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Error Estimates in Generalized Trigonometric Hölder-Zygmund Norms

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Dedicated to Professor Lothar Berg on the occasion of his 60th birthday

Wir betrachten Hölder-Zygmund-Räume 2π-periodischer Funktionen $f: \mathbb{R} \to \mathbb{C}$, wobei für die r-te Ableitung die k-te Differenz mit Schrittweite h in der L^p- oder C-Norm durch eine Funktion $\omega(h)$ vom Modul-Typ beschränkt ist. Für die Fourier-Summe und ähnliche Approximationsprozesse erhalten wir Fehlerabschätzungen in dazugehörigen Hölder-Zygmund-Normen, falls die Glattheit von f durch weitere Hölder-Zygmund-Bedingungen gegeben ist. Die Konvergenzgeschwindigkeit läßt sich auch im allgemeinen Fall in einfacher Weise angeben. Dies gestattet die Formulierung von Sätzen vom Jackson-Typ für diese Banachräume. Außerdem lassen sich die Konstanten in diesen Abschätzungen explizit berechnen.

Рассматриваются пространства типа Гёльдера-Зигмунда 2π-периодических функций $f: \mathbb{R} \to \mathbb{C}$ таких, что L^p-или C-норма k-той разности с шагом h от r-той производной ограничена по отношению к функции $\omega(h)$ типа модуль-функции. Устанавливаются оценки погрешности для частной суммы Фурье и для других процессов аппроксимации в соответствующих нормах Гёльдера-Зигмунда, если гладкость f задана в терминах других условий Гёльдера-Зигмунда. Как в известных классических результатах, так и здесь скорость сходимости выражается простым образом. Это позволяет формулировать теоремы Джексона для таких банаховых пространств. Более того, все константы в соответствующих оценках вычисляются явным образом.

We consider Hölder-Zygmund spaces of 2π-periodic functions $f: \mathbb{R} \to \mathbb{C}$, where the k-th difference with step-size h of the r-th derivative in the L^p- or C-norm is bounded by a modulus-type function $\omega(h)$. For the Fourier sum and related approximation processes we investigate error estimates in corresponding Hölder-Zygmund norms if the smoothness of f is given by other Hölder-Zygmund conditions. The convergence order for the general case can be formulated in a simple manner. This allows us to state also Jackson-type theorems for such Banach spaces. Moreover, we give explicit values for the constants appearing in these estimates.

1. Introduction

Convergence results in Hölder-Zygmund norms for given trigonometric approximation processes play an important role in the approximate solution of singular integral equations (see [11, 12]). In particular, the convergence order in such Hölder norms for the Fourier sum and the interpolatory polynomial on equidistant nodes has been treated in a series of papers in the last 15 years. In this connection, we mention Leindler [4], Stypiński [14], Krotov [3], Sickel [13] and the authors [7–12]. However, these papers contain in detail only the case of the classical growth condition h^α for the difference term.

The aim of this paper is to prove approximation results in Hölder norms based on general modulus-type functions ω. Note that our approach does not need essential restrictions on ω. In this direction our results generalize theorems obtained in [4], where ω has to fulfil very delicate inequalities. Here we establish an easier and more general convergence condition which allows us also to find the order of convergence.

Since the modulus of continuity based on the first difference does not describe the best approximation completely, we use higher differences. First results in such Hölder-Zygmund norms are contained in [5, 6].

In detail, we consider the Fourier sum and its de la Vallée Poussin means. However, the method of proof can be used in a similar way for other linear approximation methods as well. The norms based on C and L^p, $1 \leqq p < \infty$, can be handled by the same methods. The results are similar up to different operator norms of the Fourier sum. All estimates are formulated with explicit constants depending only on parameters of the spaces. Using de la Vallée Poussin means, we establish a Jackson-type theorem on the best approximation in such Hölder-Zygmund spaces. However, in our general approach it is not possible to prove converse results (see [7] for the classical Hölder spaces). Introducing some separable subspaces, we can formulate corresponding "small-o" convergence results. These estimates are of special interest in view of the nonseparability of the usual Hölder-Zygmund spaces.

