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Nonlinear Equations with Operators Satisfying Generalized Lipschitz Conditions in Scales

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Dedicated to my good collaborator Prof. L. von Wolfersdorf

Abstract. By means of the contraction principle we prove existence, uniqueness and stability of solutions for nonlinear equations $u + G_0[D,u] + L(G_1[D,u], G_2[D,u]) = f$ in a Banach space E, where G_0, G_1, G_2 satisfy Lipschitz conditions in scales of norms, L is a bilinear operator and D is a data parameter. The theory is applicable for inverse problems of memory identification and generalized convolution equations of the second kind.

Keywords: *Nonlinear operator equations, nonlinear convolution equations, scales of norms, fixed point theorems, existence, uniqueness and stability of solutions of nonlinear equations*

AMS subject classification: 47 H 15, 45 G 10, 45 D 05

0. Introduction

Recently by the author and L.v. Wolfersdorf global solvability and stability with respect to free terms has been proved for nonlinear convolution equations of the second kind [11, 12]. These results were generalized to abstract equations containing operators which are Lipschitz-continuous in scales of norms [9].

On the other hand, in the most important applications of this theory (see [7, 8, 10]) the operators involved are dependent on the data. In the present paper we generalize the results of the work [9] to the case when the Lipschitz operators depend on a data parameter D and derive stability estimates with respect to D and the free term f. In contrast to [9] the stability results obtained in this paper are global. As in [11, 12] we will use the Banach fixed point theorem in scales of norms.

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1. Equations with Lipschitz operators in scales

We will study the operator equation

$$u + G_0[D,u] + L(G_1[D,u], G_2[D,u]) = f \tag{1}$$

in a Banach space E, where $G_0 \in (\hat{E} \times E \to E)$ and $G_i \in (\hat{E} \times E \to E_i)$ $(i = 1,2)$, with $\hat{E}$ a linear space, E_1 and E_2 are normed spaces, and L is a bilinear operator from $E_1 \times E_2$ into E.

We suppose that the spaces E and E_i are endowed with scales of norms $\|\cdot\|_\sigma$ and $\|\cdot\|_{i,\sigma}$ $(\sigma \geq 0)$, respectively, which satisfy the conditions

$$\kappa(\sigma)\|u\|_0 \leq \|u\|_\sigma \leq \|u\|_0 \qquad (u \in E) \tag{2}$$

and

$$\|v_i\|_{i,\sigma} \leq \|v_i\|_{i,0} \qquad (v_i \in E_i) \tag{3}$$

where

$$\kappa \in C(\mathbb{R}_+ \to \mathbb{R}_+) \qquad (\kappa > 0), \tag{4}$$

the linear space $\hat{E}$ is endowed with a semi-norm $|\cdot|$ and for the operators G_0, G_i and L the assumptions

$$\begin{aligned} &\big\|G_0[D_1,u_1] - G_0[D_2,u_2]\big\|_\sigma \\ &\qquad \leq M_0\big(|D_1|, |D_2|, \|u_1\|_\sigma, \|u_2\|_\sigma, \sigma\big)\big(|D_1 - D_2| + \|u_1 - u_2\|_\sigma\big) \end{aligned} \tag{5}$$

$$\begin{aligned} &\big\|G_i[D_1,u_1] - G_i[D_2,u_2]\big\|_{i,\sigma} \\ &\qquad \leq M_i\big(|D_1|, |D_2|, \|u_1\|_\sigma, \|u_2\|_\sigma\big)\big(|D_1 - D_2| + \|u_1 - u_2\|_\sigma\big) \end{aligned} \tag{6}$$

for $D_1, D_2 \in \hat{E}$ and $u_1, u_2 \in E$ and

$$\|L(v_1,v_2)\|_\sigma \leq N\|v_1\|_{1,\sigma}\|v_2\|_{2,\sigma} \tag{7}$$

$$\|L(v_1,v_2)\|_\sigma \leq \lambda(\sigma)\min\big\{\|v_1\|_{1,0}\|v_2\|_{2,\sigma}; \|v_1\|_{1,\sigma}\|v_2\|_{2,0}\big\} \tag{8}$$

for $v_i \in E_i$ $(i = 1,2)$ hold. Here $N \geq 0$ and the functions M_0, M_i and λ satisfy the conditions

$$M_0 \in C(\mathbb{R}_+^5 \to \mathbb{R}_+), \quad M_0(x_1,\dots,x_4,\sigma) \text{ is increasing in } x_1,\dots,x_4 \tag{9}$$

