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Orthogonal Projection and Restricted Subordination of Hilbert-Schmidt Operator-Valued Stationary Processes

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Abstract. The paper contains two results on subordination of stationarily correlated Hilbert-Schmidt operator-valued stationary processes. First an explicit form of the spectral measure of the orthogonal projection of one process onto another is stated. On the basis of this result B. Fritzsche's and B. Kirstein's solution of the restricted subordination problem for finite-dimensional processes is generalized to Hilbert-Schmidt operator-valued processes.

Keywords: *Hilbert-Schmidt operator-valued stationary processes, subordination*

AMS subject classification: Primary 60 G 10, 60 G 25, secondary 47 B 10

1. The class of Hilbert-Schmidt operator-valued stationary processes can be identified with the class of infinite-dimensional stationary processes (cf. [10: pp. 346 - 347]). Thus, results on Hilbert-Schmidt operator-valued processes can be considered as generalizations of results on finite-dimensional stationary processes. The present short note deals with two problems of subordination of Hilbert-Schmidt operator-valued processes.

First results on subordination of stationary processes were obtained by A. N. Kolmogorov (cf. [7: Sections 4 - 5]). In the sequel Kolmogorov's results on one-dimensional processes were extended and generalized to other classes of stationary processes by several authors. In particular, V. Mandrekar and H. Salehi proved that if $\mathbf{X}$ and $\mathbf{Y}$ are stationarily correlated Hilbert-Schmidt operator-valued stationary processes, then the orthogonal projection of $\mathbf{X}$ onto $\mathbf{Y}$ is expressible with the aid of a certain stochastic integral (cf. [9: Theorem 3.14]). We specify Mandrekar's and Salehi's Theorem stating an explicit form of the function occurring in this stochastic integral (see Theorem 5). At the same time we generalize [11: Formula (1.9)]. Our second result deals with a restricted subordination problem. Such problems were first studied by D. R. Brillinger. In practice they arise if for the transmission of a signal only a limited number of channels is available. Brillinger solved the restricted subordination problem for finite-dimensional stationary processes under the additional assumption that the spectral density matrix has full rank (cf. [1: Section 3] and [2: Chapters 9 - 10]). In [3: Theorem 7] B. Fritzsche and B. Kirstein gave a solution to this problem without any additional assumptions. Theorem 8 of our paper generalizes Fritzsche's and Kirstein's result to the case of Hilbert-Schmidt operator-valued processes.

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2. Let $\mathcal{H}, \mathcal{K}$, and $\mathcal{O}$ be infinite-dimensional separable Hilbert spaces over the field of complex numbers $\mathbb{C}$, $O(\mathcal{H},\mathcal{K})$ the set of all linear operators from $\mathcal{H}$ into $\mathcal{K}$, $HS(\mathcal{H},\mathcal{O})$ the Hilbert space of all Hilbert-Schmidt operators of $\mathcal{H}$ to $\mathcal{O}$, and $T(\mathcal{H})$ the class of all non-negative definite operators in $HS(\mathcal{H},\mathcal{H})$ of finite trace. The trace of a trace class operator X is denoted by $\mathrm{tr}X$, and the Hilbert-Schmidt norm of a Hilbert-Schmidt operator X is denoted by $|X|$, i.e. $|X|^2 = \mathrm{tr}(XX^*)$. Moreover, the symbol X^+ stands for the generalized inverse of a bounded linear operator which is given, e.g., in [8: Definition 2.11], and $\bar{X}$ denotes the closure of a closable operator X.

Let G be a locally compact abelian group (under the operation +) and $\widehat{G}$ its dual group of characters. A weakly continuous map $\mathbf{X} : G \ni t \to X_t \in HS(\mathcal{H},\mathcal{O})$ is called an *$HS(\mathcal{H},\mathcal{O})$-valued stationary process* if the function $G \times G \ni (s,t) \to X_t^* X_s$ depends only on $s-t$.