2. Definitions and preliminary results

Let X be one of the usual spaces L^p $(1 \leqq p < \infty)$ or C of 2π-periodic complex-valued functions, the norm in X being

$$\|f\|_X = \begin{cases} \|f\|_p = \left\{ \int\limits_0^{2\pi} |f(x)|^p \, dx \right\}^{1/p} & \text{if} \quad X = L^p, \\ \|f\|_\infty = \max\limits_{x \in \mathbb{R}} |f(x)| & \text{if} \quad X = C. \end{cases}$$

In the following, the value $p = \infty$ will always refer to the underlying space $X = C$. For $k \in \mathbb{N}$, we define the set of modulus-type functions Ω^k as the set of all functions ω satisfying the following conditions:

a) $\omega: \mathbb{R}^+ \to \mathbb{R}^+$ (here $\mathbb{R}^+ := [0, \infty)$);

b) ω is monotonically increasing;

c) $\omega(h) \to \omega(0) = 0$ as $h \to 0+$;

d) $\omega(h)\, h^{-k}$ is monotonically decreasing.

Given $\omega \in \Omega^k$, define the seminorm $\|\cdot\|_{X^\omega}$ by

$$\|f\|_{X^\omega} = \sup \{\|\Delta_h^k f\|_X/\omega(h) : h > 0\},$$

where

$$\Delta_h^1 f(x) = f(x+h) - f(x) \qquad \text{and} \qquad \Delta_h^k f = \Delta_h^1 \Delta_h^{k-1} f, \qquad k \geqq 2.$$

In what follows, we will often use the well-known inequalities

$$\|\Delta_h^k f\|_X \leqq \begin{cases} 2^{k-l} \|\Delta_h^l f\|_X & \text{if} \quad f \in X \\ h^{k-l} \|\Delta_h^l f^{(k-l)}\|_X & \text{if} \quad f^{(k-l)} \in X \end{cases} \qquad (k \geqq l),$$

where $f^{(k-l)}$ stands for the (distributional) $(k-l)$-th derivative of f (see e.g. [15]). We denote by $X^{r,\omega}$ the set of all functions f such that $f^{(r)} \in X$ and $\|f^{(r)}\|_{X^\omega} < \infty$. Note that $X^{r,\omega}$, provided with the norm

$$\|f\|_{X^{r,\omega}} = \sum_{j=0}^{r} \|f^{(j)}\|_X + \|f^{(r)}\|_{X^\omega},$$

is a Banach space. Furthermore, let $\tilde{X}^{r,\omega}$ be the subspace of $X^{r,\omega}$ defined by

$$\tilde{X}^{r,\omega} = \{f \in X^{r,\omega} : \|\Delta_h^k f^{(r)}\|_X/\omega(h) \to 0 \text{ as } h \to 0+\}.$$

Notice that this subspace is only of interest in the case where $\omega(h)\, h^{-k} \to \infty$ as $h \to 0+$, because the condition $\|\Delta_h^k f\|_X/h^k \to 0$ as $h \to 0+$ implies that $f = \text{const}$. The spaces $X^{r,\omega}$ generalize the classical Hölder-Zygmund spaces, in which case $\omega(h) = h^\beta$ with $0 \leqq \beta \leqq k$. For $f \in X$ and $n \in \mathbb{N}_0$, we define the n-th Fourier sum of f by

$$S_n f(x) = \sum_{k=-n}^{n} f^\wedge(k)\, e^{ikx}$$

and the n-th de la Vallée Poussin mean of f by

$$\sigma_{n,v} f = \frac{1}{v+1} \sum_{k=n-v}^{n} S_k f, \qquad 0 \leqq v \leqq n.$$

Furthermore, let

$$E_n(f, Y) = \inf\{\|f - p_n\|_Y : p_n \in T_n\},$$

where T_n is the set of all trigonometric polynomials of degree less than or equal to n. To prove estimates in Hölder-Zygmund norms we need the following well-known results.

