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Comparison of Non-Commutative 2- and p-Summing Operators from $B(l_2)$ into OH

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Abstract. In the theory of p-summing operators studied by Pietsch we know that $\pi_2(C(K), H) = \pi_p(C(K), H)$ for any Hilbert space H and any p such that $2 < p < +\infty$. In this paper we prove that this equality is not true in the same notion generalized by Junge and Pisier to operator spaces, i.e. $\pi_{l_2}(B(l_2), OH)$ $(= \pi_2^0(B(l_2), OH)) \neq \pi_{l_p}(B(l_2), OH)$.

Keywords: *Operator spaces, completely bounded operators, p−summing operators*

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1. Introduction

In the recent theory of operator spaces (or non-commutative Banach spaces) developed by [1 - 6, 10 - 12], bounded operator is replaced by completely bounded operator, isomorphism by complete isomorphism and Banach space by operator space. Precisely, we view in this new category every Banach space as a subspace of $B(H)$ for some Hilbert space H ($B(H)$ is the Banach space of all bounded linear operators on H) which is non-commutative, instead of viewing them as a subspace of $C(K)$ (the space of all continuous functions on a compact K) which is commutative. The abstract characterization given in [12] signed the beginning of this theory. In [10] Pisier constructed the operator Hilbert space OH (i.e. the unique space verifying $\overline{OH^\star} = OH$ completely isometrically as in the case of Banach spaces because there are Hilbert spaces in this category which are non completely isometrically) and generalized in [11] (also Junge) the notion of p-summing operators to the non-commutative case.

In this paper we show that

$$\pi_{l_2}(B(l_2), OH) \neq \pi_{l_p}(B(l_2), OH)$$

for all p in $(2, \infty)$. In the case of completely p-summing operators the problem is raised in [11: Problem 10.2] and is still open (i.e. is every completely operator $u : B(H) \to OH$ necessarily completely 2-summing?). This question, which called

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the little Grothendieck's theorem in the case of Banach spaces, is the origin and the inspiration of this work. Le Merdy proved in [8: Theorem 4.2] that

$$cb(B(l_2)^\star, OH) \neq \pi_2^0(B(l_2)^\star, OH).$$

Let H be a Hilbert space and $X \subset B(H)$ be a closed subspace. For all $n \geq 1$ we denote by $M_n(X) = M_n \otimes X$ the space of $n \times n$ matrices $(x_{ij})_{1 \leq i,j \leq n}$ with entries $x_{ij} \in X$ equipped with the norm induced by the space $M_n(B(H)) = B(l_2^n(H)) = B(l_2^n \otimes_2 H)$ ($l_2^n \otimes_2 H$ is the Hilbert-space tensor product of l_2^n and H).

Definition 1.1. An *operator space* X is a closed subspace of $B(H)$ for some Hilbert space H.

Let X be a vector space. If for each $n \in \mathbb{N}$ there is a norm $\|\cdot\|_n$ on $M_n(X)$, then the family of norms $\{\|\cdot\|_n\}_{n\geq 1}$ is called an L_∞-*matricial structure* on X if

(i) $\|axb\|_n \leq \|a\|_{M_n(\mathbb{C})}\|x\|_n\|b\|_{M_n(\mathbb{C})}$

(ii) $\|x \oplus y\|_{n+m} = \left\|\left(\begin{smallmatrix} x & 0 \\ 0 & y \end{smallmatrix}\right)\right\|_{n+m} = \max\{\|x\|_n, \|y\|_m\}$

for all a, b in $M_n(\mathbb{C}) = B(l_2^n), x \in M_n(X)$ and $y \in M_m(X)$. We say that X is L_∞-*matricially normed* if it is equipped with an L_∞-matricial structure (which we suppose complete). Ruan proved in [12: Theorem 3.1] and simplified (with Effros) in [6] an important theorem which is the matricial norm characterization for operator spaces. This theorem sais that for any L_∞-matricial structure on a vector space X there is a Hilbert space H and an embedding of X into $B(H)$ such that for all $n \geq 1$ the norm $\|\cdot\|_n$ on $M_n(X)$ coincides with the norm induced by the space $B(l_2^n(H))$. In other words, he has given an abstract characterization of operator spaces.

