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Exposed Operators in $\mathfrak{L}(C(X), C(Y))$

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A point q_0 in a convex set Q is *exposed* if there exists a bounded linear functional ξ such that $\xi(q_0) > \xi(q)$ for all $q \in Q\setminus\{q_0\}$. Characterizations of exposed points of the unit ball and the positive part of the unit ball of $\mathfrak{L}(C(X), C(Y))$ are given. We describe the set of strongly exposed points. We also consider exposed operators on L^∞- and L^1-spaces.

Key words: *Exposed points, space of continuous functions, operators, Nice operators, strongly exposed operators*

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1. Introduction

Let B_1, B_2 be Banach spaces. By $\mathfrak{L}(B_1, B_2)$ there is denoted the Banach space of all bounded linear operators from B_1 to B_2. An operator $T \in \mathfrak{L}(B_1, B_2)$ *is called a contraction if* $\|T\| \le 1$. Throughout this paper we assume that X and Y are non-empty compact Hausdorff topological spaces. As usual, we denote by $C(X)$ the Banach space of all real-valued (or complex) continuous functions on X with supremum norm. Following Morris and Phelps [23], we call a contradiction $T \in \mathfrak{L}(C(X), C(Y))$ *nice*, if its adjoint operator T^* takes Dirac measures on Y into extreme points of the unit ball of $C(X)^*$. It is not difficult to see that every nice operator is an extreme contraction (extreme point of the unit ball). Note that any element of $C(X)^*$ can be identified both as a linear functional and a measure. Moreover, the set of all extreme points of the unit ball of $C(X)^*$ coincises with the set $\{a\delta_x : |a| = 1,\ x \in X\}$, where δ_x denotes Dirac measure (point mass) at $x \in X$ (see [2]). Thus $T \in \mathfrak{L}(C(X), C(Y))$ is nice if and only if there exists a function $r \in C(Y)$ with $|r| = 1$ and a continuous map $\varphi: Y \to X$ such that $(Tf)(y) = r(y) f(\varphi(y))$, for all $f \in C(X)$ and $y \in Y$.

Each extreme contraction in $\mathfrak{L}(C(X), C(Y))$ is nice in the following cases:

1. X is metrizable (see [4]).
2. X is Eberlein compact, Y is metrizable (see [1]).
3. X is dispersed (see [27]).
4. Y is extremally disconnected (see [27]; also [8, 18]).

It should be pointed out that Sharir [28, 29] has given counterexamples (see also [12]). Extreme operators have been studied by many authors. The first theorem of this type was given by A. and C. Ionescu Tulcea [16, 17]. Consider positive operators (an operator T is called *positive* if $Tf \ge 0$ for all $f \ge 0$). An operator $T \in \mathfrak{L}(C(X), C(Y))$ is an extreme positive contraction (extreme point of the positive part of the unit ball of operators) if and only if there exists a *clopen* (closed and open) set $Z \subset Y$ and a continuous map $\varphi: Z \to X$ such that $(Tf)(y) = 0$ if $y \in Z$ and $(Tf)(y) = f(\varphi(y))$ if $y \in Z$ (see [6, 24], also [26: III/§9]).

Recall that a point q_0 in a convex set Q is exposed if there exists a bounded real linear functional ξ such that $\xi(q_0) > \xi(q)$ for all $q \in Q\setminus\{q_0\}$. An exposed point $q_0 \in Q$ is called *strongly exposed* if for any sequence $\{q_n\} \subset Q$ the condition $\xi(q_n) \to \xi(q_0)$ implies $q_n \to q_0$. Obviously each exposed point is extreme.

The purpose of this paper is to study exposed and strongly exposed points of the unit ball of $\mathfrak{L}(C(X), C(Y))$. We also consider exposed operators acting on L^1- and L^∞- spaces. Note that exposed operators in L^p-spaces are considered in [13, 14]. We should mention here that P. Greim also obtained results in this direction for Bochner L^p-spaces [9 - 11].

We say that a compact Hausdorff space *X carries a strict positive measure* if there exists a strictly positive Radon measure μ on X (i.e. $\mu(U) > 0$ for all non-empty open subsets U of X).

