $v = u - w$
\n $v = u$
\n $v = u$
\n $v = u$
\n $\frac{dv}{dx} = 0$, $Q_z = -2\sqrt{v} = (2 - 1)\Delta w$
\n $\frac{dv}{dx} = (1 - \lambda) \frac{\partial w}{\partial n}$.
\nWe use set, combining (4) and (9),
\n $\overline{v}(x) = \dot{w}(x) + \overline{v}_0(x/\varepsilon) = \begin{cases} \frac{W[\varphi]}{(x/\varepsilon)} & (18) \\ \frac{W[\varphi]}{(x/\varepsilon)} & (19) \end{cases}$
\n $\overline{v}(x) = \dot{w}(x) + \overline{v}_0(x/\varepsilon) = \begin{cases} \frac{W[\varphi]}{(x/\varepsilon)} & (19) \\ \frac{W[\varphi]}{(x/\varepsilon)} & (19) \end{cases}$
\n $\frac{dw}{dx} = x \in \Omega_c$, respectively, where φ is the solution of the integral equation (8).
\n $G_c: -\Delta z_0 + cz_0 = -cv_0$, $S: z = -v_0$,
\n $Q_c: -\lambda \Delta z_0 + cz_0 = -c[(1 - \lambda)\lambda^{-1}w + v_0],$
\n $\Gamma_c: z_0 = -z_0^+ = 0$, $lz_0 = 0$.
\n \overrightarrow{v} .

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As v_0 is given by a double layer potential W with bounded density, we will have As v_{o} is given by a double layer potential W with bounded density, we will have, $\,$

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\nAs
$$
v_0
$$
 is given by a double layer potential W with bounded density, we will

\n
$$
|-cv_0| \leq C(\varepsilon/r)^2 \quad \text{in} \quad G_\varepsilon,
$$
\n
$$
|-c[(1-\lambda)^2\lambda^{-1}w + v_0]| \leq C \quad \text{in} \quad \Omega_\varepsilon,
$$
\nor S .

\nIf G, Q_ε are contained in balls of radii R and $-$ e.g. $-$ 2\varepsilon, then

\n
$$
Z_\varepsilon = \begin{cases} C\varepsilon^2 \ln(R/r) & \text{for} \quad r > 2\varepsilon, \\ C[\varepsilon^2 \ln(R/2\varepsilon) + 1/2(4\varepsilon^2 - r^2)] & \text{for} \quad r < 2\varepsilon \end{cases}
$$
\nis continuously differentiable in \mathbb{R}^3 and, for suitable constant C , will in the sense of the lemma, possibly with exception of the last condition of $|z_0| = 0 \leq z = C(1 - \lambda) \varepsilon^2 r^{-1} s(x)$,

\nwhere $s(x)$ denotes the scalar product between the outer unit normal and the normed radius vector $\overline{0x}/r$. As Ω is assumed star-shaped, $s(x)$ will

- .

If G, Q_e are contained in balls of radii *R* and $-$ e.g. $-$ 2 ε , then

\n Perturbation of Temperature Fields
\n given by a double layer potential
$$
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 with bounded density, we will have\n $|-cv_0| \leq C(\varepsilon/r)^2$ \n in G_t ,\n $|-c[(1-\lambda)^2 - w + v_0]| \leq C$ \n in G_t ,\n $|-(-c[(1-\lambda)^2 - w + v_0]|) \leq C$ \n in Ω_t ,\n $|-v_0| \leq C\varepsilon^2$ \n on S .\n $\mathcal{S} = \begin{cases} C\varepsilon^2 \ln(R/r) & \text{for } r > 2\varepsilon, \\ C[\varepsilon^2 \ln(R/2\varepsilon) + 1/2(4\varepsilon^2 - r^2)] & \text{for } r < 2\varepsilon \end{cases}$ \n (22)\n $\mathcal{S} = \begin{cases} C\varepsilon^2 \ln(R/2\varepsilon) + 1/2(4\varepsilon^2 - r^2) & \text{for } r < 2\varepsilon, \\ C[\varepsilon^2 \ln(R/2\varepsilon) + 1/2(4\varepsilon^2 - r^2)] & \text{for } r < 2\varepsilon, \\ C[\varepsilon^2 \ln(R/2\varepsilon) + 1/2(4\varepsilon^2 - r^2)] & \text{for } r < 2\varepsilon, \end{cases}$ \n (22)\n