By $\mathfrak{M}_{\mathbf{X}}$ we denote the subspace of $\mathcal{O}$ generated by the elements $X(t)x$ $(x \in \mathcal{H}, t \in G)$. An $HS(\mathcal{H},\mathcal{O})$-valued stationary process $\mathbf{X}$ and an $HS(\mathcal{K},\mathcal{O})$-valued stationary process $\mathbf{Y}$ are called *stationarily correlated* if $G \times G \ni (s,t) \to X_t^* Y_s$ is a function of $s-t$ only. We will always assume in the present paper that $\mathbf{X}$ and $\mathbf{Y}$ are stationarily correlated. In this case the processes $\mathbf{X}$ and $\mathbf{Y}$ have the spectral representations

$$X_t = \int_{\widehat{G}} \langle t,\lambda\rangle E(d\lambda) X_0 \qquad \text{and} \qquad Y_t = \int_{\widehat{G}} \langle t,\lambda\rangle E(d\lambda) Y_0 \qquad (t \in G),$$

respectively. Here $\langle t,\lambda\rangle$ denotes the value of the character λ on the element t and E is a spectral measure on the Borel σ-algebra $\mathfrak{B}$ of $\widehat{G}$, whose values are orthogonal projections in $\mathcal{O}$. Let $F_{\mathbf{X}} := X_0^* E X_0$, $F_{\mathbf{Y}} := Y_0^* E Y_0$, $F_{\mathbf{YX}} := Y_0^* E X_0$, $F_{\mathbf{XY}} = F_{\mathbf{YX}}^*$, and $F := \begin{pmatrix} F_{\mathbf{X}} & F_{\mathbf{XY}} \\ F_{\mathbf{YX}} & F_{\mathbf{Y}} \end{pmatrix}$. Then F is a $T(\mathcal{H}\oplus\mathcal{K})$-valued measure on $\mathfrak{B}$, which is absolutely continuous with respect to the non-negative finite measure $\tau := \mathrm{tr}F$. We remark that the results of our paper remain true if we replace the measure τ by any non-negative σ-finite measure μ such that F is absolutely continuous with respect to μ (cf. [8: Lemma 4.5]). Let $F' := \begin{pmatrix} F'_{\mathbf{X}} & F'_{\mathbf{XY}} \\ F'_{\mathbf{YX}} & F'_{\mathbf{Y}} \end{pmatrix}$ be the Radon-Nikodym derivative of F with respect to τ. The function F' is a Bochner measurable $T(\mathcal{H}\oplus\mathcal{K})$-valued function on $\widehat{G}$.

Let $L^2(F_{\mathbf{X}};\mathcal{H},\mathcal{K})$ be the Hilbert space of (equivalence classes of) measurable (in the sense of [8: Definition 2.1]) $O(\mathcal{H},\mathcal{K})$-valued functions Φ on $\widehat{G}$ such that $\Phi F_{\mathbf{X}}^{'1/2}$ is a Bochner measurable $HS(\mathcal{H},\mathcal{K})$-valued function and $\Phi F_{\mathbf{X}}^{'1/2}(\Phi F_{\mathbf{X}}^{'1/2})^*$ is Bochner integrable with respect to τ (cf. [8: Definition 4.8 and Formula (4.10)]).

Remark 1. A Hilbert-Schmidt operator-valued or a trace-class operator-valued function is Bochner measurable if and only if it is weakly measurable (cf. [6: Lemma 5]). Thus, a measurable $O(\mathcal{H},\mathcal{K})$-valued function belongs to $L^2(F_{\mathbf{X}};\mathcal{H},\mathcal{K})$ if and only if $\Phi F_{\mathbf{X}}^{'1/2}$ is a weakly measurable $HS(\mathcal{H},\mathcal{K})$-valued function and $\int_{\widehat{G}} |\Phi(\lambda) F'_{\mathbf{X}}(\lambda)^{1/2}|^2 \tau(d\lambda) < \infty$.

For each $\Phi \in L^2(F_{\mathbf{X}};\mathcal{H},\mathcal{K})$, Mandrekar and Salehi defined the stochastic integral $\int_{\widehat{G}} \Phi dEX_0$ (cf. [8: Section 6]). (Here and in the following we will often omit the integration variable.)

Remark 2. The reader shold not be confused by the fact that the range of $E(B)$ $(B \in \mathfrak{B})$ is a subset of $\mathcal{O}$, whereas the domain of definition of $\Phi(\lambda)$ $(\lambda \in \widehat{G})$ is a subset of

$\mathcal{H}$. The stochastic integral $\int_{\widehat{G}} \Phi dEX_0$ is rather to be understood as $\int_{\widehat{G}} dEX_0\Phi^*$ and its value belongs to $HS(\mathcal{K}, \mathcal{O})$.