The problem of characterization of spaces X which carry a strictly positive measure has been studied by many authors (see, e.g., [3, 7, 15, 21, 22]). In particular Kelley [19] introduced the notion of *intersection numbers* of a collection of subsets to give the characterization of spases which carry a strictly positive measure. It should be pointed out that in the case of a compact Hausdorff space the problem mentioned above is equivalent to the problem of existence of a finitely additive strictly positive measure. Note that $C(X)$ carries a strictly positive functional if and only if its dual $C(X)^*$ contains a weakly compact total subset (see [24: Theorem 4.5.b]). We refer the reader to [5: Chapter 6] for a summary of those and related results. In fact a strictly positive measure on X can be considered as a functional on $C(X)$ which exposes the function 1_X. Note that since $\mathfrak{L}(\mathbb{R}, C(Y))$ coincides with $C(Y)$ if Y carries a strict positive measure, then the extreme points of the unit ball of $\mathfrak{L}(\mathbb{R}, C(Y))$ are exposed (cf. [30]). They are strongly exposed only in the case when Y is a finite set.

2. Exposed points in $\mathfrak{L}(C(X), C(Y))$

We recommend to begin with a general sentence.

Theorem 1: *Let Y carry a strictly positive measure and suppose that*
(i) X is metric or (ii) Y is extremally disconnected.
Then each extreme point of the unit ball of $\mathfrak{L}(C(X), C(Y))$ is exposed.

Proof: Let μ be a strictly positive measure on Y with $\mu(Y) = 1$. Let T_0 be nice (= extreme), i.e. there exist $r_0 \in C(Y)$ and a continuous map $\Phi: Y \to X$ such that $(T_0 f)(y) = r_0(y) f(\Phi(y))$, $|r_0| = 1$. For $T \in \mathfrak{L}(C(X), C(Y))$ and $y \in Y$ we denote by m_y^T the measure defined by the equality $(Tf)(y) = \int_X f(x)\, dm_y^T$ for all $f \in C(X)$. This is a signed regular Borel measure on X with total variation $\|m_y^T\| \le \|T\|$. In fact $m_y^T = T^* \delta_y$.

Assume that (X, d) is a metric compact Hausdorff space. For $n \in \mathbb{N}$ and $y \in Y$ we define by $h_{n,y}(x) = r_0^{-1}(y) \max((1 - n d(x, \Phi(y)), 0)$ an element of $C(X)$. The map $h_{n,\cdot}: Y \to h_{n,y} \in C(X)$ is continuous, so for every operator $S \in \mathfrak{L}(C(X), C(Y))$ the function $y \to (S h_{n,y})(y)$ is continuous (as an element of $C(Y)$). Now we define a linear functional ξ_1 on $\mathfrak{L}(C(X), C(Y))$ by

$$\xi_1(S) = \sum_{n=1}^{\infty} \frac{1}{2^n} \int_Y (S h_{n,y})(y)\, d\mu(y) \qquad (S \in \mathfrak{L}(C(X), C(Y))).$$

If $\|S\| \le 1$, then $|S h_{n,y}(y)| \le 1$ and $\|\xi\| \le 1$. Suppose that $\xi_1(S_0) = 1 = \xi_0(T_0)$ for some contraction S_0. Then $\int (S_0 h_{n,y})(y)\, d\mu(y) = 1$ for all $n \in \mathbb{N}$. Since the map $y \to (S_0 h_{n,y})(y)$ is continuous with $|(S_0 h_{n,y})(y)| \le 1$ we get $(S_0 h_{n,y})(y) = \langle h_{n,y}, S_0^* \delta_y \rangle = 1$ for $y \in Y$. Hence $S_0^* \delta_y = r_0(y) \delta_{\varphi(y)}$, i.e. $S_0 = T_0$, what show that T_0 is exposed.