is continuously differentiable in \mathbb{R}^3 and, for suitable constant *C*, will majorize z_0 in the sense of the lemma, possibly with exception of the last condition on Γ_t :

$$
|l z_0| = 0 \leq l z = C(1 - \lambda) \varepsilon^2 r^{-1} s(x),
$$

where $s(x)$ denotes the scalar product between the outer unit normal n at $x \in \Gamma$, and the normed radius vector $0x/r$. As Ω is assumed star-shaped, $s(x)$ will be strictly positive on Γ_{ϵ} , and therefore the lemma can be applied if $\lambda \leq 1$. ase of the lemma, possibly with exception of the last condition on Γ_t :
 $|lz_0| = 0 \leq lz = C(1 - \lambda) e^{2r-1}s(x)$,
 \hat{x}) denotes the scalar product between the outer unit normal *n* at *x*

normed radius vector $0x/r$. As $\$ able in R³ and, for suitable constant, possibly with exception of the last $C(1 - \lambda) e^{2}r^{-1}s(x)$,

calar product between the outer under the outer of $\overline{\alpha}/r$. As Ω is assumed star-shap ore the lemma can be applied if

The case $\lambda > 1$ requires a modification. Let q be the harmonic function defined outside Ω , regular at infinity, and equal to 1 on *I*. There exists a positive constant c_0

so that, on \widetilde{P} , $\partial q/\partial n \leq -c_0$. Then
 $Q(x) = \begin{cases} q(x/\varepsilon) & \text{for } x \in \mathbb{R}^3 \setminus \Omega_{\varepsilon}, \\ 1 & \text{for } x \in \Omega, \end{cases}$

has the properties $\Delta Q = 0$ in Ω_{ϵ} and outside Ω_{ϵ} , $Q > 0$, bounded, and $lQ \ge c_0/\epsilon$.

Therefore $Z' = Z + c_1 \epsilon^2 Q$ with Z from (22) has all majorizing properties claimed by

the lemma, especially,
 $lZ' \geq (4C(1$ has the properties $\Delta Q = 0$ in Ω , and outside Ω , $Q > 0$, bounded, and $lQ \ge c_0/\varepsilon$.
Therefore $Z' = Z + c_1 \varepsilon^2 Q$ with Z from (22) has all majorizing properties claimed by
the lemma, especially.
 $lZ' \ge (4C(1 - \lambda) \varepsilon^2$

$$
lZ' \geq (4C(1 - \lambda) \varepsilon^2/r + c_0 c_1 \varepsilon) s(x) \geq 0
$$

in Ω . Thus we obtain, in either case, If $c_i \geq 4(\lambda - 1)$ C/r_0 , where r_0 is the radius of a ball, centered at 0, which is contained

$$
|z_0| \leq C \varepsilon^2 |\ln \varepsilon|.
$$

In order to raise the degree of approximation, next we have to expand $z_0 = w_1 + \bar{v}_1$ $+ z₁$, where $w₁$ is the solution of an unperturbed problem which compensates the values of v_0 on the outer boundary S. Looking at the problem (20) for z_0 we see that it values of v_0 on the outer boundary *S*. Looking at the problem (20) for z_0 we see that it can be interpreted as perturbation (like (2) in proportion to (1)) of the problem $G: -\Delta w_1 + cw_1 = -c(v_0 + \hat{w})$, $S: w_1 = -v_0$. (

$$
\tilde{d} \tilde{d} \tilde{d} = -\Delta w_1 + c w_1 = -c(v_0 + \hat{w}), \quad S \colon w_1 = -v_0. \tag{24}
$$