$$\lim_{\sigma\to\infty} M_0(x_1,\dots,x_4,\sigma) = 0 \quad \text{for any } (x_1,\dots,x_4) \in \mathbb{R}^4 \tag{10}$$

$$M_i \in C(\mathbb{R}_+^4 \to \mathbb{R}_+), \quad M_i(x_1,\dots,x_4) \text{ is increasing in } x_1,\dots,x_4 \tag{11}$$

$$\lambda \in C(\mathbb{R}_+ \to \mathbb{R}_+), \quad \lim_{\sigma\to\infty} \lambda(\sigma) = 0. \tag{12}$$

where as usual $\mathbb{R}_+ = [0,\infty)$.

Theorem. *Let assumptions (2) – (12) be fulfilled. Then equation (1) has a unique solution $u \in E$ for any $D \in \hat{E}$ and $f \in E$. Moreover, for solutions u_1 and u_2 corresponding to data D_1, f_1 and D_2, f_2, respectively, the stability estimate*

$$\|u_1 - u_2\|_0 \le \Lambda(Q_1, Q_2)\big(|D_1 - D_2| + \|f_1 - f_2\|_0\big) \tag{13}$$

holds where

$$Q_i = \Big(|D_i|, \|f_i\|_0, \|G_0[D_i, f_i]\|_0, \|G_1[D_i, f_i]\|_{1,0}, \|G_2[D_i, f_i]\|_{2,0}\Big) \qquad (i = 1,2)$$

and $\Lambda \in C(\mathbb{R}_+^{10} \to \mathbb{R}_+), \Lambda > 0$ and Λ - increasing in $x_1, ..., x_{10}$. If the operators G_i $(i = 0,1,2)$ satisfy the conditions $G_0[0,0] = G_1[0,0] = G_2[0,0] = 0$, then (13) has the simplified form

$$\|u_1 - u_2\|_0 \le \Lambda_1\big(|D_1|, \|f_1\|_0, |D_2|, \|f_2\|_0\big)\big(|D_1 - D_2| + \|f_1 - f_2\|_0\big), \tag{14}$$

where $\Lambda_1 \in C(\mathbb{R}_+^4 \to \mathbb{R}_+)$, $\Lambda_1 > 0$ and Λ_1 is increasing in $x_1, ..., x_4$.

The proof of Theorem is given in the next section.

The main area of applications of equation (1) and the related Theorem are inverse problems for determining memory kernels in heat flow [7], viscoelasticity [5, 8] and thermo- and poroviscoelasticity [10]. All these problems admit reductions to integral equations or systems of integral equations of the form

$$m(t) + G_0[D, m](t) + K[D, m] * m(t) = f(t) \qquad (t \in [0, T])$$

in a Banach space $E = X^n$, where X a functional space over the interval $[0, T]$ and $n \ge 1$. Here m is the memory kernel, or a vector of independent memory kernels, and

$$G_0[D, \cdot] \in (E \to E) \qquad \text{and} \qquad K[D, \cdot] \in (E \to X^{n \times n})$$

are operators of m depending on the data vector D of the inverse problem. The bilinear operator L in these cases is the convolution operator

$$L(v_1, v_2)(t) \equiv v_1 * v_2(t) = \int_0^t v_1(t - \tau) v_2(\tau)\, d\tau$$

and the scales of norms are defined using exponential weights of the form $e^{-\sigma t}$ $(\sigma \ge 0)$.

The technique of scales of norms enables to formulate statements about global existence, uniqueness and stability of solutions of these nonlinear integral equations of the second kind.

It is remarkable that the method of weighted norms applies to inverse problems of memory identification also in the case if one makes use of an approach different from the reduction to integral equations (e.g., a priori estimates [2, 3], the theory of semigroups [1], etc.). This is due to the fact that these problems, if they are constructed from linear constitutive laws, contain only nonlinearities of convolution type.

Other areas of application of the theory of the present paper are equations of autoconvolution type [1, 6, 11] arising in stochastics and spectroscopy as well as more theoretical examples of equations involving various types of generalized convolutions. Concerning the latter examples we refer the reader to the previous papers of the author and L. v. Wolfersdorf [9, 11, 12].