Definition 1.2. Let H and K be Hilbert spaces, and let $X \subset B(H)$ and $Y \subset B(K)$ be two operator spaces. A linear map $u : X \to Y$ is *completely bounded* if the maps

$$u_n : M_n(X) \to M_n(Y), \quad (x_{ij})_{1\leq i,j\leq n} \to (u(x_{ij}))_{1\leq i,j\leq n}$$

are uniformly bounded for $n \in \mathbb{N}$, i.e. $\sup_{n\geq 1}\|u_n\| < +\infty$. In this case we put $\|u\|_{cb} = \sup_{n\geq 1}\|u_n\|$ and we denote by $cb(X,Y)$ the Banach space of all completely bounded maps from X into Y which is also an operator space because $M_n(cb(X,Y)) = cb(X, M_n(Y))$ (see [3, 5]). We denote also by $X \otimes_{\min} Y$ the subspace of $B(H \otimes_2 K)$ with induced norm.

Let H be a Hilbert space. We denote by $S_p(H)$ $(1 \leq p < \infty)$ the Banach space of all compact operators $u : H \to H$ such that $\mathrm{Tr}\,(|u|^p) < \infty$, equipped with the norm

$$\|u\|_{S_p(H)} = \left(\mathrm{Tr}\,(|u|^p)\right)^{\frac{1}{p}}.$$

If $H = l_2$ or $H = l_2^n$, we denote simply $S_p(l_2)$ by S_p or $S_p(l_2^n)$ by S_p^n, respectively. We denote also by $S_\infty(H)$ and S_∞ the Banach spaces of all compact operators equipped with the norm induced by $B(H)$ and $B(l_2)$, respectively, and by S_∞^n the space $B(l_2^n)$. Recall that if $\frac{1}{p} = \frac{1}{q} + \frac{1}{r}$ for $1 \leq p, q, r < +\infty$, then $u \in B_{S_p(H)}$ if and only if there are $u_1 \in B_{S_q(H)}$ and $u_2 \in B_{S_r(H)}$ such that $u = u_1 u_2$, where $B_{S_p(H)}$ is the closed unit ball of $S_p(H)$.

Before continuing our notation we will briefly mention some properties concerning completely bounded operators. We recall that OH is homogeneous, in other words, every bounded linear operator $u : H \to OH$ is completely bounded and

$$\|u\| = \|u\|_{cb}. \tag{1.1}$$

Note also that S_2 is completely isometric to $OH \times OH$. We denote by OH_n the n-dimensional version of the Hilbert operator space OH. If now S_2^N $(N \in \mathbb{N})$ is equipped with the operator-space structure OH_{N^2}, then for any linear map $T : S_2^N \to OH_n$ we have by homogeneity of OH

$$\|T\| = \|T\|_{cb}. \tag{1.2}$$

Finally, let us recall the last property. Let Y be an operator space such that $Y \subset A \subset B(H)$, A a commutative C^*-algebra, and let X be an arbitrary operator space. Then, for all bounded linear operators $u : X \to Y$,

$$\|u\| = \|u\|_{cb}. \tag{1.3}$$

Let now X be an operator space. As usual we denote by $l_p(X)$ and $l_p^n(X)$ the spaces of infinite sequences $\{x_1, ..., x_n, ...\}$ and finite sequences $\{x_1, ..., x_n\}$ in X equipped with the norm $(\sum_{n=1}^\infty \|x_n\|^p)^{\frac{1}{p}}$ and $(\sum_{i=1}^n \|x_i\|^p)^{\frac{1}{p}}$, respectively, which become operator spaces. Let now $S_p[X]$ (for more details see [11: p. 10 and Theorem 1.5]) and $S_p^n[X]$ be subspaces of $M_\infty(X)$ and $M_n(X)$ with norms

$$\|u\|_{S_p[X]} = \inf_{u=avb} \|a\|_{S_{2p}} \|v\|_{M_\infty(X)} \|b\|_{S_{2p}}$$
$$\|u\|_{S_p^n[X]} = \inf_{u=avb} \|a\|_{S_{2p}^n} \|v\|_{M_n(X)} \|b\|_{S_{2p}^n}$$

respectively, where

$$M_\infty(X) = \Big\{ u = (u_{ij})_{1 \le i,j \le +\infty} : \|(u_{ij})_{1 \le i,j \le n}\|_{M_n(X)} \le K \quad (n \ge 1) \Big\}$$

and $\|(u_{ij})_{1 \le i,j \le \infty}\|_{M_\infty(X)} = \inf K$, which is a subspace of $B(l_2 \otimes_2 H)$.