Now suppose that Y is extremally disconnected. Then $C(Y)$ is an order complete AM-space with unit [26: Section II.7.7] and $\mathfrak{L}(C(X), C(Y))$ is a Banach lattice [26: Section IV.1.5.]. Therefore for every contraction $S \in \mathfrak{L}(C(X), C(Y))$ there exist positive contractions S_+ and S_- such that $S = S_+ - S_-$. Then $m_y^S = m_y^{S_+} - m_y^{S_-}$. The map $y \to m_y^{S_+}$ is weakly* continuous. Since for arbitrary nets $y_\alpha \to y_0$ in Y and $\beta_\alpha \to \beta_0$ in $\mathbb{R}$ with $\beta_\alpha \in [0, m_{y_\alpha}^{S_+}(\{\varphi(y_\alpha)\})]$ the condition $m_{y_\alpha}^{S_+} - \beta_\alpha \delta_{\varphi(y_\alpha)} \ge 0$ implies that $m_{y_0}^{S_+} - \beta_0 \delta_{\varphi(y_0)} \ge 0$ and the sets $\{y : m_y^{S_+}(\{\varphi(y)\}) \ge a\}$ are closed for all $a \in \mathbb{R}$. The same we have for m_y^{S-}. Therefore $\int r_0^{-1}(y) m_y^S(\{\varphi(y)\}) d\mu(y)$ exists for every $S \in \mathfrak{L}(C(X), C(Y))$:

We define a linear functional ξ_2 on $\mathfrak{L}(C(X), C(Y))$ by

$$\xi_2(S) = \int_Y r_0^{-1}(y) m_y^S(\{\varphi(y)\}) d\mu(y) \quad (S \in \mathfrak{L}(C(X), C(Y))).$$

For a contraction $S \in \mathfrak{L}(C(X), C(Y))$ and an element $y \in Y$ we have $\|m_y^S\| \le 1$. Thus $\xi_2(S) \le \mu(Y) = 1$. Moreover $\xi_2(T_0) = 1$. Suppose now that $\xi_2(S_0) = 1$ for some contraction $S_0 \in \mathfrak{L}(C(X), C(Y))$. We have $\|m_y^{S_0}\| \le 1$. Thus $m_y^{S_0}(\{\varphi(y)\}) = r_0(y)$ μ-a.e. Hence $m_y^{S_0} = r_0(y)\delta_{\varphi(y)}$ μ-a.e. Therefore by a continuity argument and the fact that $\{y\colon m_y^{S_0}(\{\varphi(y)\}) = r_0(y)\}$ is closed we obtain $(S_0 f)(y) = r_0(y) f(\varphi(y))$ for all $y \in Y$. Thus $S_0 = T_0$, i.e. T_0 is exposed by ξ_2 in the unit ball of $\mathfrak{L}(C(X), C(Y))$ ∎

Theorem 2: *Let Y carry a strictly positive measure on Y. Then each extreme point of the positive part of the unit ball of $\mathfrak{L}(C(X), C(Y))$ is exposed.*

Proof: Let μ be a strictly positive measure on Y and T_0 an extreme positive contraction. Then there exists a clopen set $Z \subset Y$ and a continuous map $\varphi\colon Z \to X$ such that $(T_0 f)(y) = 0$ if $y \notin Z$ and $(T_0 f)(y) = f(\varphi(y))$ if $y \in Z$. Now we define a linear functional ξ by

$$\xi(S) = \int_Z m_y^S(\{\varphi(y)\}) d\mu(y) - \int_{Z^c} (S 1_X)(y) d\mu(y) \quad (S \in \mathfrak{L}(C(X), C(Y))).$$

This functional exposes T_0. Indeed, for a positive contraction S we have $\xi(S) \le \mu(Z) = \xi(T_0)$. Suppose $\xi(S_0) = \mu(Z)$ for some positive contraction S_0. Then $m_y^{S_0}(\{\varphi(y)\}) = 1$ for $y \in Z$ and $(S_0 1)(y) = 0$ for $y \in Z^c$. Using the same arguments as in the proof of Theorem 1 we have $(S_0 f)(y) = f(\varphi(y))$ for $y \in Z$. If $0 \le f \le 1$, then $0 \le S_0 f \le S_0 1$, so $(S_0 f)(y) = 0$ for $y \in Z^c$. Therefore $S_0 = T_0$, i.e. T_0 is exposed by ξ ∎

3. Strongly exposed operators

Now we consider the strongly exposed points of the unit ball and the positive part of the unit ball of $\mathfrak{L}(C(X), C(Y))$.