2. Proof of Theorem

The proof uses the contraction principle in balls

$$B_{\rho,\sigma}(w) = \{u \in E : \|u - w\|_\sigma \leq \rho\} \qquad (\rho > 0,\, \sigma \geq 0,\, w \in E). \tag{15}$$

Step 1. At first we show that the auxiliary equation

$$g + G_0[D, g] = f \tag{16}$$

has a solution in $B_{R,\sigma}(f)$, where $R = 2\|G_0[D,f]\|_0$ and σ is chosen large enough. We define the operator $A_1 g = f - G_0[D,g]$. Then equation (16) reads $g = A_1 g$. In view of (2), (5) and the monotonicity of M_0 we have

$$\begin{aligned} &\|A_1 g - f\|_\sigma \\ &\quad \leq \|G_0[D,g] - G_0[D,f]\|_\sigma + \|G_0[D,f]\|_\sigma \\ &\quad \leq M_0\big(|D|, |D|, \|g - f\|_\sigma + \|f\|_0, \|f\|_0, \sigma\big)\, \|g - f\|_\sigma + \|G_0[D,f]\|_0 \end{aligned} \tag{17}$$

and

$$\begin{aligned} &\|A_1 g_1 - A_1 g_2\|_\sigma \\ &\quad = \|G_0[D,g_1] - G_0[D,g_2]\|_\sigma \\ &\quad \leq M_0\big(|D|, |D|, \|g_1 - f\|_\sigma + \|f\|_0, \|g_2 - f\|_\sigma + \|f\|_0, \sigma\big)\, \|g_1 - g_2\|_\sigma. \end{aligned} \tag{18}$$

In the case $g, g_1, g_2 \in B_{R,\sigma}(f)$ inequalities (17) and (18) yield

$$\begin{aligned} \|A_1 g - f\|_\sigma &\leq M_0\big(|D|, |D|, R + \|f\|_0, \|f\|_0, \sigma\big) R + \frac{R}{2} \\ \|A_1 g_1 - A_1 g_2\|_\sigma &\leq M_0\big(|D|, |D|, R + \|f\|_0, R + \|f\|_0, \sigma\big)\, \|g_1 - g_2\|_\sigma. \end{aligned}$$

By the continuity of M_0 and the limit condition (10) there exists a quite large $\sigma = \sigma_0$ depending continuously on $\|f\|_0, |D|$ and R, so that

$$A_1 B_{R,\sigma_0}(f) \subseteq B_{R,\sigma_0}(f), \qquad A_1 \text{ is a contraction in } B_{R,\sigma_0}(f).$$

Therefore, equation (16) has for every $f \in E$ a unique solution g in the ball $B_{R,\sigma_0}(f)$.

Next we derive some estimates for the solution g of equation (16). To this end we introduce the vector

$$Q = \Big(|D|, \|f\|_0, \|G_0[D,f]\|_0, \|G_1[D,f]\|_{1,0}, \|G_2[D,f]\|_{2,0}\Big) \in \mathbb{R}^5_+.$$

By definition (15) of the ball $B_{R,\sigma_0}(f)$ the solution g satisfies the inequality

$$\|g - f\|_{\sigma_0} \leq R = 2\|G_0[D,f]\|_0.$$

Thus, by means of (2) and (4) we obtain

$$\|g\|_0 \le \mu_0(Q) \tag{19}$$

where

$$\mu_0(Q) = 2\|G_0[D,f]\|_0\,(\kappa(\sigma_0(Q)))^{-1} + \|f\|_0$$

is a positive continuous function of Q. Further, using (2) and (6) we can estimate

$$\begin{aligned}\|G_i[D,g]\|_{i,0} &\le \|G_i[D,g]-G_i[D,f]\|_{i,0} + \|G_i[D,f]\|_{i,0}\\ &\le M_i(|D|,|D|,\|g\|_0,\|f\|_0)\,\|g-f\|_0 + \|G_i[D,f]\|_{i,0}.\end{aligned}$$

Applying here (11) and (19) we obtain

$$\|G_i[D,g]\|_{i,0} \le \mu_i(Q) \qquad (i=1,2) \tag{20}$$

where

$$\mu_i(Q) = 2M_i\big(|D|,|D|,\mu_0(Q),\|f\|_0\big)\frac{\|G_0[D,f]\|_0}{\kappa(\sigma_0(Q))} + \|G_i[D,f]\|_{i,0}$$

are again positive continuous functions of Q.

Step 2. Let us return to equation (1). In view of (16) we write it in the operator form $u = Au$, where

$$Au = g - L(G_1[D,u],G_2[D,u]) + G_0[D,g] - G_0[D,u].$$

We are going to show that equation (1) has a solution in the ball $B_{\rho,\sigma}(g)$, where ρ is small enough and σ is large enough.