values of v_0 on the outer boundary S. Looking at the problem (20) for z_0 we see
can be interpreted as perturbation (like (2) in proportion to (1)) of the proble
 $G: -\Delta w_1 + cw_1 = -c(v_0 + \hat{w})$, $S: w_1 = -v_0$.
Therefore th instead of *h*, $-v_0$ instead of g, and in (18), (19) \bar{v} , w , \hat{w} , v_0 replaced by \bar{v}_1 , w_1 , \hat{w}_1 (cf. (4)), v_1 , *respectively.* For the remainder z_1 in $z_0 = w_1 + \hat{w}_1 + v_1 + z_1$ we now obtain, analogously, again (20), with z_1, v_1, w_1 instead of $z_0, v_0, w_0 = w$. As the estimate (23) for z_0 will also hold, the more, for w_1 (using the lemma with $\lambda^- = \lambda^+ = 1$), the right-hand sides of (18), (19), and (8), rewritten with the new quantities, will be of order $O(\epsilon^2\,|\!\ln\varepsilon|)$ $|z_0| \leq Ce^2 |\ln \varepsilon|$.

In order to raise the degree of approximation, next we have to ex
 $+ z_1$, where w_1 is the solution of an unperturbed problem which

values of v_0 on the outer boundary S. Looking at the proble $\mathbf{e} \text{ right-hand} \ \text{and} \ \mathbf{e} \mathbf{f} \left(\frac{\partial \mathbf{f}}{\partial \mathbf{f}} \mathbf{f} \right) \mathbf{e} \left(\frac{\partial \mathbf{f}}{\partial \mathbf{f}} \mathbf{f} \right)$ In order to raise the degree of a
 $+ z_1$, where w_1 is the solution

values of v_0 on the outer bounda

can be interpreted as perturbati
 $G: -\Delta w_1 + cw_1 = -c($

Therefore the construction just of $h, -v_0$ instead of g \hat{w}) instead

(cf. (4)), v_1 ,

btain, ana-

ate (23) for

right-hand
 $O(\varepsilon^2 |\ln \varepsilon|)$,

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$$
|z_1| \leq C \epsilon^4 (\ln \epsilon)^2
$$

(21)

 $\frac{1}{2}$ **(25**

 (23)

Proposition 1: *The solution u,of the perturbed problem* (17) *admits expansions*

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\nD. GöHDE

\nProposition 1: The solution
$$
u
$$
 of the perturbed problem (17) admits expansions

\n
$$
u = w_0 + \hat{w}_0 + v_0 + z_0
$$

\n
$$
= w_0 + \hat{w}_0 + v_0 + w_1 + \hat{w}_1 + v_1 + z_1
$$

\nwith $w_0 = w$ solution of the unperturbed problem (16), $\hat{w}_0 + v_0$ and $\hat{w}_1 + v_1$ (cf. (19))

corrections; previously in the vicinity of Ω *,, while* w_1 *(cf. (24)) corrects on S; the remain-*. *ders* z_0 , z_1 *are uniformly bounded according to (23), (25).* 210 D. Gönne

Proposition 1: The solution u of the perturbed problem (17) admits expansions
 $u = w_0 + w_0 + v_0 + z_0$
 $= w_0 + w_0 + w_0 + w_1 + w_1 + z_1$

with $w_0 = w$ solution of the unperturbed problem (16), $w_0 + v_0$ and $w_1 + v_1$

I

Finally, it should be remarked that in the special case where Ω_{ϵ} is a ball K_{ϵ} of radius ε , centered at 0, an asymptotic expansion of the solution of the model problem (2) would better be established more directly, based on the Taylor expansion of the unperturbed solution w, at $x=0$, ϵ , centered at 0, an asy
 ϵ , centered at 0, an asy

perturbed solution w, at
 $w = w^0 + w_i^0$

where $w_i^0 = \frac{\partial w}{\partial x_i}$, tal

the equations (3) for the

to the estimation of the
 G_{ϵ} : $-\Delta \tilde{v} = 0$
 K_{ϵ} : $-\lambda \Delta \$

$$
w = w^0 + w_i{}^0 x_i + \frac{1}{2!} w_{ij}^0 x_i x_j + \cdots,
$$

the equations (3) for the difference $v = u - w$ turns out to be reasonable, with regard to the estimation of the remainder,

