On the Singular Behaviour of Fluid in a Vertical Wedge

E. MIERSEMANN

Dedicated to the memory of Johannes Maul¹)

Es werden Lösungen der Gleichung für Kapillarflächen über Gebieten mit Ecken betrachtet. Dabei wird angenommen, daß die Ecke durch zwei Kurven begrenzt wird, die einen inneren Winkel 2α haben mit $0 < 2\alpha < \pi$ und $\alpha + \gamma < \pi/2$, wobei γ , $0 < \gamma < \pi/2$, der Kontaktwinkel zwischen der Fläche und der Containerwand ist. Es wird eine asymptotische Formel für Lösungen in der Umgebung der Ecke angegeben.

Исследуются решения уравнения капиллярности в областях с угловыми точками. Предполагается, что угловая точка окаймляется двумя кривыми имеющими внутренний угол 2 α такой, что $\sqrt{0} < 2\alpha < \pi$ и $\dot{\alpha} + \gamma < \pi/2$, где γ , $0 < \gamma < \pi/2$, является углом между контактной поверхностью и границей области. Доказывается асимптотическая формула для решений в окрестности угловой точки.

Solutions of capillary surface equation over domains with corners are considered. It is assumed. that the corner is bounded by curves which make an interior angle 2α with $0 < 2\alpha < \pi$ and $\alpha + \gamma < \pi/2$, where $\gamma, 0 < \gamma < \pi/2$ is the contact angle between the surface and the container wall. An asymptotic formula for the solutions near the corner is given.

1. Introduction. We consider the non-parametric capillary problem in the presence of gravity. One seeks a sufface $S: u = u(x)$, $x = (x_1, x_2)$, defined over a bounded base domain $\Omega \subset \mathbb{R}^2$, such that S meets vertical cylinder walls over the boundary $\partial\Omega$ in a prescribed constant angle γ , $0 \leq \gamma \leq \pi/2$. The problem when a tube of cross-section Ω is placed into an infinite reservoir leads to the equations (see FINN [3])

 $div T u = xu$ in Ω , (1.1)

 $v \cdot Tu = \cos y$ on the smooth parts of $\partial \Omega$,

 (1.2)

where $Tu = Du/\sqrt{1 + |Du|^2}$, $\varkappa = \text{const} > 0$ and v is the exterior unit normal on $\partial\Omega$. By Du we denote the gradient of u.

Let the origin $x = 0$ be a corner of Ω with the interior angle 2α satisfying $0 < 2\alpha$ $\leq \pi$. We assume that the corner is bounded by two sufficiently regular curves and that each curve makes an angle, α with the positive x_1 -axis, see Figure 1.

In fact, it is enough that the curves belong to $C^{2,\mu}$ for some $\mu \in (0, 1)$. When the curves are lines near the origin, then CONCUS and FINN [2] have shown that u is unbounded at the origin if and only if $\alpha + \gamma < \pi/2$ holds. In this paper we are interested in this singular case. Thus, we suppose that $\alpha + \gamma < \pi/2$ in what follows. Let r, θ be polar coordinates centred at $x = 0$, set $k = \sin \alpha / \cos \gamma$ and define

$$
u_0(r,\theta) = (\cos\theta - \sqrt{k^2 - \sin^2\theta})/x}
$$
 (1.3)

 $'$) See foot-note on p. 433.,

30

Then, using a method of Concus and Finn, we have shown in [4] that'

$$
u(x) = u_0(r, \theta) + O(r^{\epsilon})
$$
\n(1.4)

ERSEMANN

method of Concus and Finn, we have shown in [4] that
 $= u_0(r, \theta) + O(r^{\epsilon})$ (1.4)

corner for an $\varepsilon > 0$ when the corner is bounded by lines near the origin.

tan $\tilde{\alpha} \cdot x_1$ is the upper curve and $x_2 = -\tan \alpha \cdot x$ holds near the corner for an $\epsilon > 0$ when the corner is bounded by lines near the origin. That is, $x_2 = \tan \alpha \cdot x_1$ is the upper curve and $x_2 = -\tan \alpha \cdot x_1$ the lower one which define the corner. The leading singular term $u_0(r, \theta)$ was discovered by Concus and FINN ([2] and [3, Theorem 5.5]). The expansion (1.4) shows that for fixed θ the function $u(r)$ is asymptotically a hyperbola. For $\theta = \pm \alpha$ one obtains the curves of contact on the container wall, compare FINN [3, Note 4, p. 131] with respect to an. **experiment performed** by **TAYLOR [5].**