Lemma 1 [15–17]: *If $f^{(m)} \in X$, then $E_n(f, X) \leqq \frac{\pi}{2} (n+1)^{-m} \|f^{(m)}\|_X$ and, for all $l \in \mathbb{N}$,*

$$E_n(f, X) \leqq C_l (n+1)^{-m} \sup_{0<h<1/(n+1)} \|\Delta_h^l f^{(m)}\|_X.$$

The constant C_l can be estimated by

$$C_l \leqq \begin{cases} 3 & \text{if } l = 1, 2, \quad m \geqq 0, \\ 18 & \text{if } l = 3 \\ 53 & \text{if } l = 4 \\ 2^l & \text{if } l \geqq 5 \end{cases} \quad m \geqq 1.$$

Lemma 2 [2, 17]: *The estimate $\|f - S_n f\|_X \leqq B_p E_n(f, X)$ holds with*

$$B_p = \begin{cases} 2{,}5 + 4\pi^{-2} \ln n & \text{if } 1 \leqq p \leqq \infty, \\ 4(p/(p-1))^{1/p} + 2 & \text{if } 1 < p < 2, \\ 1 & \text{if } p = 2, \\ 4p^{1-1/p} + 2 & \text{if } 2 < p < \infty. \end{cases}$$

Furthermore,

$$\|f - \sigma_{n,v} f\|_X \leqq 36 \sum_{i=0}^{n+v} E_{i+n-v}(f, X)/v^{i+1}. \tag{1}$$

3. Approximation by the Fourier sum

For simplicity of the representation, we here restrict ourselves to the approximation of f by the Fourier sum $S_n f$. By the same method of proof one obtains analogous error estimates for interpolatory polynomials on equidistant nodes (see [9, 12]).

Theorem 1: *Let $r, m \in \mathbb{N}_0$, $k, l, n \in \mathbb{N}$, $\omega_\alpha \in \Omega^l$ and $\omega_\beta \in \Omega^k$. Assume one of the following two conditions to be satisfied:*

(i) $0 \leqq m - r \leqq k - l$ *and $q(h) = h^{m-r}\omega_\alpha(h)/\omega_\beta(h)$ is monotonically increasing*

or

(ii) $k \leqq m - r$ (implying that $q(h)$ is monotonically increasing).

Then for $f \in X^{m,\omega_\alpha}$ we have the estimate

$$\|f - S_nf\|_{X^{r,\omega_\beta}} \leqq DB_pq\big(1/(n+1)\big) \sup_{0<h<1/(n+1)} \{\|\Delta_h{}^l f^{(m)}\|_X/\omega_\alpha(h)\},$$

where

$$D = \begin{cases} C_l 2^{k+1} & \text{if } k \leqq m - r, \\ \max\big(C_l 2^{k+1}, 2C_l + 2^{k-m+r-l-1}\big) & \text{if } m - r \leqq k - l, \end{cases}$$

or, shortly,

$$\|f - S_nf\|_{X^{r,\omega_\beta}} = \begin{cases} O\big(q(1/n)\big) & \text{if } 1 < p < \infty \\ O\big(q(1/n) \log n\big) & \text{if } p = 1 \text{ or } p = \infty \end{cases} \quad (n \to \infty). \tag{2}$$

Remarks: **1.** There is a gap $m - r < k < m - r + l$ in the conditions (i) and (ii) of Theorem 1. To make this clear we quote the following example. Let $m = 1$, $r = 0$, $k = l = 2$, $\omega_\alpha(h) = h$, $\omega_\beta(h) = h^2$. Then

$$X^{r,\omega_\beta} = \{f: \|\Delta_h{}^2 f\|_X = O(h^2),\, h \to 0+\} \subsetneqq X^{m,\omega_\alpha} = \{f: E_n(f, X) = O(n^{-2}),\, n \to \infty\},$$

so that Theorem 1 does not make sense.