\n- 6. Spherical inclusions
\n- Finally, it should be remarked that in the special case where
$$
\Omega
$$
 is a ball K_t of radius ε , centered at 0, an asymptotic expansion of the solution of the model problem (2) would better be established more directly, based on the Taylor expansion of the unperturbed solution w , at $x = 0$,
\n- $w = w^0 + w_i^0 x_i + \frac{1}{2!} w_{ij}^0 x_i x_j + \cdots$,
\n- where $w_i^0 = \partial w/\partial x_i$, taken at $x = 0$, and analogously. As a first approximation to the equations (3) for the difference $v = u - w$ turns out to be reasonable, with regard to the estimation of the remainder,
\n- $G_t: -\Delta \tilde{v} = 0$
\n- $K_t: -\lambda \Delta \tilde{v} = (1 - \lambda) h(0) = (2 - 1) \Delta w(0)$
\n- $\partial K_t: \tilde{v}^* - \tilde{v}^+ = 0$, $l\tilde{v} = (\lambda - 1) [w_i^0 x_i/r + w_{ij}^0 x_i x_j/r]$
\n- with $\tilde{v}(\infty) = 0$. Suitable expansion functions will be, for $\tilde{v} = \tilde{v}^+$ in G_t , linear combination of the equation $\tilde{v} = \tilde{v}^+$.
\n

with $\tilde{v}(\infty) = 0$. Suitable expansion functions will be, for $\tilde{v} = \tilde{v}^+$ in G_i , linear combinations of derivatives of the principal solution, and for $\tilde{v} = \tilde{v}$ in the ball K_t , homogeneous polynomials, some of which solve the approximating equation on *K,,* and the other ones are harmonic. For the latter it is best to take those which arise in the the other ones are harmonic. For the latter it is best to take those which arise in the
nominators $H(x)$ of the derivatives just mentioned if they are represented by $H(x)/r^m$.
After inserting these ansatzes into (26) and After inserting these ansatzes into (26) and equating at $r = \varepsilon$ the polynomials, or the exact solution of (26) , in the three-dimensional case: 0
 $=(1 - \lambda) h(0) = (2 - 1) \Delta w(0)$
 $= 0, \quad l\tilde{v} = (\lambda - 1) [w_i^0 x_i/r + w_{ij}^0 x_i x_j]$

ble expansion functions will be, for $\tilde{v} = 0$

of the principal solution, and for $\tilde{v} = 0$

some of which solve the approximation

monic. For 2 $\Delta \tilde{v} = (1 - \lambda) h(0) = (2 - 1) \Delta w(0)$
 $- - \tilde{v}^+ = 0$, $l\tilde{v} = (\lambda - 1) [w_1^0 x_i / \tilde{r} + w_1^0 x_i x_i / r]$

Suitable expansion functions will be, for $\tilde{v} = \tilde{v}^+$ in G_c , linear contributed by the principal solution, and f

genous polynomials, some of which solve the approximating equation of A, and
the other ones are harmonic. For the latter it is best to take those which arise in the
nonintators
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H(x)
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 of the derivatives just mentioned if they are represented by $H(x)/r^m$.
After inserting these ansatzes into (26) and equating at $r = \varepsilon$ the polynomials, or
nominator polynomials, respectively, of the same order, some calculations will provide,
the exact solution of (26), in the three-dimensional case:

