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Erratum to

$C^{1,\alpha}$ Local Regularity for the Solutions of the p-Laplacian on the Heisenberg Group for $2 \leq p < 1+\sqrt{5}$

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Abstract. The paper in object dealt with some equations approximating the p-Laplacian on the Heisenberg group. We proved the existence of the second derivatives of their solutions for $2 \leq p < 1+\sqrt{5}$. But the differentiation of the approximating equations is incorrect. So the regularity for the solutions of the p-Laplacian obtained from it as a by-product must be considered as not proved.

Keywords: *Degenerate elliptic equations, weak solutions, regularity of solutions, higher differentiability*

AMS subject classification: 35D10, 35J60, 35J70

The purpose of the paper was to prove local Hölder continuity of the gradient of local weak solutions $u \in W^{1,p}_{loc}(\Omega, X)$ $(2 \leq p < 1+\sqrt{5})$ of the p-Laplacian on the Heisenberg group

$$\mathrm{div}_{\mathbb{H}} \vec{a}(Xu) = 0 \tag{1}$$

where $a^k(q) = |q|^{p-2} q_k$ $(k = 1, ..., 2n)$. On this aim we introduced regularized equations

$$\mathrm{div}_{\mathbb{H}} \vec{a}_\varepsilon(Xu) = 0 \tag{2}$$

for small $\varepsilon > 0$, where $a^k_\varepsilon(q) = [(\varepsilon + |q|^2)^{\frac{p-2}{2}} q_k]$ $(k = 1, ..., 2n)$, and we proved that the local weak solutions $u_\varepsilon \in W^{1,p}_{loc}(\Omega, X)$ of equations (2) are twice differentiable with respect to the horizontal fields. In particular, u_ε satisfies for $2 \leq p < 1+\sqrt{5}$ the inequalities

$$\int_{\Omega'} |T(g^4 u_\varepsilon)|^p dx \leq C R^{-4p} \int_{\Omega'} (V^p_\varepsilon + |u_\varepsilon|^p)\, dx \tag{3}$$

$$\int_{\Omega'} g^6 V^{p-2}_\varepsilon |X^2 u_\varepsilon|^2 dx \leq C R^{-4p} \int_{\Omega'} (V^p_\varepsilon + |u_\varepsilon|^p)\, dx \tag{4}$$

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where g is a cut-off function with support in the ball $B(2R)$ and $V_\varepsilon^2 = \varepsilon + |Xu_\varepsilon|^2$ (see Sections 2 - 4 and 7 of the paper). This enabled us to differentiate equation (2) and this should be, in turn, the main tool to prove the uniform (with respect to ε) boundedness and the Höder continuity of the gradient. A limit argument for $\varepsilon \to 0$ should yield the same result for the solutions of equation (1).

But equation (30) of the paper obtained by differentiating (2) with respect to X_i is incorrect, because it does not as account for the non-commutativity of the basic vector fields. Likely, the right equation would be

$$\int_{\Omega'} a_j^k X_j X_i u_\varepsilon X_k \varphi \, dx + \int_{\Omega'} a_j^k T_{ij} u_\varepsilon X_k \varphi \, dx + \int_{\Omega'} a^k T_{ik} \varphi \, dx = 0 \tag{5}$$

where $T_{lm} = [X_l, X_m]$. But, specially due to the second integral in the left-hand side of (5), for the moment we are able to obtain only higher integrability for the gradient, not the boundedness. For this reason, **the following theorems in the paper must be considered as not proved**:

Theorems 1.1 and 5.2 (local boundedness of the gradient for u and u_ε)
Theorems 1.2 and 6.5 (local Hölder continuity of the gradient for u and u_ε).

Therefore, the real content of the paper concerns the existence and regularity of the second derivatives of local weak solutions of equations (2) (Sections 2 - 4 and 7) with constants independent on ε. Moreover, an iterative application of the method from (63) - (68) easily improves the result until to $p < 4$ instead of $p < 1 + \sqrt{5}$ only.

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