The aim of this note is to obtain an expansion like (1.4) in the case when the corner is bounded by curves instead by lines. Under the stronger assumption $0 < y < \pi/2$ we obtain by the same method that

$$
u(x) = u_0(s, \theta) + q(\theta) + O(s^{\epsilon})
$$

holds. Here *s*, θ denote curvilinear coordinates and q is the (unique) solution of a twopoint boundary value problem fora regular second order ordinary differential equation, see the next sections.

Acknowledgement. I would like to thank Professor Robert Finn for initiating my interest in capillary problems and for useful discussions.

2. Curvilinear coordinates. We use curvilinear coordinates $x_1 = x_1(s, \theta)$ and $x_2 = x_2(s, \theta)$ ($-\alpha \le \theta \le \alpha$; $0 \le s \le s_0$, s_0 small enough). Here $\theta = \text{const}$ yield the **• •** *• • • • • • <i>• • • <i>• • • <i>• • • • • • • • • • • <i>• • • • • • • • • • • <i>• •* curves parsing through the origin and *s* denotes the are length on these curves méasured from the origin, see Figure 1. More precisely, let **x**_{**1**} $(x, y) = x_0(x, y) + x_1(y) + x_2(y)$

re *s*, θ denote curvilinear coordinates and q is the (unique) solution of a two-

the next sections.
 i kledgement. I would like to thank Professor Robert Finn for initiating holds. Here s, θ denote curvilinear coordinates and q is the (upoint boundary value problem for a regular second order or
tion, see the next sections.
Acknowledgement. I would like to thank Professor Robe
interest in coordinates. We use curvilinear coordine $-\alpha \leq \theta \leq \alpha$; $0 \leq s \leq s_0$, s_0 small enough). H
through the origin and s denotes the arc leng
origin, see Figure 1. More precisely, let
 $f_1(x_1) = \tan \alpha \cdot x_1 + a_1 x_1^2 + O(x_1^3)$
u *x* Finn for i
 $x_1 =$
 $\sec \theta = \cos \theta$
 $\sin \theta = \sin \theta$
 $\sin \theta = \sin \theta$
 $\sin \theta = \sin \theta$ $u(x) = u_0(s, \theta) + q(\theta) + O(s^*)$

holds. Here s, θ denote curvilinear coordinates and q is the (unique) solution of a

point boundary value problem for a regular second order ordinary differential c
 Acknowledgement. I woul $x_2 = x_2(s, \theta)$ $(-\alpha \leq \theta \leq \alpha; 0 \leq s \leq s_0, s_0$ small enough). Here $\theta = c$
curves passing through the origin and s denotes the arc length on the
sured from the origin, see Figure 1. More precisely, let
 $x_2 = f_1(x_1) = \tan \alpha \cdot$

$$
x_2 = f_1(x_1) = \tan \alpha \cdot x_1 + a_1 x_1^2 + O(x_1^3) \tag{2.1}
$$

$$
x_2 = f_2(x_1) = -\tan \alpha \cdot x_1 + a_2 x_1^2 + O(x_1^3) \tag{2.2}
$$

the lower one which define the corner. We set

using through the origin and s denotes the arc length on these curves mean, the origin, see Figure 1. More precisely, let

\n
$$
x_2 = f_1(x_1) = \tan \alpha \cdot x_1 + a_1 x_1^2 + O(x_1^3)
$$
\nper curve and

\n
$$
x_2 = f_2(x_1) = -\tan \alpha \cdot x_1 + a_2 x_1^2 + O(x_1^3)
$$
\none which define the corner. We set

\n
$$
x_2(x_1, \theta) = \frac{1}{2} \left(1 + \frac{\tan \theta}{\tan \alpha} \right) f_1(x_1) + \frac{1}{2} \left(1 - \frac{\tan \theta}{\tan \alpha} \right) f_2(x_1)
$$
\nduce the arc length instead of x_1 through

\n
$$
s = \int \sqrt{1 + x_{2,x_1}^2(\xi, \theta)} \, d\xi
$$
\nwhere x_1 is the point x_1 and x_2 is the point x_1 and x_2 is the point x_1 and x_2 .