Observing estimates (2), (3) and (5) - (8), the bilinearity of L as well as the monotonicity of the functions M_0, M_1 and M_2 we obtain

$$\begin{aligned}&\|Au-g\|_\sigma\\ &\quad\le \big\|L\big(G_1[D,u]-G_1[D,g],G_2[D,u]-G_2[D,g]\big)\big\|_\sigma\\ &\qquad+\big\|L\big(G_1[D,u]-G_1[D,g],G_2[D,g]\big)\big\|_\sigma\\ &\qquad+\big\|L\big(G_1[D,g],G_2[D,u]-G_2[D,g]\big)\big\|_\sigma\\ &\qquad+\big\|L\big(G_1[D,g],G_2[D,g]\big)\big\|_\sigma\\ &\qquad+\big\|G_0[D,g]-G_0[D,u]\big\|_\sigma\\ &\quad\le NM_1\big(|D|,|D|,\|u-g\|_\sigma+\|g\|_0,\|g\|_0\big)\\ &\qquad\times M_2\big(|D|,|D|,\|u-g\|_\sigma+\|g\|_0,\|g\|_0\big)\,\|u-g\|_\sigma^2\\ &\qquad+\lambda(\sigma)\sum_{i=1}^{2} M_i\big(|D|,|D|,\|u-g\|_\sigma+\|g\|_0,\|g\|_0\big)\,\|G_{j_i}[D,g]\|_{j_i,0}\,\|u-g\|_\sigma\\ &\qquad+\lambda(\sigma)\|G_1[D,g]\|_{1,0}\|G_2[D,g]\|_{2,0}\\ &\qquad+M_0\big(|D|,|D|,\|u-g\|_\sigma+\|g\|_0,\|g\|_0,\sigma\big)\,\|u-g\|_\sigma\end{aligned} \tag{21}$$

and

$$
\begin{aligned}
\|Au_1 - Au_2\|_\sigma
&\le \Big\|L(G_1[D,u_1] - G_1[D,u_2], G_2[D,u_1] - G_2[D,g] + G_2[D,g])\Big\|_\sigma \\
&\quad + \Big\|L(G_1[D,u_2] - G_1[D,g] + G_1[D,g], G_2[D,u_1] - G_2[D,u_2])\Big\|_\sigma \\
&\quad + \big\|G_0[D,u_1] - G_0[D,u_2]\big\|_\sigma \\
&\le \Bigg\{ \sum_{i=1}^{2} M_i\Big(|D|,|D|,\|u_1 - g\|_\sigma + \|g\|_0, \|u_2 - g\|_\sigma + \|g\|_0\Big) \\
&\quad \times \Big[N M_{j_i}\Big(|D|,|D|,\|u_i - g\|_\sigma + \|g\|_0, \|g\|_0\Big)\|u_i - g\|_\sigma + \lambda(\sigma)\|G_{j_i}[D,g]\|_{j_i,0}\Big] \\
&\quad + M_0\Big(|D|,|D|,\|u_1 - g\|_\sigma + \|g\|_0, \|u_2 - g\|_\sigma + \|g\|_0, \sigma\Big)\Bigg\}\|u_1 - u_2\|_\sigma
\end{aligned}
\tag{22}
$$

where j_i is defined so that $j_1 = 2$ and $j_2 = 1$. Further, assuming that $u, u_1, u_2 \in B_{\rho,\sigma}(g)$ and applying estimates (19) and (20) in (21) and (22) we have

$$
\begin{aligned}
\|Au - g\|_\sigma
&\le \rho^2 N M_1(|D|,|D|,\rho + \mu_0(Q), \mu_0(Q)) M_2(|D|,|D|,\rho + \mu_0(Q), \mu_0(Q)) \\
&\quad + \rho\lambda(\sigma) \sum_{i=1}^{2} M_i(|D|,|D|,\rho + \mu_0(Q), \mu_0(Q))\mu_{j_i}(Q) \\
&\quad + \lambda(\sigma)\mu_1(Q)\mu_2(Q) + \rho M_0(|D|,|D|,\rho + \mu_0(Q), \mu_0(Q), \sigma)
\end{aligned}
\tag{23}
$$

and

$$
\begin{aligned}
\|Au_1 - Au_2\|_\sigma
&\le \Bigg\{ \sum_{i=1}^{2} M_i(|D|,|D|,\rho + \mu_0(Q), \rho + \mu_0(Q)) \\
&\quad \times \Big[\rho N M_{j_i}(|D|,|D|,\rho + \mu_0(Q), \mu_0(Q)) + \lambda(\sigma)\mu_{j_i}(Q)\Big] \\
&\quad + M_0(|D|,|D|,\rho + \mu_0(Q), \rho + \mu_0(Q), \sigma)\Bigg\}\|u_1 - u_2\|_\sigma.
\end{aligned}
\tag{24}
$$