2. Assertion (2) remains true, without any restrictions, for all $k, l \in \mathbb{N}$, $m, r \in \mathbb{N}_0$, $m \geqq r + 1$, $\omega_\alpha \in \Omega^l$, $\omega_\beta \in \Omega^{m-r} \cap \Omega^k$ and q monotonically increasing. This is an immediate consequence of Theorem 1 and the inequality

$$\sup \{\|\Delta_h{}^k f^{(r)}\|_X/\omega_\beta(h): h > 0\} \leqq 2^{k-k'} \sup \{\|\Delta_h{}^{k'} f^{(r)}\|_X/\omega_\beta(h): h > 0\}$$

if $k' < k$ and $\omega_\beta \in \Omega^{k'} \subset \Omega^k$. However, in the case $m = r$ we must assume $k \geqq l$, in general, because

$$\{f \in X: \|\Delta_h{}^1 f\|_X = O(h),\, h \to 0+\} \subsetneqq \{f \in X: \|\Delta_h{}^2 f\|_X = O(h),\, h \to 0+\}. \tag{3}$$

3. Note that if the conditions formulated in Theorem 1 or in Remark 2 are fulfilled, then $X^{m,\omega_\alpha} \subseteqq X^{r,\omega_\beta}$.

4. Similarly one can find analogous approximation results for the interpolatory polynomial $L_nf \in T_n$, which interpolates f at equidistant nodes (see [9, 11]).

Before proving the theorem we add some simple facts.

Corollary 1. *Under the assumptions of Theorem 1 (see also Remark 2) we have, for $f \in \tilde{X}^{m,\omega_\alpha}$,*

$$\|f - S_nf\|_{X^{r,\omega_\beta}} = \begin{cases} o\big(q(1/n)\big) & \text{if } 1 < p < \infty \\ o\big(q(1/n) \log n\big) & \text{if } p = 1 \text{ or } p = \infty. \end{cases} \quad (n \to \infty).$$

The following two corollaries illustrate what happens for particular choices of the parameters.

Corollary 2. *Let $\omega_\alpha \in \Omega^2$, $\omega_\beta \in \Omega^2$, $m - r \geqq 2$, $2 \leqq p < \infty$ and $f \in X^{m,\omega_\alpha}$. Then $\|f - S_nf\|_{X^{r,\omega_\beta}} \leqq 120pq\big(1/(n+1)\big) \|f^{(m)}\|_{X^{\omega_\alpha}}$.*

In [5, 6] asymptotic estimates are obtained for $\|f - S_nf\|_{\mathcal{H}^s}$ if $f \in \mathcal{H}^t$, $s < t$, where $\mathcal{H}^s$ is the Banach space of all functions $f \in X$ with finite norm

$$\|f\|_{\mathcal{H}^s} = \sum_{j=0}^{s'} \|f^{(j)}\|_X + \sup_{h>0} h^{s'-s} \|\Delta_h^v f^{(s')}\|_X,$$

with $s' \in \mathbb{N}_0$, $s - 1 \leqq s' < s$, $v = 1$ if $s \notin \mathbb{N}$ and $v = 2$ if $s \in \mathbb{N}$. Except for the case $t \in \mathbb{N}$, $t - 1 < s < t$ (where $k = 1$, $l = 2$, $m = r$), Theorem 1 gives good constants for these estimates. In view of (3) it is clear that Theorem 1 does not include this particular case. However, in the special situation of $\mathcal{H}^s$ spaces we can compute a constant by standard methods (see [12]). It is natural that this constant tends to infinity if $t - s$ goes to zero.