Using [8: Theorem 6.9] and some results from [9: pp. 121 - 122], one obtains that the map $\Phi \rightarrow \int_{\widehat{G}} \Phi dEX_0$ is an isometry from $L^2(F_{\mathbf{X}}; \mathcal{H}, \mathcal{K})$ onto the space $HS(\mathcal{K}, \mathfrak{M}_{\mathbf{X}})$, where

$$\left(\int_{\widehat{G}} \Phi dEX_0\right)^* \left(\int_{\widehat{G}} \Psi dEX_0\right) = \int_{\widehat{G}} \Phi F_{\mathbf{X}}^{\prime 1/2} (\Psi F_{\mathbf{X}}^{\prime 1/2})^* d\tau.$$

3. Let P be the orthogonal projection in $\mathcal{O}$ onto the subspace $\mathfrak{M}_{\mathbf{Y}}$. The map $\mathbf{X}_P : G \ni t \rightarrow PX_t$ is called the *orthogonal projection of* $\mathbf{X}$ *onto* $\mathbf{Y}$ (cf. [9: p. 122] and [10: p. 348/Definition 5]). The map $\mathbf{X}_P$ is an $HS(\mathcal{H}, \mathcal{O})$-valued stationary process and from [9: Theorem 3.14] it follows that there exists a function $\Phi_P \in L^2(F_{\mathbf{Y}}; \mathcal{K}, \mathcal{H})$ such that

$$PX_t = \int_{\widehat{G}} \langle t, \lambda \rangle \Phi_P(\lambda) E(d\lambda) Y_0 \qquad (t \in G).$$

In order to determine an explicit form of Φ_P, we need two lemmas.

Lemma 3. *For τ-a.a. $\lambda \in \widehat{G}$, the operator $F'_{\mathbf{XY}}(\lambda) F'^{+}_{\mathbf{Y}}(\lambda)^{1/2}$ is densely defined and bounded, its closure $(F'_{\mathbf{XY}}(\lambda) F'^{+}_{\mathbf{Y}}(\lambda)^{1/2})^{-}$ belongs to $HS(\mathcal{K}, \mathcal{H})$ and*

$$F'_{\mathbf{X}}(\lambda) \geq \left(F'_{\mathbf{XY}}(\lambda) F'^{+}_{\mathbf{Y}}(\lambda)^{1/2}\right)^{-} \left(F'_{\mathbf{XY}}(\lambda) F'^{+}_{\mathbf{Y}}(\lambda)^{1/2}\right)^{*}. \tag{1}$$

Proof. By [5: Korollar 5] the operator $F'_{\mathbf{XY}}(\lambda) F'^{+}_{\mathbf{Y}}(\lambda)^{1/2}$ is densely defined and bounded and by [5: Formula (3) of Section 6] the operator $F'^{+}_{\mathbf{X}}(\lambda)^{1/2} F'_{\mathbf{XY}}(\lambda) F'^{+}_{\mathbf{Y}}(\lambda)^{1/2}$ is a densely defined contraction. It follows

$$\left(F'^{+}_{\mathbf{X}}(\lambda)^{1/2} F'_{\mathbf{XY}}(\lambda) F'^{+}_{\mathbf{Y}}(\lambda)^{1/2}\right)^{-} \left(F'^{+}_{\mathbf{X}}(\lambda)^{1/2} F'_{\mathbf{XY}}(\lambda) F'^{+}_{\mathbf{Y}}(\lambda)^{1/2}\right)^{*} \leq I, \tag{2}$$

where I denotes the identity operator in $\mathcal{H}$. If we assume that $F'_{\mathbf{X}}(\lambda)$ has a bounded inverse, the left-hand side of (2) can be written as

$$F'_{\mathbf{X}}(\lambda)^{-1/2} \left(F'_{\mathbf{XY}}(\lambda) F'^{+}_{\mathbf{Y}}(\lambda)^{1/2}\right)^{-} \left(F'_{\mathbf{XY}}(\lambda) F'^{+}_{\mathbf{Y}}(\lambda)^{1/2}\right)^{*} F'_{\mathbf{X}}(\lambda)^{-1/2},$$

which implies (1). In the general case we obtain from the result just proved

$$F'_{\mathbf{X}}(\lambda) + \varepsilon I \geq \left(F'_{\mathbf{XY}}(\lambda) F'^{+}_{\mathbf{Y}}(\lambda)^{1/2}\right)^{-} \left(F'_{\mathbf{XY}}(\lambda) F'^{+}_{\mathbf{Y}}(\lambda)^{1/2}\right)^{*}$$

for each $\varepsilon > 0$. Letting $\varepsilon \rightarrow 0$, we get (1). Since $F'_{\mathbf{X}}(\lambda) \in T(\mathcal{H})$, $(F'_{\mathbf{XY}}(\lambda) F'^{+}_{\mathbf{Y}}(\lambda)^{1/2})^{-} \in HS(\mathcal{K}, \mathcal{H})$ follows ∎