Theorem 3: *Let Y carry a strictly positive measure and X be metric or extremally disconnected.*

(a) *If* card $Y < \infty$, *then all extreme points of the unit ball of the space $\mathfrak{L}(C(X), C(Y))$ are strongly exposed.*

(b) *If* card $Y = \infty$, *then there are no strongly exposed points in the unit ball of the space $\mathfrak{L}(C(X), C(Y))$.*

Proof: (a) Let $Y = \{y_1, y_2, \dots, y_n\}$, $n \in \mathbb{N}$ and T_0 be an extreme contraction. Then $(T_0 f)(y_j) = r(y_j) f(\varphi(y_j))$, where $|r(y_j)| = 1$. Put $\xi(S) = \sum_{j=1}^n r(y_j) m_{y_j}^S(\{\varphi(y_j)\})$, where $m_{y_j}^S$ denotes the measure on X defined in the proof of Theorem 1. Obviously ξ exposes T_0. Suppose that $\xi(S_k) \to \xi(T_0) = n$ for some sequence of contractions $S_k \in \mathfrak{L}(C(X), C(Y))$. Then $\|m_{y_j}^{S_k}\| \le 1$ and $m_{y_j}^{S_k}(\{\varphi(y_j)\}) \to r(y_j)$ as $k \to \infty$. Thus

$$\left\| m_{y_j}^{S_k} - r(y_j)\delta_{\varphi(y_j)} \right\| \le \left| m_{y_j}^{S_k} - r(y_j)\delta_{\varphi(y_j)} \right|(\{\varphi(y_j)\}) = \left| m_{y_j}^{S_k} \right|(\{\varphi(y_j)\}^c) \longrightarrow 0$$

as $k \to \infty$. Now we obtain

$$\|S_k - T_0\| = \sup_{\|f\|\le 1} \sup_{j\le n} \left|\int_X f d\left(m_{y_j}^{S_k} - r(y_j)\delta_{\varphi(y_j)}\right)\right|$$
$$\le \sup_{\|f\|\le 1} \|f\| \sup_{j\le n} \left\|m_{y_j}^{S_k} - r(y_j)\delta_{\varphi(y_j)}\right\| \longrightarrow \text{ as } k\to\infty.$$

Thus T_0 is strongly exposed.

(b) Suppose now that card$Y = \infty$. Let T_0 be exposed by a functional ξ, i.e. $\xi(S) \le \xi(T_0) = 1$ for all contractions S. There exists a sequence $\{U_j\}_{j=1}^{\infty}$ of disjoint non-empty open subsets of Y. Let $g_j \in C(Y)$ be such that $\|g_j\| = 1$, and supp$g_j \subset U_j$ for all $j \in \mathbb{N}$. Now we define operators R_j by $R_j f = g_j T_0 f$. Put $\gamma_j = \xi(R_j)$ and let $k \in \mathbb{N}$. The operator $\sum_{j=1}^{k} R_j$ is a contraction. Thus $\sum_{j=1}^{k}\gamma_j = \xi(\sum_{j=1}^{k} R_j) \le 1$. Therefore $\lim_j \gamma_j = 0$. Consider the operators $T_j = T_0 - R_j$. We have $\|T_0 - T_j\| = 1$, i.e. $\{T_j\}$ does not converge to T_0. But $\xi(T_j) = \xi(T_0) - \xi(R_j) = 1 - \gamma_j \to 1 = \xi(T_0)$ as $j\to\infty$. Thus T_0 is not strongly exposed ∎

Theorem 4: *The following statements are true.*

(a) *If* card$Y < \infty$, *then all extreme points of the positive part of unit ball of the space* $\mathfrak{L}(C(X), C(Y))$ *are strongly exposed.*

(b) *If* card$Y = \infty$, *then there are no strongly exposed points in the positive part of the unit ball of the space* $\mathfrak{L}(C(X), C(Y))$.