\nSince x_1 and x_2 are the point x_1

b

and intro
 $\overline{\mathcal{L}}$

and introduce the arc length instead of x_1 through
 $s = \int \sqrt[2n]{1 + x_{2,x_1}^2(\xi, \theta)} d\xi$

On the Singular Behavior of Fluid
\nwhich defines
$$
x_1 = x_1(s, \theta)
$$
 and $x_2 = x_2(s, \theta)$, where we denote $x_2(x_1(s, \theta), \theta)$ by
\n $x_2(s, \theta)$ again. We find the coefficients g_1, g_2 in the expansions
\n $x_1(s, \theta) = s \cos \theta + s^2 g_1(\theta) + O(s^3)$ (2.4)
\n $x_2(s, \theta) = s \sin \theta + s^2 g_2(\theta) + O(s^3)$ (2.5)
\nas follows. Inserting (2.1) and (2.2) into (2.3) and then (2.4) for x_1 , we obtain $x_2(s, \theta)$.
\nComparison of coefficients with (2.5) yields
\n $g_2(\theta) = g_1(\theta)$ tan $\theta + 2^{-1} \cos^2 \theta \cdot G(a_1, a_2, \alpha, \theta)$, (2.6)
\nwhere G is defined by
\n $G(a_1, a_2, \alpha, \theta) = (1 + \tan \theta/\tan \alpha) a_1 + (1 - \tan \theta/\tan \alpha) a_2$ (2.7)
\nFrom (2.4), (2.5) and $x_2 \cdot x_3 = 1$ it follows that
\n $g_1(\theta) = -g_2(\theta)$ tan θ (2.8)
\nholds. Combining this equation with (2.6), we obtain
\n $g_1(\theta) = 2^{-1} \cos^4 \theta \cdot G(a_1, a_2, \alpha, \theta)$ (2.9)
\nand
\n $g_2(\theta) = 2^{-1} \cos^4 \theta \cdot G(a_1, a_2, \alpha, \theta)$ (2.10)
\nSet $x = (x_1, x_2)$ and $D = \det \begin{pmatrix} x_{1, s} & x_{2, s} \\ x_{3, s} & x_{3, s} \end{pmatrix}$. From (2.4), (2.5) we see that
\n $x_3 \cdot x_3 = s^2 + 2e(\theta)s^3 + O(s^4)$,
\n $x_3 \cdot x_3 = f(\theta) s^2 + O(s^3)$,
\n $D = s + e(\theta)s^2 + O(s^3)$,
\n $D = s + e(\theta)s^2 + O(s^3)$,

We mention that
$$
e = f'
$$
 holds because (2.8). Finally, we obtain from (2.9), (2.10) for
 f and e
\n
$$
f = (1/2) \cos^3 \theta \cdot G(a_1, a_2, \alpha, \theta),
$$
\n
$$
e = -(3/2) \sin \theta \cos^2 \theta \cdot \hat{G}(a_1, a_2, \alpha, \theta) + (\cos \theta/2 \tan \alpha) (a_1 - a_2),
$$
\n(2.12)

where G is defined by (2.7) .

3. The asymptotic formula. For $0 < \varrho < \varrho_0$, ϱ_0 small enough, we set $\Omega_{\varrho} = \Omega \cap \widetilde{B_{\varrho}}$, $\sum_{e} = (\partial \Omega \cap B_e) \setminus \{0\}$ and $\Gamma_{\varrho} = \Omega \cap \partial B_{\varrho}$. Here B_{ϱ} denotes a disc with radius ϱ and the centre at the origin. The proof of the asymptotic formulas (1.4) and (1.5) is based on a method of Concus and Finn, see [3, proof of Theorem 5.5], which relies on the following comparison principle. We give here a special version which we need in our case. For the constant $x>0$ let $Nv = \text{div } Tv - xv$.

Theorem 3.1 (Concus and FINN [1]): Suppose that $Nw \geq Nv$ in Ω_p , $v \geq w$ on Γ_e and $\nu \cdot Tv \geq \nu \cdot Tw$ on Σ_e hold. Then $v \geq w$ in Ω_e .