Consider the function

$$\Phi_P := \big(F'_{\mathbf{XY}} F_{\mathbf{Y}}^{'+1/2}\big)^{\tilde{}} F_{\mathbf{Y}}^{'+1/2}. \tag{3}$$

Lemma 4. *The function Φ_P belongs to $L^2(F_{\mathbf{Y}}; \mathcal{K}, \mathcal{H})$.*

Proof. From [6: Corollary 2] the weak measurability of $\Phi_P F_{\mathbf{Y}}^{'1/2} = (F'_{\mathbf{XY}} F_{\mathbf{Y}}^{'+1/2})^{\tilde{}}$ follows and (1) gives

$$\int_{\widehat{G}} |\Phi_P F_{\mathbf{Y}}^{'1/2}|^2 d\tau = \int_{\widehat{G}} \big|(F'_{\mathbf{XY}} F_{\mathbf{Y}}^{'+1/2})^{\tilde{}}\big|^2 d\tau \le \int_{\widehat{G}} \operatorname{tr} F'_{\mathbf{X}} d\tau = \operatorname{tr} F_{\mathbf{X}}(\widehat{G}) < \infty.$$

In view of Remark 1 the proof is finished ∎

Theorem 5. *Let Φ_P be the function of (3). Then, for all $t \in G$,*

$$PX_t = \int_{\widehat{G}} \langle t, \lambda \rangle \Phi_P(\lambda) E(d\lambda) Y_0.$$

Proof. Because of Lemma 4 the stochastic integral $\int_{\widehat{G}} \langle t, \lambda \rangle \Phi_P(\lambda) E(d\lambda) Y_0$ exists and it remains to show that the range of $X_t - \int_{\widehat{G}} \langle t, \lambda \rangle \Phi_P(\lambda) E(d\lambda) Y_0$ is orthogonal to $\mathfrak{M}_{\mathbf{Y}}, t \in G$. But

$$\begin{aligned}
&\Big(X_t - \int_{\widehat{G}} \langle t, \lambda \rangle \Phi_P(\lambda) E(d\lambda) Y_0\Big)^* \int_{\widehat{G}} \langle s, \lambda \rangle E(d\lambda) Y_0 \\
&= \int_{\widehat{G}} \langle t - s, \lambda \rangle F'_{\mathbf{XY}}(\lambda) \tau(d\lambda) - \int_{\widehat{G}} \langle t - s, \lambda \rangle \Phi_P(\lambda) F'_{\mathbf{Y}}(\lambda)^{1/2} (F'_{\mathbf{Y}}(\lambda)^{1/2})^* \tau(d\lambda) \\
&= 0,
\end{aligned}$$

and the assertion follows ∎

4. Let $\mathcal{F}$ be a separable Hilbert space over $\mathbb{C}$, whose dimension d can be finite or infinite. If $d = \infty$, then the symbol $\mathbb{N}_d$ stands for the set $\mathbb{N}$ of positive integers, if $d < \infty$, then $\mathbb{N}_d$ denotes the set of the first d positive integers.

For $\Phi \in L^2(F_{\mathbf{Y}}; \mathcal{K}, \mathcal{F})$, denote by $F_{\mathbf{Y};\Phi}$ the $T(\mathcal{F})$-valued measure $\Phi dF_{\mathbf{Y}} \Phi^*$. Then for $\Psi \in L^2(F_{\mathbf{Y};\Phi}; \mathcal{F}, \mathcal{H})$ the stochastic integral $\int_{\widehat{G}} \Psi \Phi dE Y_0$ exists and defines an operator of the class $HS(\mathcal{H}, \mathfrak{M}_{\mathbf{Y}})$.