$$
\tilde{v} = \begin{cases} \tilde{v}^+ = \frac{1-\lambda}{3+2\lambda} \left[\frac{h(0)}{3} \varepsilon^2 \bar{v}^3 + \frac{3+2\lambda}{2+\lambda} \bar{v}^3 w_i^0 x_i + \bar{v}^5 w_{ij}^0 x_i x_j \right], \\ \tilde{v}^- = \frac{1-\lambda}{3+2\lambda} \left[\frac{h(0)}{3} \varepsilon^2 + \frac{3+2\lambda}{2+\lambda} w_i^0 x_i + w_{ij}^0 x_i x_j + \frac{h(0)}{2\lambda} \varepsilon^2 (1 - \bar{v}^2) \right], \end{cases}
$$

where $\bar{v} = \varepsilon/r$. It should be pointed out that in \tilde{v}^+ the principal solution $1/r$ does not
appear itself. This will hold also for other dimensions; especially for $n = 2$ no loga-
rithmic term will trouble – and this will confirm the fact stated above, in Section 4,
that the expansion procedure will also run in the two-dimensional case.
In order to estimate the degree of approximation to *u* given by $w + \tilde{v}$ it is estab-
lished, for $z = u - (w + \tilde{v})$,
 $G_e: -\Delta z = 0$, $S: z = O(\varepsilon^3)$
 $K_e: -\lambda\Delta z = O(\varepsilon)$, $\partial K_e: z = -z^+ = 0$, $lz = O(\varepsilon^2)$.
(28)

where $\bar{\rho} = \varepsilon/r$. It should be pointed out that in \tilde{v}^+ the principal solution $1/r$ does not appear itself. This will hold also for other dimensions; especially for $n = 2$ no logarithmic term will trouble $-$ and this will confirm the fact stated above, in Section 4, that the expansion procedure will also run in the two-dimensional case. $3 + 2\lambda \begin{bmatrix} 3 & 2 + \lambda \end{bmatrix}$

should be pointed out that in \tilde{v}^+ the principal solution

is will hold also for other dimensions; especially for

trouble – and this will confirm the fact stated about

on procedure wi 2. $\begin{cases} (27) \\ (27) \end{cases}$
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 \therefore (28)
 (28)

at the expansion procedure will also run in the two-dimensional case.
In order to estimate the degree of approximation to u given by $w + \tilde{v}$ it is established, for $z = u - (w + \tilde{v})$,

Here,
$$
c
$$
 is the degree of approximation to u given by $w + v$ it is is the equation:

\n
$$
G_{\epsilon} : -\Delta z = 0, \quad S : z = O(\epsilon^3)
$$

\n
$$
K_{\epsilon} : -\lambda \Delta z = O(\epsilon), \quad \partial K_{\epsilon} : z^- - z^+ = 0, \quad iz = O(\epsilon^2).
$$

\n(28)

 \sim

. .

A majorant function Z which satisfies the conditions of the lemma, except that on S , will be given by A majorant function Z w
will be given by
 $Z = \begin{cases} Z^+ = C_0 \varepsilon \end{cases}$

$$
Z = \begin{cases} 2^+ = C_0 \epsilon^4 / r, \\ Z^- = 1/6C_1 \epsilon (\epsilon^2 - r^2) + C_0 \epsilon^3 \end{cases}
$$

where constants C_0 , C_1 . Merely to be able to apply
the added to \tilde{v} where

• Perturbation

A majorant function Z which satisfies the conditions of

will be given by
 $Z = \begin{cases} Z^+ = C_0 \varepsilon^4 / r, \\ Z^- = 1/6C_1 \varepsilon (\varepsilon^2 - r^2) + C_0 \varepsilon^3 \end{cases}$

with proper constants C_0 , C_1 . Merely to be able to app
 with proper constants C_0 , C_1 . Merely to be able to apply the lemma, formally a function $\varepsilon^3 w_1$ must be added to \tilde{v} , where Perturbation of Temperature Fields