By virtue of the limit conditions (10) and (12) and the monotonicity of M_0, M_1 and M_2 there exist a quite small but positive $\rho = \rho_1$ and quite large $\sigma = \sigma_1$ so that

$$
AB_{\rho_1,\sigma}(g) \subseteq B_{\rho_1,\sigma}(g), \qquad A \text{ is a contraction in } B_{\rho_1,\sigma}(g)
$$

for every $\sigma \ge \sigma_1$.

Therefore, equation (1) has a unique solution in every ball $B_{\rho_1,\sigma}(g)$, where $\sigma \ge \sigma_1$. Particularly, this proves the existence assertion of Theorem. Since the functions

μ_0, μ_1, μ_2, M_0, M_1, M_2 and λ are continuous, the quantities ρ_1 and σ_1 depend also continuously on Q.

Step 3. Let us show the uniqueness of the solution of equation (1) in the whole space E. Suppose that $u_1 \in E$ and $u_2 \in E$ are two arbitrary solutions of (1). Then from (1) in view of (2), (3), (5), (8) and (16) we obtain

$$\begin{aligned}\|u_i - g\|_\sigma &\le \big\|L(G_1[D,u_i], G_2[D,u_i])\big\|_\sigma \\ &\quad + \big\|G_0[D,u_i] - G_0[D,g]\big\|_\sigma \\ &\le \lambda(\sigma)\|G_1[D,u_i]\|_{1,0}\|G_2[D,u_i]\|_{2,0} \\ &\quad + M_0\big(|D|,|D|,\|u_i\|_0,\|g\|_0,\sigma\big)\,\|u_i - g\|_0.\end{aligned}$$

Due to the limit conditions (10) and (12) the relations $\lim_{\sigma\to\infty}\|u_i - g\|_\sigma = 0$ hold for $i = 1,2$. Thus, there exists a quite large $\sigma \ge \sigma_1$ so that $\|u_i - g\|_\sigma \le \rho_1$ for $i = 1,2$. Observing definition (15) we see that u_1 and u_2 belong to a ball $B_{\rho_1,\sigma}(g)$, where the uniqueness has already been show. Consequently, $u_1 = u_2$.

Step 4. Finally, let us derive the stability estimates (13) and (14). To this end we need a bound for $\|u\|_0$ in terms Q. In the second part of the proof we have shown that the solution of equation (1) belongs to the ball $B_{\rho_1,\sigma_1}(g)$. Thus, by definition (15) we get the inequality $\|u - g\|_{\sigma_1} \le \rho_1$. Using here (2) and (19) we obtain

$$\|u\|_0 \le \mu(Q) \tag{25}$$

where $\mu(Q) = \rho_1(Q)\kappa(\sigma_1(Q))^{-1} + \mu_0(Q)$.

Since ρ_1, σ_1, κ and μ_0 are positive continuous functions of Q, μ is also positive and continuous.

Further we denote the solutions of equation (16) coresponding the data D_i and f_i by g_i $(i = 1,2)$, respectively, and subtract the equations (1) with D_1, f_1 and D_2, f_2. We get the relation

$$\begin{aligned}u_1 - u_2 = f_1 - f_2 &+ G_0[D_2,u_2] - G_0[D_1,u_1] \\ &+ L\Big(G_1[D_2,u_2] - G_1[D_1,u_1], G_2[D_2,u_2] - G_2[D_2,g_2] + G_2[D_2,g_2]\Big) \\ &+ L\Big(G_1[D_1,u_1] - G_1[D_1,g_1] + G_1[D_1,g_1], G_2[D_2,u_2] - G_2[D_1,u_1]\Big).\end{aligned}$$