Proof: Let T_0 be a positive contraction in $\mathfrak{L}(C(X), C(Y))$. Then $(Tf)(y) = 0$ for $y \notin Z$ and $(Tf)(y) = f(\varphi(y))$ for $y \in Z$.

(a) Suppose that card$Y < \infty$. We define a functional ξ_0 by

$$\xi_0(S) = \sum_{y_j\in Z} m_{y_j}^{S}(\{\varphi(y_j)\}) - \sum_{y_j\in Z^c}(S1_X)(y_j).$$

This functional exposes T_0 since $\xi_0(S) \le$ card $Z = \xi_0(T_0)$. Suppose now that $\xi_0(S_k) \to \xi_0(T_0)$ as $k\to\infty$, for some sequence of positive S_k. We have

$$\left\|m_{y_j}^{S_k} - \delta_{\varphi(y_j)}\right\| \xrightarrow[k\to\infty]{} 0 \text{ for } y_j \in Z \text{ and } (S_k 1)(y_j) \xrightarrow[k\to\infty]{} 0 = (T_0 1)(y_j) \text{ for } y_j \in Z^c.$$

Therefore $\|S_n - T_0\| \to 0$ as $n\to\infty$, i.e. T_0 is strongly exposed.

(b) Suppose that a functional ξ exposes a positive contraction T_0, i.e. $\xi(S) \le \xi(T_0) = 1$ for all positive contractions S. If card$Y = \infty$, then using arguments from the proof of Theorem 4/(b) we get that T_0 is not strongly exposed. Consider now the case card$Z^c = \infty$. Let $\{U_j\}_{j=1}^{\infty}$ be a family of disjoint non-empty open sets and let g_j be such that $0 \le g_j \le 1$, supp$g_j \subset U_j$, $\|g_j\| = 1$. Fix $x_0 \in X$. We define operators R_j by $(R_j f)(y) = g_j(y)f(x_0)$. Put $\gamma_j = \xi(R_j)$ and let $k \in \mathbb{N}$. Since $\sum_{j=1}^{k} R_j$ is a positive contraction, using the same arguments as in the proof of Theorem 3/(b) we obtain $\gamma_j \to 0$ and $\xi(T_j) \to \xi(T_0)$ as $j\to\infty$, where $T_j = T_0 + R_j$, though $\|T_j\| \le 1$, $T_j \ge 0$, and $\|T_j - T_0\| \le 1$. Thus T_0 is not strongly exposed ∎

Theorem 5: *If* Y *does not carry a strictly positive measure, then there are not exposed points in the unit ball and in the positive part of the unit ball of* $\mathfrak{L}(C(X), C(Y))$.

Proof: First consider the case of the whole unit ball. Suppose that ξ_0 exposes an extreme contraction $T_0 \in \mathfrak{L}(C(X), C(Y))$. We define a functional m on $C(Y)$ by $m(h) = \xi_0(hT_0)$, $h \in C(Y)$. Suppose that there exists a non-zero h_0, $0 \le h_0 \le 1$ and $m(h_0) \le 0$. Then $\xi_0((1-h_0)T_0) = m(1-h_0) \ge m(1) = \xi_0(T_0)$. Since $\|(1-h_0)T_0\| \le 1$, we have $(1-h_0)T_0 = T_0$. Fix $x_0 \in X$. Because $\|(1-h_0)T_0 f$

$\pm hf(x_0)\| \le \|f\|$ for all $f \in C(X)$. The operator $(1-h_0)T_0$ is not extreme. This contradictions proves that $m(h) > 0$ for all $h \in C(Y)$ with $0 \le h \le 1$. Therefore if there exists an exposed point in the unit ball, then Y carries a strictly positive measure, what ends the proof for the unit ball.