2. Non-commutative p-summing operators

We first give the following definition which was introduced in [11: p. 31].

Definition 2.1. Let $1 \le p < \infty$, let X and Y be operator spaces, and let $u : X \to Y$ be a linear operator. We will say that u is *completely p-summing* if there is a constant $C > 0$ such that, for all $n \ge 1$ and all $(x_{ij})_{1 \le i,j \le n} \in M_n(X)$, $\|u(x_{ij})\|_{S_p^n[X]} \le C \|(x_{ij})\|_{S_p^n \otimes_{\min} X}$.

We will denote by $\pi_p^0(u)$ the smallest constant with this property and by $\pi_p^0(X, Y)$ the space of all completely p-summing operators equipped with the norm $\pi_p^0(\cdot)$ for which it becomes a Banach space.

The classical definition in the sens of Pietsch [9] is the following: If X and Y are Banach spaces, an operator $u : X \to Y$ is absolutely p-summing if there exists a constant $C > 0$ such that, for all $n \geq 1$ and for all $\{x_i\}_{1\leq i\leq n} \subset X$, $\|u(x_i)\|_{l_p^n(X)} \leq C\|(x_i)\|_{l_p^n \check{\otimes} X}$ where $l_p^n \check{\otimes} X$ is the injective tensor product.

After this outline and to facilitate the comprehension we will use a definition due to Junge intermediate between absolutely p-summing operators in the case of Banach spaces and completely p-summing in the case of operator spaces. All these definitions are rejoining on certain operator spaces, particularly those which interest us.

The next definition is due to Junge [7].

Definition 2.2. Let H be a Hilbert space, let $X \subset B(H)$ be an operator space, and let $u : X \to Y$ be a linear operator from X into a Banach space Y. We will say that u is l_p-*summing* $(1 \leq p < +\infty)$ if there is a constant $C > 0$ such that for all finite sequences $\{x_i\}_{1\leq i\leq n}$ in X

$$\left(\sum_{i=1}^{n} \|u(x_i)\|^p\right)^{\frac{1}{p}} \leq C \sup_{a,b\in B^+_{S_{2p}}} \left(\sum_{i=1}^{n} \|ax_ib\|^p_{S_p(H)}\right)^{\frac{1}{p}}.$$

We denote by $\pi_{l_p}(u)$ the smallest constant C for which this holds and by $\pi_{l_p}(X,Y)$ the space of all l_p-summing operators with the norm $\pi_{l_p}(\cdot)$ which becomes a Banach space. We can show that

$$\sup_{a,b\in B_{S_{2p}(H)}} \left(\sum_{n=1}^{\infty} \|ax_ib\|^p_{S_p(H)}\right)^{\frac{1}{p}} = \|\{x_i\}\|_{l_p^n\otimes_{\min} X} = \sum_{i=1}^{n} \|e_i \otimes x_i\|_{cb(l_q^n,X)} \tag{2.1}$$

where q is the conjugate of p and $\{e_i\}_{1\leq i\leq n}$ is the canonical basis of l_q^n. By (2.1) Definition 2.2 is equivalent to the following: For all $n \in \mathbb{N}$, $\{x_i\}_{1\leq i\leq n} \in X$ and $T \in cb(l_q^n, X)$ such that $T(e_i) = x_i$,

$$\left(\sum_{i=1}^{n} \|uT(e_i)\|^p\right)^{\frac{1}{p}} \leq C\|T\|_{cb}. \tag{2.2}$$

The following remark will be needed to prove Lemma 2.3.

Remark 1. Let $C > 0$ be a constant. Then u is l_p-summing and $\pi_{l_p}(u) \leq C$ if and only if

$$\left\|I \otimes u : l_p^n \otimes_{\min} X \to l_p^n(Y)\right\| \leq C \tag{2.3}$$

for all $n \geq 1$.

Remark 2.

1) Let E and X be operator spaces and let Y and F be Banach spaces. Let $E \xrightarrow{v} X \xrightarrow{u} Y \xrightarrow{w} F$ such that v is completely bounded, u is l_p-summing and w is a bounded operator. Then $\pi_{l_p}(wuv) \leq \|w\|\pi_{l_p}(u)\|v\|_{cb}$.

