Zeitschrift für Analysis
und ihre Anwendungen
Vol. 10 (1991) 2, 263–264

Correction to Products of Distributions: Nonstandard Methods

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The purpose of this note is to correct a faulty structure theorem for limited distributions, claimed to hold in [1] in the setting of Internal Set Theory. Though the assertion could not be recovered completely, a weaker version suffices to prove those results in [1] which depended on it.

Key words: *Structure theorems for distributions, Internal Set Theory, multiplication of distributions*

AMS subject classification: 46 F 10, 03 H 05

I am grateful to T.D. TODOROV, who pointed out the following mistake in [1] to me: The sentence after equation (2.4) is wrong; it does not follow that $\sup\{|g(x)| : |x| \le k\}$ is infinitesimal. A counterexample is obtained by taking θ as in (2.1) with $\theta(0) = 1$ and letting $T(x) = e^{n/\rho}\,\theta(e^{1/\rho}x)$; then $g(x) = T(x) - \rho^{-n}\,\theta(x/\rho)$ is not infinitesimal at $x = 0$. This invalidates the proof of (a) $\Rightarrow$ (b) in Proposition 2.10. For the same reason, Corollary 2.11 is wrong; the same $T(x)$ as above supplies a counterexample. The implication (b) $\Rightarrow$ (a) in Proposition 2.10 remains correct. Corollary 2.11 is not a consequence of Proposition 2.10, but rather of its proof. Thus (a) $\Rightarrow$ (b) may still be true, though I do not have a proof at the moment.

The only effect of the invalidation of Corollary 2.11 occurs in Section 4. To remedy this, Proposition 2.10 and Corollary 2.11 should be replaced by

Proposition 2.12: *Let $T \in {}^{st}\mathcal{D}'$ and $\rho \in \mathbb{R}$ be a positive infinitesimal, $\theta \in {}^{st}\mathcal{D}$ and θ_ρ as in* (2.1). *Then*

$$\forall\, {}^{st}k \in \mathbb{N}\ \exists\, {}^{st}j \in \mathbb{N} \text{ such that } \sup\{|(T \star \theta_\rho)(x)| : |x| \le k\} \le \rho^{-j}.$$

Proof: By the classical structure theorem for distributions and transfer we have: Given $k \in {}^{st}\mathbb{N}$ there is $\alpha \in {}^{st}\mathbb{N}_0^n$ and a standard continuous function f with compact support such that $\langle T, \varphi\rangle = (-1)^{|\alpha|}\langle f, \partial^\alpha\varphi\rangle$ for all $\varphi \in \mathcal{D}_{k+1}$. Now if $\operatorname{supp}(\varphi) \subset \{x : |x| \le k + \frac{1}{2}\}$, then $\varphi \star \check{\theta}_\rho \in \mathcal{D}_{k+1}$, so

$$\langle T \star \theta_\rho, \varphi\rangle = \langle T, \varphi \star \check{\theta}_\rho\rangle = (-1)^{|\alpha|}\langle f, \partial^\alpha(\varphi \star \check{\theta}_\rho)\rangle = \langle \partial^\alpha(f \star \theta_\rho), \varphi\rangle .$$

It follows that $(T \star \theta_\rho)(x) = \partial^\alpha(f \star \theta_\rho)(x)$ for all x with $|x| \le k$. The assertion that $|\partial^\alpha(f \star \theta_\rho)(x)| \le \rho^{-|\alpha|-1}$ for all $|x| \le k$ is verified just as in the proof of Corollary 2.11 ∎

Turning to Section 4, we infer from the discussion above that the first sentence after Definition 4.1 is incorrect: $\mathbf{D}'$ is not contained in $\mathbf{E}_\rho$. However, thanks to Proposition 2.12, it remains true that the map $S \to S \star \theta_\rho$ defines an imbedding of ${}^{st}\mathcal{D}'$ into $\mathbf{E}_\rho$. Finally, Proposition 4.4 remains true, but a slight modification of its proof is required. To show that $S \star \theta_\rho \in \mathrm{E}_\rho$ for $S \in {}^{st}S'$ and $\theta \in {}^{st}S$, a similar argument as in Proposition 2.12 may be used, but this time involving the structure theorem for tempered distributions.

I may take the opportunity to point out that two open questions in the article have been answered by now. R. WAWAK [2] has shown that the products M_2, M_3, M_4 actually are equivalent. In particular, the existence of either M_2, M_3, or M_4 implies the existence of M_5. On the other hand, J. JELÍNEK [2] has shown that M_5 is strictly more general than $M_2 - M_4$. I would also like to call the readers' attention to an interesting recent characterization of the products of type (P1) for homogeneous distributions by P. WAGNER [3].

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Received 01. 08. 1990

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