A majorant function *Z* which satisfies the conditions of the lemma, except t

will be given by
 $Z = \begin{cases} Z^+ = C_0 \varepsilon^4 / r, \\ Z^- = 1/6 C_1 \varepsilon (\varepsilon^2 - r^2) + C_0 \varepsilon^3 \end{cases}$

with proper constant majorant f

1 be given
 $Z =$

² *b* proper c
 $\epsilon^3 w_1$ mus
 $G:$ Perturbation of Temperature Fields
 nnt function Z which satisfies the conditions of the lemma, except that of
 $Z = \begin{cases} Z^+ = C_0 \epsilon^4 / r, \\ Z^- = 1/6C_1 \epsilon (\epsilon^2 - r^2) + C_0 \epsilon^3 \end{cases}$

per constants C_0 , C_1 . Merely to be able t Perturbation of Tempe
the conditions of the lemi
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o be able to apply the lem
 $\frac{1 - \lambda}{2 + \lambda} \frac{w_i^0 x_i}{r^3}$.
 S – the other conditions innetion. $Z = \begin{cases} Z^+ = C_0 \epsilon^4 / r, \\ Z^- = 1/6C_1 \epsilon (\epsilon^2 - r^2) + C_0 \epsilon^3 \end{cases}$,
with proper constants C_0 , C_1 . Merely to be able to apply the lemma,
 $\epsilon^3 w_1$ must be added to $\tilde{\nu}$, where
 $G: -\Delta w_1 = 0, \quad S: w_1 = -\frac{1-\lambda}{2+\lambda} \frac{w_i^0 x_i}{r$

$$
G: -\Delta w_1 = 0, \quad S: w_1 = -\frac{1-\lambda}{2+\lambda} \frac{w_i^0 x_i}{r^3}
$$

Then in (28) we will have $z = O(\varepsilon^5)$ on S - the other conditions are not concerned - and Z is now an admissible majorant function.

Proposition 2: *In the case* $n = 3$ *and if the inclusion occupies the ball K_t, an asymp-• totic expansion of the solution of* (2) *is*
 $u(x) = w(x) + \tilde{v}(x) + \varepsilon^3 z_1(x)$ $\frac{1}{2}$ (29) *G*: $-\Delta w_1 = 0$, *S*: $w_1 = -\frac{1}{2 + \lambda} \frac{w_1}{r^3}$.

Then in (28) we will have $z = O(\varepsilon^5)$ on *S* – the other conditions are not concerned –

and *Z* is now an admissible majorant function.

Proposition 2: *In the case n*

 $u(x) = w(x) + \tilde{v}(x) + \varepsilon^3 z_1(x, \varepsilon)$

If higher approximations are, desired, then, of course, the function w_1 compensating $u(x) = w(x) + \tilde{v}(x) + \varepsilon^3 z_1(x, \varepsilon)$ (29)
with \tilde{v} given by (27) and $|z_1(x, \varepsilon)| \leq C$ min $\{\varepsilon/r, 1\}$.
If higher approximations are desired, then, of course, the function w_1 compensating
on the outer boundary S, Then in (28) we will have $z = O(\varepsilon^5)$ on S – the other conditions
and Z is now an admissible majorant function.

Proposition 2: In the case $n = 3$ and if the inclusion occupies

totic expansion of the solution of (2) If higher approximations are desired, then, of course, the function w_1 compensating

on the outer boundary *S*, will be essential and must, further, be expanded itself simi-

larly as w .
 Remark: The limit case λ

 $u(x) = w(x) + \tilde{v}(x)$

with \tilde{v} given by (27) and $|z_1(x)|$

If higher approximations a

on the outer boundary S, will

larly as w.

Remark: The limit case λ

has been treated in [2]; the

(27).