By means of assumptions (2), (5), (6), (8) and the monotonicity of M_0, M_1, M_2 we derive the estimate

$$\begin{aligned}&\|u_1 - u_2\|_\sigma \\ &\quad\le M_0\big(|D_2|,|D_1|,\|u_2\|_0,\|u_1\|_0,\sigma\big)\big(|D_1 - D_2| + \|u_1 - u_2\|_\sigma\big) \\ &\qquad + \lambda(\sigma)\sum_{i=1}^{2} M_i\big(|D_{j_i}|,|D_i|,\|u_{j_i}\|_0,\|u_i\|_0\big) \\ &\qquad \times \Big[M_{j_i}\big(|D_{j_i}|,|D_{j_i}|,\|u_{j_i}\|_0,\|g_{j_i}\|_0\big)\,\|u_{j_i} - g_{j_i}\|_0 + \|G_{j_i}[D_{j_i},g_{j_i}]\|_{j_i,0}\Big] \\ &\qquad \times \big(|D_1 - D_2| + \|u_1 - u_2\|_\sigma\big) + \|f_1 - f_2\|_0.\end{aligned}$$

Applying here inequalities (19), (20) and (25) we have

$$
\begin{aligned}
&\|u_1 - u_2\|_\sigma \\
&\le \Big\{ M_0\big(|D_2|, |D_1|, \mu(Q_2), \mu(Q_1), \sigma\big) \\
&+ \lambda(\sigma) \sum_{i=1}^{2} M_i\big(|D_{j_i}|, |D_i|, \mu(Q_{j_i}), \mu(Q_i)\big) \\
&\times \Big[M_{j_i}\big(|D_{j_i}|, |D_{j_i}|, \mu(Q_{j_i}), \mu_0(Q_{j_i})\big)\big(\mu(Q_{j_i}) + \mu_0(Q_{j_i})\big) \\
&+ \mu_{j_i}(Q_{j_i})\Big] \Big\} \|u_1 - u_2\|_\sigma \\
&+ \Big\{ M_0\big(|D_2|, |D_1|, \mu(Q_2), \mu(Q_1), \sigma\big) \\
&+ \lambda(\sigma) \sum_{i=1}^{2} M_i\big(|D_{j_i}|, |D_i|, \mu(Q_{j_i}), \mu(Q_i)\big) \\
&\times \Big[M_{j_i}\big(|D_{j_i}|, |D_{j_i}|, \mu(Q_{j_i}), \mu(Q_{j_i})\big)\big(\mu(Q_{j_i}) + \mu_0(Q_{j_i})\big) \\
&+ \mu_{j_i}(Q_{j_i})\Big] \Big\} |D_1 - D_2| + \|f_1 - f_2\|_0
\end{aligned}
\tag{26}
$$

where, as before, $j_1 = 2$ and $j_2 = 1$. Due to the limit conditions (10) and (12) there exists a quite large σ_3 so that the coefficient of $\|u_1 - u_2\|_\sigma$ in the right-hand side of (26) becomes less than one if $\sigma = \sigma_3$. Hence, there holds the relation

$$\|u_1 - u_2\|_{\sigma_3} \le \mu_3(Q_1, Q_2)\big(|D_1 - D_2| + \|f_1 - f_2\|_0\big) \tag{27}$$

with some coefficient μ_3. Since μ, μ_0, μ_1, μ_2, M_0, M_1, M_2 and λ are continuous, the quantities σ_3 and μ_3 are positive continuous functions of Q_1 and Q_2. Applying in (27) the left inequality (2) we derive the stability estimate (13) with

$$\Lambda(Q_1, Q_2) = \mu_3(Q_1, Q_2)\kappa(\sigma_3(Q_1, Q_2))^{-1}.$$

Since μ_3, σ_3 and κ are positive and continuous functions of Q_1 and Q_2, the coefficient Λ is also positive and continuous. Without loss of generality we may assume $\Lambda(x_1, \ldots, x_{10})$ to be increasing in each of its arguments.

In order to prove estimate (14) we observe that inequalities (5) and (6) in view of the assumptions $G_i[0,0] = 0$ $(i = 0,1,2)$ yield the following bounds for components of the vectors Q_j:

$$
\begin{aligned}
\|G_0[D_j, f_j]\|_0 &\le M_0\big(|D_j|, 0, \|f_j\|_0, 0, 0\big)\big(|D_j| + \|f_j\|_0\big) \\
\|G_i[D_j, f_j]\|_{i,0} &\le M_i\big(|D_j|, 0, \|f_j\|_0, 0\big)\big(|D_j| + \|f_j\|_0\big) \quad (i = 1,2).
\end{aligned}
$$

Using these relations in (13) we obtain assertion (14). Theorem is proved ∎

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