Consider the following restricted subordination problem:

Calculate

$$\rho := \inf \Big\{ \Big| X_0 - \int_{\widehat{G}} \Psi \Phi dE Y_0 \Big| : \Phi \in L^2(F_{\mathbf{Y}}; \mathcal{K}, \mathcal{F}),\ \Psi \in L^2(F_{\mathbf{Y};\Phi}; \mathcal{F}, \mathcal{H}) \Big\} \tag{4}$$

and determine functions $\Phi_0 \in L^2(F_{\mathbf{Y}}; \mathcal{K}, \mathcal{F})$ and $\Psi_0 \in L^2(F_{\mathbf{Y};\Phi_0}; \mathcal{F}, \mathcal{H})$ such that the minimum is attained.

Since the range of $X_0 - PX_0$ is orthogonal to $\mathfrak{M}_{\mathbf{Y}}$, we have

$$|X_0 - Z|^2 = |X_0 - PX_0|^2 + |PX_0 - Z|^2 \qquad \text{for } Z \in HS(\mathcal{H}, \mathfrak{M}_{\mathbf{Y}}).$$

Thus ρ^2 can be written in the form

$$\rho^2 = |X_0 - PX_0|^2 + \min\left\{\left|PX_0 - \int_{\widehat{G}} \Psi\Phi dEY_0\right|^2 : \Phi \in L^2(F_{\mathbf{Y}}; \mathcal{K}, \mathcal{F}), \Psi \in L^2(F_{\mathbf{Y};\Phi}; \mathcal{F}, \mathcal{H})\right\}. \tag{5}$$

By Theorem 5, the first summand on the right-hand side of (5) can be calculated and it remains to discuss the second summand.

For $\lambda \in \widehat{G}$, let $l_j(\lambda) \downarrow$ be the eigenvalues of $(F'_{\mathbf{XY}}(\lambda)F'^{+}_{\mathbf{Y}}(\lambda)^{1/2})(F'_{\mathbf{XY}}(\lambda)F'^{+}_{\mathbf{Y}}(\lambda)^{1/2})^*$ counted as often as their multiplicities and $\{v_j(\lambda)\}_{j\in\mathbb{N}}$ a corresponding orthonormal system of eigenvectors. Recall that the functions l_j $(j \in \mathbb{N})$ are measurable (cf. [8: Theorem 2.10]).

Lemma 6. *The function v_j $(j \in \mathbb{N})$ can be chosen Bochner measurable.*

Proof. Let $\{x_n\}_{n\in\mathbb{N}}$ be a countable dense subset of $\mathcal{H}$ and let $Q(\lambda)$ be the orthogonal projection onto the eigenspace corresponding to the largest eigenvalue of $(F'_{\mathbf{XY}}(\lambda)F'^{+}_{\mathbf{Y}}(\lambda)^{1/2})(F'_{\mathbf{XY}}(\lambda)F'^{+}_{\mathbf{Y}}(\lambda)^{1/2})^*$. According to [8: Theorem 2.10] the function Q is strongly measurable. Hence the sets

$$B_n := \left\{\lambda \in \widehat{G} : Q(\lambda)x_n \neq 0 \text{ and } Q(\lambda)x_j = 0 \text{ for } j < n,\ j \in \mathbb{N}\right\} \qquad (n \in \mathbb{N})$$

belong to $\mathfrak{B}$. Since $Q(\lambda)$ is a bounded linear operator and the set $\{x_n\}_{n\in\mathbb{N}}$ is dense in $\mathcal{H}$, we have $\cup_{n\in\mathbb{N}} B_n = \widehat{G}$.

Let $v_1(\lambda) := \|Q(\lambda)x_n\|^{-1}Q(\lambda)x_n$, if $\lambda \in B_n$, $n \in \mathbb{N}$. Here $\|\cdot\|$ denotes the norm in $\mathcal{H}$. Clearly, the function v_1 is Bochner measurable and for $\lambda \in \widehat{G}$ the vector $v_1(\lambda)$ is an eigenvector to the eigenvalue $l_1(\lambda)$.