REFERENCES

[1] Acers

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 $\mathbf{v} \in \mathbb{R}^{N \times N}$

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- [1] ACERBI, E., BUTTAZZO, G., and D. PERCIVALE: Thin inclusions in linear elasticity: a variational approach. J. reine angew. Math. 386 (1988), $99-115$.
- [2] GöнDE, D.: Singuläre Störung von Randwertproblemen durch ein kleines Loch im Gebiet. *Z.* Anal. Anw. 4 (1985), 467-477.
- [3] GÖHDE, D.: Störung stationärer Temperaturfelder durch kleine Einschlüsse mit großer Wärmcleitfähigkeit. Wiss. Beiträge Ingenieurhochschule Zwickau 12 (1986) 5, 91-96. **[4] ACERBI, E., BUTTAZZO, G., and D. PERCIVALE: Thin inclusions in linear elasticity: a variational approach.** J. reine angew. Math. 386 (1988), 99 – 115.
 [2] GÖHDE, D.: Singuläre Störung von Randwertproblemen durch e
- области-с узкой щелью. 1. Двумерный случай. Мат. Сборник. 99 (1976), 514—537.
[5] Ильин, А. М.: Краевая задача для эллиптического уравнения второго порядка в
- [4] Ийгтеleitfähigkeit. Wiss. Beiträge Ingenieurhochschule Zwickau 12 (1986) 5, 91-96.
[4] Ильин, А. М.: Краевая задача для эллиптического уравнения второго порядка в области с узкой щелью. 1. Двумерный случай. Мат. Сбор [3] Göнре, D.:

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[4] Ильин, A. M

области с уз

[5] Ильин, A. M

области с уз

265-284.

[6] Ильин, A. M

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SUTTAZZO, G., and D. PERCIVALE: Thin inclusions in linear elasticity

ch. J. reine angew. Math. 386 (1988), 99-115.

Singuliare Storung von Randwertproblemen durch ein kleines Loch i

4 (1985), 467-477.

Egicius statio Göнne, D.: Singuläre Störung von Randwertproblemen' durch ein kleines Loch im Gebiet.
Z. Anal. Anw. 4 (1985), 467-477.
Gönne, D.: Störung stationärer Temperaturfelder durch kleine Einschlüsse mit großer
Wärmeleitfähigkeit (2) GoHDE, D.: Singulare

Z. Anal. Anw. 4 (1985)

[3] GöHDE, D.: Störung :

Wärmeleitfähigkeit. W

[4] Ильин, A. M.: Крае

области с узкой щел

области с узкой щел

265—284.

[6] Ильин, A. M.: Иссле

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itfähigkeit. Wiss. Beiträge Ingenieurhochs

A. M.: Краевая задача для эллиптиче

с узкой щелью. 1. Двумерный случай.

A. M.: Краевая задача для эллиптиче

с узкой щелью. 2. Область
	- [6] Ильин, А. М.: Исследование асимптотики решения эллиптической краевой задачи в области с малым отверстием. Труды Сем. им. Петровского 6 (1981), 57-82.
- [7] Cu1oIiEiiao' H. 13.: 3ajta'ui OJICKrpocTaTnKu a **IIeolL** Iiopoa Ii ^o n cpege.. Ciy'ian TOI.Woro **•**
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- [8] Шишкин, Г. И.: Решение краевой задачи для уравнения второго порядка с малым отверстием в случае круговой симметрии. В сб-е: Дифференцияльные уравнения с малым параметром. Свертловск: Уральский Научный Центр Акад. Наук СССР 1984, стр. $104 - 118$.
- [9] WANKA, G., und J. WANKA: Ein Existenztheorem für eine Klasse von allgemeinen Dirichletschen Randkontaktproblemen bei linearen Differentialgleichungen zweiter Ordnung. Math. Nachr. 110 (1983), 215-229.

Manuskripteingang: 18.05.1988; in revidierter Fassung 26.08.1988

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Added in proof: Solving (8), at first, only for the constant term of Taylor's expansion of the right-hand side, the properties of double layer potential W, with constant density (Gauss' integral – e.g., vanishing outside Ω_{ϵ} !), will allow the constant C in the basic estimate (10) to be replaced by $C \cdot \varepsilon$, so that all powers of ε , in the sequel, can be lifted respectively; especially, (13) is true even with ε^{3k} instead of ε^{2k} , and in (13') ε^k may be replaced by ε^{2k} .