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A slight modification to: On some Improperly Posed Problem for a Degenerate Nonlinear Parabolic Equation

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Abstract.

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The conclusion (2.5) of our Theorem in [1] is revised as follows :

Theorem. *Under the assumptions of our Theorem in* [1],

$$\begin{aligned}
&\int_{D_{\theta\eta}} \left(u^2 + |\nabla_x u|^2 + \alpha |u|^{2\alpha} u_t^2\right) dxdt \\
&\qquad \le C \frac{\kappa^2}{\delta^{10}} (1+\alpha)^2 (1+|\gamma|)(1+M^{2\alpha})^2 \\
&\qquad\quad \times \frac{1}{(1-\theta)^3} \exp\left[\frac{2(1-\theta'^2)}{\theta'^2(\eta^2-1)} \log\varepsilon\right] \exp\left[\frac{2(\theta'^2\eta^2-1)}{\theta'^2(\eta^2-1)} \log M\right]
\end{aligned}$$

where C does not depend on θ, ε and M.

Though there is no mistake in [1], the essential term

$$\alpha \int_{D_{\theta\eta}} |u|^{2\alpha} u_t^2 dxdt$$

in the above estimate has dropped in our previous theorem. So we add it as above. Instead the coefficients on the right-hand side of (2.5) have to be changed as follows :

$$\begin{aligned}
\delta^8 &\longrightarrow \delta^{10} \\
1+\alpha &\longrightarrow (1+\alpha)^2 \\
1+M^{2\alpha} &\longrightarrow (1+M^{2\alpha})^2 \\
(1-\theta)^2 &\longrightarrow (1-\theta)^3.
\end{aligned}$$

In order to prove the above new theorem, we need to replace our Proposition in [1] with the following

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Proposition. *Under the assumptions of our Proposition in* [1],

$$\frac{\alpha}{1+\alpha}\frac{1}{\delta^2\tau}\int_{D_\eta}|u|^{2\alpha}u_t^2dxdt+\int_{D_\eta}|\nabla_x v|^2dxdt+\frac{1}{1+\alpha}\Big(\frac{\delta}{\kappa}\Big)^2\int_{D_\eta}v^2dxdt$$
$$\le C\frac{1+M^{2\alpha}}{\delta^2}\int_{\partial D_\eta}\big(v_t^2+|\nabla_x v|^2+\tau^2v^2+|\gamma|e^{\beta\tau\varphi}|v|^{2-\beta}\big)d\sigma$$

where the constant C does not depend on τ and M.

Thus our Proposition in [1] is revised too as above. In [1] the first term

$$\frac{1}{1+\alpha}\frac{1}{\delta^2\tau}\int_{D_\eta}|u|^{2\alpha}u_t^2dxdt$$

has dropped. Our revised theorem is derived from the above proposition in the same manner as in [1]. In particular, we have the inequality

$$\int_{D_{\theta\eta}}e^{2\tau\varphi}\big(u^2+|\nabla u|^2+\alpha|u|^{2\alpha}u_t^2\big)\,dxdt$$
$$\le C\frac{\kappa^2}{\delta^4}\tau^3(1+\alpha)^2(1+|\gamma|)(1+M^{2\alpha})^2\int_{\partial D_\eta}e^{2\tau\varphi}\big(u^2+|\nabla u|^2+u_t^2+|u|^{2-\beta}\big)d\sigma.$$

instead of (4.1): If we follow carefully the proof of our proposition in [1], we can conclude the new proposition along the previous line. In the different point from our previous method, we make the most of I_2 in (3.2), which was neglected before. We sketch the outline of our new proof as follows:

We leave I_2 and add it to the right-hand side of (3.21). First we use the inequality

$$2\alpha\tau|(\nabla\phi\cdot\nabla v,e^{\tau\varphi}f)|\le 2\alpha\tau^2(1,(\nabla\varphi\cdot\nabla v)^2)+\frac{1}{2}\alpha\|e^{\tau\varphi}f\|^2$$

whose right-hand side was $4\alpha\tau^2(1,(\nabla\varphi\cdot\nabla v)^2)+\frac{1}{4}\alpha\|e^{\tau\varphi}f\|^2$ in [1]. Then the half of the term $4\alpha\tau^2(1,(\nabla\varphi\cdot\nabla v)^2)$ in (3.21) is left, though it vanished in the previous proof. Further we use

$$\frac{\alpha}{2+\alpha}\|e^{-\alpha\tau\varphi}|v|^\alpha v_t\|^2\le I_2+2\alpha\tau^2(1,(\nabla\varphi\cdot\nabla v)^2)$$

which is due to the trivial inequality $\frac{\alpha}{2+\alpha}B^2\le(A+B)^2+\frac{\alpha}{2}A^2$.

Thus we can add the new term $\frac{\alpha}{2+\alpha}\|e^{-\alpha\tau\varphi}|v|^\alpha v_t\|$ to the right-hand side of (3.22) and (3.23), respectively. We have finally the inequality

$$\alpha(|u|^{2\alpha},v_t^2)+(\alpha+1)\delta^2\tau(1,|\nabla v|^2)+\Big(\frac{\delta^2}{\kappa}\Big)^2\tau^3(1,v^2)\le C\big(|J_4|+(1+\alpha)\|e^{\tau\varphi}f\|^2\big)$$

instead of (3.32) from which we obtain the conclusion of our new p roposition.

References

[1] Hayasida, K.: *On some improperly posed problem for a degenerate nolinear parabolic equation.* Z. Anal. Anw. 19 (2000), 395 – 413.

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