470 E. MIERSEMANN
\nWith the abbreviation
\n
$$
R = D^2 + w_0^2 + x_0 \cdot x_0 w_s^2 - 2x_s \cdot x_0 w_s w_0
$$
\nwe have in curvilinear coordinates s, θ
\n
$$
= \frac{1}{\sqrt{2\pi}} \left[\int x_0 \cdot x_0 w_s - x_s \cdot x_0 w_s \right] + \int w_0 \cdot x_s.
$$

E. MIERSEMANN
\nthe abbreviation
\n
$$
R = D^2 + w_0^2 + x_0 \cdot x_0 w_s^2 - 2x_s \cdot x_0 w_s w_0
$$
\nin curvilinear coordinates s, θ
\ndiv $Tw = \frac{1}{|D|} \left[\left(\frac{x_0 \cdot x_0 w_s - x_s \cdot x_0 w_0}{\sqrt{R}} \right)_s + \left(\frac{w_0 - x_s \cdot x_0 w_s}{\sqrt{R}} \right)_0 \right]$
\n $v \cdot Tw = \begin{cases} (w_0 - x_s \cdot x_0 w_s) / \sqrt{R} & \text{on the upper curve } (\theta = \infty) \\ (-w_0 + x_s' \cdot x_0 w_s) / \sqrt{R} & \text{on the lower curve } (\theta = -\infty) \end{cases}$

and

With the abbreviation
\n
$$
R = D^2 + w_0^2 + x_0 : x_0w_3^2 - 2x_s \cdot x_0w_sw_0
$$
\nwe have in curvilinear coordinates s, θ
\n
$$
\text{div } Tw = \frac{1}{|D|} \left[\left(\frac{x_0 \cdot x_0w_s - x_s \cdot x_0w_s}{\sqrt{R}} \right)_s + \left(\frac{w_0 - x_s \cdot x_0w_s}{\sqrt{R}} \right)_\theta \right]
$$
\nand
\n
$$
v \cdot Tw = \left\{ \frac{(w_0 - x_s \cdot x_0w_s)}{(-w_0 + x_s' \cdot x_0w_s)} \right| \sqrt{\frac{R}{R}} \quad \text{on the upper curve } (\theta = \alpha),
$$
\nFor
\n
$$
h(\theta) = (\cos \theta - \sqrt{k^2 - \sin^2 \theta})/kx,
$$
\nwhere $k = \sin \alpha/\cos \gamma$, let $w = s^{-1}h(\theta) + q(\theta) - As^{\lambda}$ for a function $q \in C^2[-\alpha, \alpha]$,
\na constant $A \neq 0$ and a constant $\lambda > 0$. We define
\n
$$
Lq = (A_2(\theta) q')' + A_1(\theta) q',
$$
\nwhere $A_2 = h^2(h^2 + h^2)^{-3/2}$ and $A_1 = 2hh'(h^2 + h^2)^{-3/2}$, and set
\n
$$
F(\theta) = -2(h^2 + h^2)^{-3/2} (eh^3 + 2ehh^2 + fh^3)
$$
\n(3.2)

For

•

where $k = \sin \alpha / \cos \gamma$, let $w = s^{-1}h(0) + q(0) - As^{\lambda}$ for a function $q \in C^{2}[-\alpha, \alpha]$, a constant $A \neq 0$ and a constant $\lambda > 0$. We define *•*

$$
Lq = (A_2(0) q')' + A_1(0) q', \qquad (3.2)
$$

where $A_2 = h^2(h^2 + h^2)^{-3/2}$ and $A_1 = 2hh'(h^2 + h^2)^{-3/2}$, and set

div
$$
Tw = \frac{1}{|D|} \left[\left(\frac{x_0 + x_0w_s - x_s \cdot x_0w_o}{\sqrt{R}} \right)_s + \left(\frac{w_0 - x_s \cdot x_0w_o}{\sqrt{R}} \right)_0 \right]
$$