Now consider the positive part of the unit ball. Suppose that a functional ξ_0 exposes a positive contraction T_0. The operator T_0 is of the form $T_0(f)(y) = 0$ for $y \notin Z$ and $T_0(f)(y) = f(\varphi(y))$ for $y \in Z$. Using arguments presented in the first part of the proof one can see that the clopen set Z carries a strictly positive measure. Fix $x_0 \in X$ and put $Rf = 1_{Z^c} f(x_0)$, $f \in C(X)$. We define a functional n on $C(Z^c)$ by $n(h) = -\xi_0(hR)$, $h \in C(Z^c)$. Let $0 \le h \le 1$ and $h \ne 0$. Then because $T_0 + hR$ is a positive contraction and $hR \ne 0$ we have $\xi_0(T_0) > \xi_0(T_0 + hR) = \xi_0(T_0) - n(h)$, so $n(h) > 0$. Therefore n is a strictly positive measure on Z^c ∎

4. The case of L^∞- and L^1-spaces

Let $(Q,\mathfrak{B},\mu)$ be a σ-finite measure space. Denote by $L^\infty(\mu)$ the space of all essentially measurable functions on (Q,μ), with essential supremum norm. The space $L^\infty(\mu)$ is the dual of the AL-space $L^1(\mu)$, and is isomorphic to $C(X)$, where X is the Stone representation space of β/N (N denotes the ideal of measure zero sets). In this case the space X must be hyperstonian (see [26: Chap. II, Sec. 9]). Thus X is also Stonean (extremally disconnected). Since μ is σ-finite, there exists a strictly positive $f \in L^1(\mu)$. Hence X carries a strictly positive measure.

Let $(Q_i,\mathfrak{B}_i,\mu_i)$ be σ-finite measure spaces, $i=1,2$. Consider now extreme operators in $\mathfrak{L}(L^\infty(\mu_1),L^\infty(\mu_2))$. We can identify this space with the space $\mathfrak{L}(C(X),C(Y))$, where X and Y are suitable hyperstonean spaces. Note that the representation of an extreme operator in $\mathfrak{L}(L^\infty(\mu_1),L^\infty(\mu_2))$ by means of a measurable transformation φ is not always possible (see [18: p. 152]). The extreme positive contractions in the space $\mathfrak{L}(L^\infty(\mu_1),L^\infty(\mu_2))$ can be characterized as operators which carry characteristic functions, or equivalently, which are multiplicative (see [24: Theorem 2.2]). The set of extreme contractions in $\mathfrak{L}(L^\infty(\mu_1),L^\infty(\mu_2))$ coincides with the set of all lattice homomorphisms taking the function 1 into itself, multiplied by functions of absolute value one (see [18, 20]). Using Theorem 2 and 3 we obtain the following

Theorem 6: *Extreme positive contractions and extreme contractions in $\mathfrak{L}(L^\infty(\mu_1),L^\infty(\mu_2))$ are exposed. Moreover the exposed operators are strongly exposed if and only if $L^\infty(\mu_2)$ is finite-dimensional.*

Let us consider extreme operators in $\mathfrak{L}(L^1(\mu_1),L^1(\mu_2))$. The extreme contractions can be characterized as those operators whose adjoints are extreme contractions in $\mathfrak{L}(L^\infty(\mu_2),L^\infty(\mu_1))$ (see [18]). As we mentioned above, in general extreme operators cannot be represented by measurable transformations. But in some cases this is possible, for example, if μ_2 is a σ-finite Borel measure on $\mathbb{R}$ (see [18: Theorem 2]). Also extreme positive contractions are characterized by duality.

Theorem 7: *Let μ_2 be a σ-finite Borel measure on $\mathbb{R}$. Then extreme positive contractions and extreme contractions in $\mathfrak{L}(L^1(\mu_1),L^1(\mu_2))$ are exposed. Moreover the exposed operators are strongly exposed if and only if $L^1(\mu_1)$ is finite-dimensional.*

Proof: Let T_0 be an extreme operator in $\mathfrak{L}(L^1(\mu_1),L^1(\mu_2))$. Then T_0^* is also extreme, so exposed. Suppose that the functional ξ_0 defined on $\mathfrak{L}(L^\infty(\mu_2),L^\infty(\mu_1))$ exposes T_0^*. We define a functional η on $\mathfrak{L}(L^1(\mu_1),L^1(\mu_2))$ by $\eta(T) = \xi_0(T^*)$. It is easy to see that η exposes T_0 ∎

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