Corollary 3: *Let $f \in \mathcal{H}^t$. Then, for $t > s$,*

$$\|f - S_nf\|_{\mathcal{H}^s} \leqq B_p(n + 1)^{s-t} \|f\|_{\mathcal{H}^t} \cdot \begin{cases} 18 + 12/(1 - 2^{s-t}) & \textit{if } t \in \mathbb{N}, t - 1 < s, \\ 24 & \textit{otherwise}. \end{cases}$$

Proof of Theorem 1: Set $H = \{h: 0 < h \leqq 1/(n + 1)\}$, $G = \{h: h > 1/(n + 1)\}$. In view of the Lemmas 1 and 2 we obtain for $0 \leqq j \leqq r$ that

$$\begin{aligned}\|(f - S_nf)^{(j)}\|_X &\leqq B_pE_n(f^{(j)}, X) \\ &\leqq B_pC_l(n + 1)^{j-m} \omega_\alpha(1/(n + 1)) \sup \{\|\Delta_h^l f^{(m)}\|_X/\omega_\alpha(h): h > 0\},\end{aligned}$$

where we have used that $S_nf^{(j)} = (S_nf)^{(j)}$. Hence, by summation, we find that

$$\sum_{j=0}^{r} \|(f - S_nf)^{(j)}\|_X \leqq 2(n + 1)^{r-m} B_pC_l\omega_\alpha\left(\frac{1}{n + 1}\right) \sup_{h\in H} \frac{\|\Delta_h^l f^{(m)}\|_X}{\omega_\alpha(h)}. \tag{4}$$

To estimate the second term of the norm we split the supremum into two parts. First write

$$A := \sup_{h\in H} \{\|\Delta_h^k (f - S_nf)^{(r)}\|_X/\omega_\beta(h)\} \leqq B_p \sup_{h\in H} \{E_n(\Delta_h^k f^{(r)}, X)/\omega_\beta(h)\}.$$

If condition (i) is fulfilled, then

$$\begin{aligned}2E_n(\Delta_h^k f^{(r)}, X) &\leqq \pi \|\Delta_h^k f^{(r)}\|_X \leqq \pi \|\Delta_h^{k-m+r} f^{(m)}\|_X h^{m-r} \\ &\leqq \pi 2^{k-m+r-l} h^{m-r} \|\Delta_h^l f^{(m)}\|_X,\end{aligned} \tag{5}$$

which gives that

$$A \leqq B_p\pi 2^{k-m+r-l-1} \sup_{h\in H} q(h) \sup_{h\in H} \{\|\Delta_h^l f^{(m)}\|_X/\omega_\alpha(h)\}. \tag{6}$$

Let now $k \leqq m - r$. Using Lemma 1 once more and taking into account that $\Delta_\delta^l\Delta_h^k f = \Delta_h^k\Delta_\delta^l f$, we get

$$\begin{aligned}E_n(\Delta_h^k f^{(r)}, X) &\leqq C_l \sup_{\delta\in H} \|\Delta_h^k\Delta_\delta^l f^{(m-k)}\|_X (n + 1)^{r+k-m} \\ &\leqq C_l(n + 1)^{r+k-m} h^k \sup_{\delta\in H} \{\|\Delta_\delta^l f^{(m)}\|_X/\omega_\alpha(\delta)\} \sup_{\delta\in H} \omega_\alpha(h).\end{aligned} \tag{7}$$

Hence,

$$A \leqq B_pC_l(n + 1)^{r-m} \omega_\beta\left(\frac{1}{n + 1}\right)^{-1} \omega_\alpha\left(\frac{1}{n + 1}\right) \sup_{\delta\in H} \{\|\Delta_\delta^l f^{(m)}\|_X/\omega_\alpha(\delta)\}.$$

Combining this estimate and (6) we obtain

$$A \leqq B_pC_lq\left(\frac{1}{n + 1}\right) \sup_{h\in H} \frac{\|\Delta_h^l f^{(m)}\|_X}{\omega_\alpha(h)} \cdot \begin{cases} \pi 2^{k-m+r-l-1} & \text{if } m - r \leqq k - l, \\ C_l & \text{if } k \leqq m - r. \end{cases}$$