\nand
\n $v \cdot Tw = \left\{ \frac{(w_0 - x_s \cdot x_0w_s)/\sqrt{R}}{(-w_0 + x_s' \cdot x_0w_s)/\sqrt{R}} \text{ on the upper curve } (\theta = \infty). \right.$
\nFor
\n $h(\theta) = (\cos \theta - \sqrt{k^2 - \sin^2 \theta})/kx$, (3.1)
\nwhere $k = \sin \alpha/\cos \gamma$, let $w = s^{-1}h(\theta) + q(\theta) - As^{\lambda}$ for a function $q \in C^2[-\infty, \infty]$,
\na constant $A \neq 0$ and a constant $\lambda > 0$. We define
\n $Lq = (A_2(\theta) q')' + A_1(\theta) q'$, (3.2)
\nwhere $A_2 = h^2(h^2 + h'^2)^{-3/2}$ and $A_1 = 2hh'(h^2 + h'^2)^{-3/2}$, and set
\n $F(\theta) = -2(h^2 + h'^2)^{-3/2} (eh^3 + 2ehh'^2 + fh'^3)$
\n $+ [(h^2 + h'^2)^{-3/2} (fh^3 - eh'h^2)]' + eh(h^2 + h'^2)^{-1/2}$
\n $-e[h'(h^2 + h^2)^{-1/2}]'$ (3.3)
\nwith e and f from (2.12) and (2.11). After some calculation, we obtain that
\ndiv $Tw = xs^{-1}h(\theta) + Lq + F + O(A\lambda s^2) + O(s)$
\nholds, provided that $\lambda \leq 1$ and $|A| \lambda \leq K_0$ are satisfied for a constant $K_0 > 0$.
\nHence, since $s^{-1}h(\theta) = w - q(\theta) + As^{\lambda}$, it follows
\ndiv $Tw = xw + xAs^{\lambda} + O(A\lambda s^{\lambda}) + O(s)$
\nif q is a solution of
\n $Lq - xq + F = 0$ on $(-\alpha, \alpha)$.

with *e* and *f* from (2.12) and (2.11). After some calculation, we obtain that

$$
\operatorname{div} T w = \kappa s^{-1} h(\theta) + Lq + F + O(A\lambda s^2) + O(s)
$$

holds, provided that $\lambda \le 1$ and $|A| \lambda \le K_0$ are satisfied for a constant $K_0 > 0$.
Hence, since $s^{-1}h(\theta) = w - q(\theta) + As^{\lambda}$, it follows (3.3)

some calculation, we obtain that
 $O(A^{2s^2}) + O(s)$
 \vdots K_0 are satisfied for a constant $K_0 > 0$:

follows
 $+ O(s)$
 \vdots (3.4)
 (3.5)
 $)^{-3/2} s^{1+\lambda} + O(s^2)$
 (3.6)
 $)^{-3/2} s^{1+\lambda} + O(s^2)$
 (3.7) $T w = \kappa s^{-1} h(\theta) + Lq + F + O(A\lambda s^{\lambda}) + O(s)$

ided that $\lambda \leq 1$ and $|A| \lambda \leq K_0$ are satisfied for a covertible $s^{-1} h(\theta) = w - q(\theta) + As^{\lambda}$, it follows
 $T w = \kappa w + \kappa As^{\lambda} + O(A\lambda s^{\lambda}) + O(s)$
 $\kappa w = \kappa w + \kappa As^{\lambda} + O(A\lambda s^{\lambda}) + O(s)$

some cal

$$
\operatorname{div} Tw = xw + xAs^2 + O(AAs^2) + O(s) \qquad \qquad (3.4)
$$

since
$$
s^{-1}h(\theta) = w - q(\theta) + As^{\lambda}
$$
, it follows
\ndiv $Tw = \kappa w + \kappa As^{\lambda} + O(A\lambda s^{\lambda}) + O(s)$ (3.4)
\nsolution of
\n $Lq - \kappa q + F = 0$ on $(-\alpha, \alpha)$.
\n(3.5).
\nHere some calculation, one finds
\n $\nu \cdot Tw = \cos \gamma - A\lambda h h'(h^2 + h'^2)^{-3/2} s^{1+\lambda} + O(s^2)$ (3.6)
\npper curve $(\theta = \alpha)$ and
\n $\nu \cdot Tw = \cos \gamma + A\lambda h h'(h^2 + h'^2)^{-3/2} s^{1+\lambda} + O(s^2)$ (3.7)
\nower one $(\theta = -\alpha)$, provided that q satisfies the boundary conditions
\n $q' + fh - eh' = 0$ for $\theta = -\alpha$ and $\theta = \alpha$.
\ne, we assume also here that $|A| \lambda \le K_0$ and $\lambda \le 1$ hold.
\nna 3.1: There exists a unique solution to the two-point boundary value problem