2) Let X and Y be operator spaces and consider u in $\pi_p^0(X,Y)$. Then $u \in \pi_{l_p}(X,Y)$ and $\pi_{l_p}(u) \leq \pi_p^0(u)$.

3) Let X be an operator space, Y a Banach space, and consider u in $\pi_p(X,Y)$. Then $u \in \pi_{l_p}(X,Y)$ and $\pi_{l_p}(u) \le \pi_p(u)$.

4) Let $X \subset A \subset B(H)$ with A a commutative C^*-algebra and Y be operator spaces. Then by (1.3) and (2.2) $\pi_{l_p}(X,Y) = \pi_p(X,Y)$.

5) If $X = OH$, then by (1.1) and (2.2) $\pi_{l_2}(OH,Y) = \pi_2^0(OH,Y)$.

The following theorem is the extension of the Pietsch factorization for l_p-summing operators. The proof is exactly the same than that used in [11: Theorem 3.1].

Theorem 2.3. *Let $X \subset B(H)$ be an operator space, Y a Banach space and $u : X \to Y$ a linear operator. Let $1 \le p < +\infty$. The following properties of a constant $C > 0$ are equivalent:*

(i) *u is l_p-summing and $\pi_{l_p}(u) \le C$.*

(ii) *There are a set I, families a_α and b_α in $B^+_{S_{2p}}$ and an ultrafilter $\mathcal{U}$ on I such that $\|u(x)\| \le C \lim_{\mathcal{U}} \|a_\alpha x b_\alpha\|_{S_p(H)}$ for all $x \in X$.*

(iii) *u factors of the form $u = \tilde{u}(M/E_\infty)i$ and $\|\tilde{u}\| \le C$ as*

where

- $i(x) = \{x_\alpha\}_{\alpha\in I}$ *with* $x_\alpha = x$ *for all* $\alpha \in I$
- $E_\infty = i(X)$ *which is a closed subspace of* $\hat{B}(H)$ *and* $\|i\|_{cb} = 1$
- $\hat{B}(H) = (B_\alpha(H))/\mathcal{U}$ *with* $B_\alpha(H) = B(H)$ *for all* $\alpha \in I$
- M *is the operator associated to* $\{M_\alpha\}_{\alpha\in I}$, $M_\alpha : B(H) \to S_p(H), M_\alpha(x) = a_\alpha x b_\alpha$ *and* $\pi_{l_p}(M) \le 1$
- $\hat{S}_p(H) = (S_\alpha(H))/\mathcal{U}$ *with* $S_\alpha(H) = S_p(H)$ *for all* $\alpha \in I$
- $E_p = M(E_\infty)$ *which is a closed subspace of* $\hat{S}_p(H)$
- *—$\hat{B}(H)$ and $\hat{S}_p(H)$ are operator spaces.*

Remark 3.

1) Let $u : X \to Y$ be a linear operator between operator spaces. Then the property u to be l_p-summing not implies that $\tilde{u}$ is completely bounded. On the other hand, if $p = 2$ and $Y = OH$, then by (1.1) u is completely bounded and $\pi_{l_2}(X,OH) = \pi_2^0(X,OH)$.

2) Let $Y \subset A \subset B(H)$ with A a commutative C^*-algebra and X be operator spaces. Then by (1.3) and (2.2) $\pi_{l_p}(X,Y)) = \pi_p^0(X,Y)$.

3) If $X = Y = OH$, then clearly $\pi_{l_2}(OH,OH) = \pi_2^0(OH,OH) = HS(OH,OH)$.

Lemma 2.4. *Let $X \subset B(H)$ be an operator space. Consider $a, b \in B^+_{S_{2p}}$ and $1 \le p \le q < +\infty$. Then $\|axb\|_{S_p(H)} \le \|a^{\frac{p}{q}} x b^{\frac{p}{q}}\|_{S_q(H)}$ for all $x \in X$.*