The case $h \in G$ can be handled in a similar way. This gives

$$\sup_{h\in G} \{\|\Delta_h^k(f - S_nf)^{(r)}\|_X/\omega_\beta(h)\} \leqq B_p 2^k \sup_{h\in G} \{E_n(f^{(r)}, X)/\omega_\beta(h)\}$$

$$\leqq B_p 2^k C_l (n+1)^{r-m} \,\omega_\beta\left(\frac{1}{n+1}\right)^{-1} \omega_\alpha\left(\frac{1}{n+1}\right) \sup_{h\in H} \{\|\Delta_h^l f^{(m)}\|_X/\omega_\alpha(h)\}. \quad (8)$$

Putting together (4) with the last two estimates we arrive at the assertion ∎

4. Best approximation

By a well-known result of BARI and STECKIN [1], the properties

$$E_n(f, X) = O(\omega(1/n)) \ (n \to \infty) \quad \text{and} \quad \|\Delta_h^k f\|_X = O(\omega(h)) \ (h \to 0+)$$

are equivalent if and only if $\omega \in \Omega^k$ satisfies the estimate

$$1 < \varliminf_{h\to 0+} \omega(ch)/\omega(h) \leqq \varlimsup_{h\to 0+} \omega(ch)/\omega(h) < c^k$$

for some c. By this reason we do not try to characterize the best approximation in X^{r,ω_β} for the general choice of ω_α and ω_β. However, with the help of the de la Vallée Poussin means of f we can remove the $\log n$ term in the estimates. Then we obtain an order of convergence which is best possible at least for ω_α and ω_β from some subclasses of Ω^l and Ω^k containing $\omega_\alpha(h) = h^\alpha$, $\omega_\beta(h) = h^\beta$, $0 < \beta < k$, $0 < \alpha < l$.

Lemma: *Under the assumptions of Theorem 1 we have*

$$\|f - \sigma_{n,v}f\|_{X^{r,\omega_\beta}} \leqq 36D \sum_{i=0}^{n+v} \frac{1}{v+i+1}\, q\left(\frac{1}{i+n-v+1}\right) \sup_{0<h<1/(n-v+1)} \frac{\|\Delta_h^l f^{(m)}\|_X}{\omega_\alpha(h)}.$$

For $q(h)\,h^{-\gamma} < C$, a simple calculation yields that

$$\|f - \sigma_{n,v}f\|_{X^{r,\omega_\beta}} = \begin{cases} O\left(n^{-\gamma} \log \dfrac{n}{v+1}\right) & \text{if } 0 \leq \gamma < 1 \\ O\left(\dfrac{1}{n} \log \dfrac{n^2}{(v+1)\,(n-v+1)}\right) & \text{if } \gamma = 1 \\ O\left(\dfrac{1}{n} (n-v+1)^{1-\gamma} \log \dfrac{n}{v+1}\right) & \text{if } \gamma > 1 \end{cases} \quad (n \to \infty).$$

Proof: By (1),

$$\|f - \sigma_{n,v}f\|_{X^{r,\omega_\beta}} \leqq 36 \sum_{i=0}^{n+v} \frac{1}{v+i+1} \left(\sum_{j=0}^{r} E_{i+n-v}(f^{(j)}, X) + \sup_{h>0} \frac{E_{i+n-v}(\Delta_h^k f^{(r)}, X)}{\omega_\beta(h)}\right).$$

Estimating the terms in the parantheses in the same way as in (4), (5), (7) and (8) gives the assertion immediately ∎

To have the best possible order of convergence let $v = [n/2]$. Using that $q(2/n) \leqq 2^{r-m+k} q(1/n)$, we obtain the following theorem.

Theorem 2: *Assume the conditions of Theorem 1 are fulfilled. Then*

$$E_n(f, X^{r,\omega_\beta}) \leqq 108 \cdot 2^{r-m+k} D q(1/n) \|f^{(m)}\|_{X^{\omega_\alpha}}.$$

In particular, if $f \in \tilde{X}^{m,\omega_\alpha}$, then

$$E_n(f, X^{r,\omega_\beta}) = o(q(1/n)), \qquad n \to \infty.$$

From the last result it follows immediately that the spaces $\tilde{X}^{m,\omega_\alpha}$ are separable for any choice of $\omega_\alpha \in \Omega^l$.

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