Again, after some calculation, one finds

$$
\nu \cdot Tw = \cos \gamma - A\lambda h h'(h^2 + h'^2)^{-3/2} s^{1+\lambda} + O(s^2) \tag{3.6}
$$

on the upper curve $(\theta = \alpha)$ and

$$
\nu \cdot Tw = \cos \gamma + A \lambda h h'(h^2 + h'^2)^{-3/2} s^{1+\lambda} + O(s^2) \tag{3.7}
$$

on the lower one $(\theta = -\alpha)$, provided that q satisfies the boundary conditions $\frac{1}{2}$ in the loop

As above, we assume also here that $|A| \lambda \leq K_0$ and $\lambda \leq 1$ hold.

1'

Lemma 3.1 There exists a unique solution to the two-point boundary value problem

 $Lq - \kappa q + F = 0$

Again, after some calculatic
 $\gamma \cdot Tw = \cos \gamma - A$

on the upper curve $(\theta = \alpha)$
 $\gamma \cdot Tw = \cos \gamma + A$

on the lower one $(\theta = -\alpha)$,
 $q' + fh - eh' = 0$

As above, we assume also h

Lemma 3.1: 'There exists

(3.5), (3.8).

Pr Proof: It is enough to show that the homogeneous problem has only the solution $q = 0$. Let q_0 be a solution to the homogeneous problem associated to (3.5), (3.8) and u a solution to (1.1) , (1.2) , when the origin is a corner which is bounded by lines and each line makes an angle α with the positive x_1 -axis. By the same argument as in $[4]$ one finds $u = r^{-1}h(\theta) + q_0(\theta) + O(r^{\epsilon})$. Here *r*, θ denote polar coordinetes. Thus, since (1.4) holds too, it follows that $q_0(\theta) = 0$ on $[-\alpha, \alpha]$

Theorem 3.2: Let u be a solution to (1.1), (1.2) and suppose that $0 < 2\alpha < \pi$, $0 < y < \pi/2$ and $\alpha + \gamma < \pi/2$. Then, for an $\varepsilon > 0$, $u = u_0(s, \theta) + q(\theta) + O(s^{\epsilon})$ near *the corner, where q is the solution to the boundary value problem (3.5), (3.8) and* u_0 *is defined through (1.3).*

Proof: Since $0 < y < \pi/2$ holds it follows from the definition (3.1) of *h* that *h*egined *hrough* (1.3).
 Proof: Since $0 < y < \pi/2$ holds it follows from the definition (3.1) of h that $h \in C^{\infty}[-\alpha, \alpha]$ and $h'(\alpha) = -h'(-\alpha) > 0$. Let, $w = s^{-1}h(\theta) + q(\theta) - As^{\lambda}$, where the constant A is positive, then one $h \in C^{\infty}[-\alpha, \alpha]$ and $h'(\alpha) = -h'(-\alpha) > 0$. Let $w = s^{-1}h(\theta) + q(\theta) - As^{\lambda}$, where the constant *A* is positive, then one obtains from (3.4), (3.6) and (3.7) by the same argument as in [4, proof of the theorem] that there are positive constants A , ρ and λ not depending on the particular solution *u* considered such that div $Tw - xw \geq 0$ in Ω_e , $w \leq u$ on Γ_e and $v \cdot Tw \leq \cos \gamma$ on Σ_e hold. Then, Theorem 3.1 implies. $u \ge u_0(s, \theta) + q(\theta) - As^{\lambda}$ in Ω_e . By the same argument it follows $u \le u_0(s, \theta) + q(\theta)$ $+A s¹$ for possibly other positive constants A, ϱ and λ . Here the comparison function $w = s^{-1}h(\theta) + q(\theta) + As^{\lambda}, A > 0$, is used. Thus, the theorem is proved \blacksquare net through (1.9).