Proof. Let $x \in X$ and consider $a, b \in B^+_{S_{2p}}$. We have

$$\begin{aligned}\|axb\|_{S_p(H)} &= \big\|a^{1-\frac{p}{q}} a^{\frac{p}{q}} x b^{\frac{p}{q}} b^{1-\frac{p}{q}}\big\|_{S_p(H)} \\ &\leq \big\|a^{1-\frac{p}{q}}\big\|_{S_{\frac{2pq}{q-p}}} \big\|a^{\frac{p}{q}} x b^{\frac{p}{q}} b^{1-\frac{p}{q}}\big\|_{S_{\frac{2pq}{q-p}}} \\ &\leq \big\|a^{1-\frac{p}{q}}\big\|_{S_{\frac{2pq}{q-p}}} \big\|a^{\frac{p}{q}} x b^{\frac{p}{q}}\big\|_{S_q(H)} \big\|b^{1-\frac{p}{q}}\big\|^{1-\frac{p}{q}}_{S_{\frac{2pq}{q-p}}} \\ &\leq \big\|a^{\frac{p}{q}} x b^{\frac{p}{q}}\big\|_{S_q(H)}\end{aligned}$$

because $\|a^{1-\frac{p}{q}}\|_{S_{\frac{2pq}{q-p}}} = \|a\|^{\frac{q-p}{q}}_{S_{2p}} \leq 1$ and this illustrates that the diagram

is commutative ∎

Proposition 2.5. *Let $1 \leq p \leq q < +\infty$. Let $X \subset B(H)$ be an operator space, Y a Banach space and $u : X \to Y$ an l_p-summing operator. Then u is l_q-summing and $\pi_{lq}(u) \leq \pi_{lp}(u)$.*

Proof. We have

$$\|u(x)\| \leq \pi_{lp}(u) \lim_{\mathcal{U}} \|a_\alpha x b_\alpha\|_{S_p(H)} \leq \pi_{lp}(u) \lim_{\mathcal{U}} \|a_\alpha^{\frac{p}{q}} x b_\alpha^{\frac{p}{q}}\|_{S_q(H)}$$

by Lemma 2.4 where $a_\alpha^{\frac{p}{q}}$ and $b_\alpha^{\frac{p}{q}}$ are in $B^+_{S_{2q}}$ and the comparison is obtained ∎

3. The finite-dimensional case

Let us now give the following finite-dimensional version of Theorem 2.3.

Proposition 3.1. *Consider $N \in \mathbb{N}$ and $1 \leq p < +\infty$. Let $X \subset M_N$ be a finite-dimensional operator space, Y a Banach space and $u : X \to Y$ an l_p-summing operator. Then there are $a, b \in B^+_{S^N_{2p}}$ such that $\|u(x)\| \leq \pi_{lp}(u)\|axb\|_{S^N_p}$.*

Proof. Let

$$S = \big\{s = (a, b) \in S^N_{2p} : a, b \geq 0\big\}.$$

From the proof of [11: Theorem 3.1] there are a set I, an ultrafilter $\mathcal{U}$ on I and a family $\{\lambda_\alpha\}_{\alpha \in I}$ of probabilities on S such that

$$\|u(x)\|^p \leq \pi^p_{lp}(u) \lim_{\mathcal{U}} \int_S \|a_\alpha x b_\alpha\|^p_{S^N_p} d\lambda_\alpha(s).$$

As S is compact, there is a probability λ on S such that $\lambda_\alpha \to \lambda$ in the weak topology of measures on S and

$$\lim_{\mathcal{U}} \int_S \|a_\alpha x b_\alpha\|^p_{S_p^N} d\lambda_\alpha(s) = \int_S \|axb\|^p_{S_p^N} d\lambda(s).$$

Since the application

$$C(S) \to S_p^N \to \mathbb{R}, \quad s = (a,b) \to axb \to \|axb\|$$

as composition of two continuous applications is continuous, then for all $\varepsilon > 0$ there are $n_\varepsilon \in \mathbb{N}, \{s_i\}_{1\le i\le n_\varepsilon} \subset S$ and $\{\lambda_i\}_{1\le i\le n_\varepsilon}$ with $\lambda_i \ge 0$ and $\sum \lambda_i = 1$ such that

$$\int_S \|axb\|^p_{S_p^N} d\lambda(s) \le (1+\varepsilon)\Big(\sum \lambda_i \|a_i x b_i\|^p\Big)^{\frac{1}{p}}.$$

Using [11: Lemma 1.4] we have

$$\int_S \|axb\|^p_{S_p^N} d\lambda(s) \le (1+\varepsilon)\Big\|\Big(\sum \lambda_i a_i^{2p}\Big)^{\frac{1}{2p}} x \Big(\sum \lambda_i b_i^{2p}\Big)^{\frac{1}{2p}}\Big\|_{S_p^N}$$

for all $\varepsilon > 0$. Hence there are $a, b \in S_{2p}^N$ such that $\|u(x)\| \le \pi_{lp}(u)\|axb\|_{S_p^N}$ and this gives the announced result ∎

The next lemma will be crucial to prove our main result in Section 4.