Now assume that the functions $v_1, \dots, v_n$ with the desired properties have been constructed. To obtain v_{n+1}, repeat the considerations above with

$$\left(F'_{\mathbf{XY}}(\lambda)F'^{+}_{\mathbf{Y}}(\lambda)^{1/2}\right)\left(F'_{\mathbf{XY}}(\lambda)F'^{+}_{\mathbf{Y}}(\lambda)^{1/2}\right)^*$$

replaced by

$$Q_n(\lambda)\left(F'_{\mathbf{XY}}(\lambda)F'^{+}_{\mathbf{Y}}(\lambda)^{1/2}\right)\left(F'_{\mathbf{XY}}(\lambda)F'^{+}_{\mathbf{Y}}(\lambda)^{1/2}\right)^* Q_n(\lambda),$$

where $Q_n(\lambda)$ is the orthoprojector onto the orthogonal complement of the space spanned by $v_1(\lambda), \dots, v_n(\lambda)$, $\lambda \in \widehat{G}$ ∎

Let $\{e_j\}_{j\in\mathbb{N}_d}$ be an orthonormal basis in $\mathcal{F}$ and let v_j $(j \in \mathbb{N})$ be chosen as in Lemma 6. For $\lambda \in \widehat{G}$, let $V(\lambda)$ be the isometric linear operator such that $V(\lambda)e_j = v_j(\lambda)$ $(j \in \mathbb{N}_d)$. Let

$$\Phi_0 := V^*\Phi_P \qquad \text{and} \qquad \Psi_0 := V,$$

where Φ_P was defined in (3).

Lemma 7. *The function Φ_0 belongs to $L^2(F_{\mathbf{Y}};\mathcal{K},\mathcal{F})$ and the function Ψ_0 belongs to $L^2(F_{\mathbf{Y};\Phi_0};\mathcal{F},\mathcal{H})$.*

Proof. Use [12: Theorem 13.2], the weak measurability of $(F'_{\mathbf{XY}}F'^{+1/2}_{\mathbf{Y}})^{\check{}}$, Lemma 6, and [6: Lemma 1] to obtain the weak measurability of the functions $\Phi_0 F'^{1/2}_{\mathbf{Y}}$ and $\Psi_0 F'^{1/2}_{\mathbf{Y};\Phi_0}$. Moreover, by (1),

$$\int_{\widehat{G}} |\Phi_0 F'^{1/2}_{\mathbf{Y}}|^2 d\tau = \int_{\widehat{G}} |V^*(F'_{\mathbf{XY}}F'^{+1/2}_{\mathbf{Y}})^{\check{}}|^2 d\tau \le \int_{\widehat{G}} \mathrm{tr} F'_{\mathbf{X}} d\tau < \infty$$

and

$$\int_{\widehat{G}} |\Psi_0 F'_{\mathbf{Y};\Phi_0}|^2 d\tau = \int_{\widehat{G}} |(F'_{\mathbf{XY}}F'^{+1/2}_{\mathbf{Y}})^{\check{}}|^2 d\tau \le \int_{\widehat{G}} \mathrm{tr} F'_{\mathbf{X}} d\tau < \infty.$$

An appeal to Remark 1 concludes the proof ∎

Theorem 8. *Let ρ be the minimum in* (4). *Then*

$$\rho^2 = \begin{cases} |X_0 - PX_0|^2 & \text{if } d = \infty \\ |X_0 - PX_0|^2 + \sum_{j=d+1}^{\infty} \int_{\widehat{G}} l_j d\tau & \text{if } d < \infty. \end{cases}$$

The minimum is attained for $\Phi_0 = V^*\Phi_P$ and $\Psi_0 = V$.

Proof. For $\Phi \in L^2(F_{\mathbf{Y}};\mathcal{K},\mathcal{F})$ and $\Psi \in L^2(F_{\mathbf{Y};\Phi};\mathcal{F},\mathcal{H})$ we have

$$\left|PX_0 - \int_{\widehat{G}} \Psi\Phi\, dEY_0\right|^2 = \int_{\widehat{G}} \left|(F'_{\mathbf{XY}}F'^{+1/2}_{\mathbf{Y}})^{\check{}} - \Psi\Phi F'^{1/2}_{\mathbf{Y}}\right|^2 d\tau.$$

Moreover, it is not hard to see that $\Psi_0(\lambda)\Phi_0(\lambda)F'_{\mathbf{Y}}(\lambda)^{1/2}$ is equal to the sum of the first d summands of the Schmidt expansion of the operator $(F'_{\mathbf{XY}}(\lambda)F'^{+}_{\mathbf{Y}}(\lambda)^{1/2})^{\check{}}$, if $d < \infty$ and equal to the Schmidt expansion, if $d = \infty$ (cf. [4: Subsection 2.2 of Chapter II]). Thus, Theorem 8 follows from [2: Chapter III/Lemma 6.1] (compare also [2: Chapter III/Theorem 7.1]) ∎

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