Proof: Since $0 < \gamma < \pi/2$ holds it follows from the definition (3.1) of h that $h \in C^\infty[-\alpha, \alpha]$ and $h'(\alpha) = -h'(-\alpha) > 0$. Let $w = s^{-1}h(\theta) + q(\theta) - As^{\lambda}$, where the constant A is positive, then one obtains fr **Proof:** Since $0 < \gamma < \pi/2$ holds it follows from $h \in C^{\infty}[-\alpha, \alpha]$ and $h'(\alpha) = -h'(-\alpha) > 0$. Let $w =$
the constant A is positive, then one obtains from $(3.4$
argument as in [4, proof of the theorem] that there are
not dep the constant A is positive, then one obtains from (3.4), (3.6) and
argument as in [4, proof of the theorem] that there are positive coi
not depending on the particular solution u considered such that
in Ω_e , $w \le u$ on Lepending on the particular solution u consi
 Ω_e , $w \leq u$ on Γ_e and $v \cdot Tw \leq \cos \gamma$ on Σ_e $\geq u_0(s, \theta) + q(\theta) - As^{\lambda}$ in Ω_e . By the same argu
 As^{λ} for possibly other positive constants A, θ an
 $= s^{-1}h(\theta) + q(\$

• Note added in proof. More recently, the correction term $q(\theta)$ was being calculated numerically by Dr. Berndt and Dr. Janassary from the University of Leipzig. Vote added in proof. More recently, the correction term $q(\theta)$ was being cancrically by Dr. Berndt and Dr. Janassary from the University of Leipz

HERENCES

CONCUS, P., and R. FINN: On capillary free surfaces in the absen

- [1] Concus, P., and R. FINN: On capillary free surfaces in the absence of gravity. Acta Math. mmerically by Dr. Berndt an

FEFERENCES

(1] Concus, P., and R. FINN: Or

132 (1974), 177–198.

(2) Concus, P., and R. FINN: On

(1974), 207–223.

(3) FINN, R.: Equilibrium Cap

Verlag 1986.

(4) MIERSEMANN, E.: On the be

- [2] CoNcus, P., and R. **FINN:** On capillary free surfaces in a gravitational field. Acta Math. 132 132 (1974), 177-198.

[2] Concus, P., and R. FINN: On capillary free surfaces in a gravitational field. Acta Math. 132

(1974), 207-223.

[3] FINN, R.: Equilibrium Capillary Surfaces. Berlin-Heidelberg-New York: Springer-S. F., and R. FINN: On capillary free surfaces in the absence of gravity. Acta Mi

1974), 177–198.

S. P., and R. FINN: On capillary free surfaces in a gravitational field. Acta Math.

207–223.

R.: Equilibrium Capillary S
	- [3] FINN, R.: Equilibrium Capillary Surfaces. Berlin-Heidelberg-New York: Springer-/
	- [4] MIERSEMANN, E.: On the behaviour of capillaries at a corner. Pac. J. Math. 140 (1989), $149-153$. 207-223.

	R.: Equilibrium Capillary Surfaces. Berlin-Heidelberg-Ne

	1986.

	EMANN, E.: On the behaviour of capillaries at a corner. Pac.

	53.
 F. B.: Concerning the ascent of water between two glass plane

	condon 27 (1712
	- [5] TAYLOR, B.: Concerning the ascent of water between two glass planes. Phil. Trans. Roy. Soc. London 27 (1712), 538. *.• ' .*

Dr. ERICH MIERSEMANN
Sektion Mathematik der Universität Leipzig Section Mathematik der Universität Leipzigy.

Sektion Mathematik der Universität Leipzigy.

Sektion Manuskripteingang: 20. 12. 1988

The Manuskripteingang: 20. 12. 1988

The Manuskripteingang: 20. 12. 1988

The Manuskript S. P., and R. Frive: On capillary free surfaces in a gravitational field. Acta Math. 13

207-223.

R.: Equilibrium Capillary Surfaces. Berlin - Heidelberg - New York: Springer

1986.

EMANN, E.: On the behaviour of capilla (a) $207 - 223$.

R.: Equilibrium Capillary Surfaces. If 1986.

EMANN, E.: On the behaviour of capillation

53.

R. B.: Concerning the ascent of water boomdon 27 (1712), 538.

Manuskripteingang: 20. 12. 1988

(BREASSER

Dr - • .. . - " . - $\frac{1}{2}$ 712), 538.

•.

•. **112)**, 538.

•.

•. **FRE**
 FRECTION CONTENTS AND SET ASSESSMANT
 FRECTION

•.

•. **1**

•

• • . *•* •

T -

• /

' •

I

-•