Lemma 3.2. *Consider $N \in \mathbb{N}$ and $2 < p < +\infty$. Let Y be a Banach space and $u : M_N \to Y$ be a linear operator. Then $\pi_{lp}(u) \le \pi_{l_2}^{\frac{2}{p}}(u)\|u\|^{1-\frac{2}{p}}$.*

Proof. Let $u : M_N \to Y$ be a linear operator. As u is of finite rank, it is automatically l_2-summing and by (2.3) we have for all $n \ge 1$

$$\big\|I \otimes u : l_2^n \otimes_{\min} M_N \to l_2^n(Y)\big\| \le \pi_{l_2}(u). \tag{3.1}$$

Apart from that we have abviously

$$\big\|I \otimes u : l_\infty^n \otimes_{\min} M_N \to l_\infty^n(Y)\big\| \le \|u\|. \tag{3.2}$$

We will now interpolate these two estimates. Precisely, by putting $\theta = \frac{2}{p}$, we have

$$[l_\infty^n, l_2^n]_\theta = l_p^n \qquad \text{and} \qquad [l_\infty^n(Y), l_2^n(Y)]_\theta = l_p^n(Y).$$

We have also

$$l_2^n \otimes_{\min} M_N = M_N(l_2^n), \quad l_\infty^n \otimes_{\min} M_N = M_N(l_\infty^n), \quad l_p^n \otimes_{\min} M_N = M_N(l_p^n).$$

Hence, by definition of operator space structures on l_2^n, l_∞^n and l_p^n,

$$\big[l_\infty^n \otimes_{\min} M_N, l_2^n \otimes_{\min} M_N\big]_\theta = l_p^n \otimes_{\min} M_N.$$

Therefore by interpolation of (3.1) and (3.2) we obtain

$$\big\|I \otimes u : l_p^n \otimes_{\min} M_N \to l_p^n(Y)\big\| \le \pi_{l_2}^\theta(u)\|u\|^{1-\theta}.$$

As this is true for all $n \ge 1$ whence the desired result ∎

4. Main result

We now prove that the converse of Proportion 2.5 is not true in the non-commutative case for certain spaces. This is the main result of this paper.

Theorem 4.1. *Let $2 < p < +\infty$. Then $\pi_{l_2}(B(l_2), OH) \neq \pi_{l_p}(B(l_2), OH)$.*

Proof. We proceed by contradiction. Assume that $\pi_2^0(B(l_2), OH)$ $(= \pi_{l_2}(B(l_2), OH)) = \pi_{l_p}(B(l_2), OH)$. Then there is a constant $C > 0$ such that, for all $u \in \pi_{l_p}(B(l_2), OH)$, $\pi_2^0(u) \leq C\pi_{l_p}(u)$. Let $N \in \mathbb{N}$ and consider $u \in \pi_{l_p}(M_N, OH)$. Using Lemma 3.2 we obtain $\pi_{l_2}(u) = \pi_2^0(u) \leq C\pi_{l_2}^{\frac{2}{p}}(u)\|u\|^{1-\frac{2}{p}}$. Hence $\pi_2^0(u) \leq C\|u\|$. Since by [11: Corollary 3.5] $\|u\|_{cb} \leq \pi_2^0(u)$, $\|u\|_{cb} \leq C\|u\|$ for all $u : M_N \to OH$ which is a contradiction by [10: Theorem 2.10] ∎

Corollary 4.2. *Let $B(B(l_2), OH)$ be the space of all bounded operators from $B(l_2)$ into OH. Then $\pi_{l_2}(B(l_2), OH) \neq B(B(l_2), OH)$.*

Remark. Corollary 4.2 implies that the little (the variant of) Grothendieck's theorem is not true.

We end this paper by mentioning the interesting question raised in [11: Problem 10.2] whether, considering $u \in cb(B(l_2), OH)$, $u \in \pi_{2,OH}(B(l_2), OH)$ is true.

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