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F-Implicit Complementarity Problems in Banach Spaces

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Abstract. In this paper, the F-implicit complementarity problem (F-ICP) and F-implicit variational inequality problem (F-IVIP) are introduced and studied. The equivalence between (F-ICP) and (F-IVIP) is presented under certain assumptions. Furthermore, we derive some new existence theorems of solutions for (F-ICP) and (F-IVIP) by using the Fan-Knaster-Kuratowski-Mazurkiewicz theorem [K. Fan: Math. Ann. 142 (1961), 305 – 310] and Lin's result [T. C. Lin: Bull. Austral. Math. Soc. 34 (1986), 107 – 117] under some suitable assumptions without the monotonicity.

Keywords: *F-implicit complementarity problem, F-implicit variational inequality, equivalence, Knaster-Kuratowski-Mazurkiewicz-mapping*

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1. Introduction

It is well known that the complementarity problem and variational inequality theory are very powerful tools of the current mathematical technology. The classical complementarity problem can be considered as equivalent form of the variational inequality problem (see [4, 8, 9, 16, 17]). In recent years, the classical complementarity problem and variational inequality theory have been extended and generalized to study a wide class of problems arising in mechanics, physics, optimization and control, nonlinear programming, game theory, economics, finance, regional, structural, transportation, elasticity, and applied sciences, etc. (see, for example, [1-4, 6-24] and the references therein).

Let X be a real Banach space with dual space X^*, and $\langle t, x\rangle$ denote the value of a linear continuous function $t \in X^*$ at x. Let K be a closed convex cone of

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X. In 2001, Yin, Xu and Zhang [24] introduced a class of a F-complementarity problem (F-CP), which consists of finding $x \in K$ such that

$$\langle Tx, x\rangle + F(x) = 0, \quad \langle Tx, y\rangle + F(y) \geq 0$$

for all $y \in K$, where $T : K \to X^*$ and $F : K \to \mathbb{R}$. They proved that problem (F-CP) is equivalent to the following generalized variational inequality problem:

(GVIP): Find $x \in K$ such that

$$\langle Tx, y - x\rangle + F(y) - F(x) \geq 0 \quad \forall y \in K,$$

where K is a non-empty closed convex cone and F is a positively homogeneous and convex function. They also proved the existence of solutions for the problem (F-CP) under some assumptions with the F-pseudo-monotonicity.

In this paper, the F-implicit complementarity problem (F-ICP) and F-implicit variational inequality problem (F-IVIP) are introduced and studied. The equivalence between the problems (F-ICP) and (F-IVIP) is presented under certain assumptions. Furthermore, we derive some new existence theorems of solutions for (F-ICP) and (F-IVIP) by using the Fan-Knaster-Kuratowski-Mazurkiewicz theorem [5] and Lin's result [22] under some different assumptions without the F-pseudomonotonicity.

2. Preliminaries

We first recall some definitions and Lemmas which needed in the main results of this paper. In this, by conv the convex hull is denoted.

Definition 2.1. Let K be a non-empty subset of topological vector space X. A point-to-set mapping $T : K \to 2^X$ is called a Knaster-Kuratowski-Mazurkiewicz-mapping (for short, KKM-mapping) if, for every finite subset $\{x_1, x_2, \ldots, x_n\}$ of K, $\text{conv}\{x_1, x_2, \ldots, x_n\}$ is contained in $\bigcup_{i=1}^n T(x_i)$.

Lemma 2.1. [5] *Let K be a non-empty subset of the Hausdorff topological vector space X. Let $G : K \to 2^X$ be a KKM-mapping, such that for any $y \in K$, $G(y)$ is closed and $G(y^*)$ is compact for some $y^* \in K$. Then, there exists $x^* \in K$ such that $x^* \in G(y)$ for all $y \in K$, i.e. $\cap_{y\in K} G(y) \neq \emptyset$.*

Lemma 2.2. [22] *Let K be a non-empty, convex subset of a Hausdorff topological vector space X, and A be a non-empty subset of $K \times K$. Suppose that the following assumptions hold:*

(i) $(x, x) \in A$ *for each* $x \in K$

(ii) $A_y = \{x \in K : (x, y) \in A\}$ *is closed in* K *for each* $y \in K$

(iii) $A_x = \{y \in K : (x, y) \notin A\}$ *is convex or empty for each* $x \in K$

(iv) *there exists a non-empty compact convex subset* C *of* K *such that* $B = \{x \in K : (x, y) \in A, \forall y \in C\}$ *is compact in* K.

Then there exists an $x^* \in K$ *such that* $\{x^*\} \times K \subseteq A$.

3. F-Implicit complementarity problems and variational inequality problems

Let X be a real Banach space with dual space X^* and K be a non-empty closed convex cone of X. Let $f : K \to X^*$, $g : K \to K$ and $F : K \to R$ be a function. In this section, we consider the following F-implicit complementarity problem:

(F-ICP): Find $x^* \in K$ such that

$$\langle f(x^*), g(x^*)\rangle + F(g(x^*)) = 0 \quad \text{and} \quad \langle f(x^*), y\rangle + F(y) \geq 0 \quad \forall y \in K.$$

Examples of (F-ICP).

(1) If g is the identity mapping on K, then (F-ICP) collapses to the F-complementary problem (in short F-CP) of finding $x^* \in K$ such that

$$\langle f(x^*), x^*\rangle + F(x^*) = 0 \quad \text{and} \quad \langle f(x^*), y\rangle + F(y) \geq 0, \quad \forall y \in K,$$

which has been studied by Yin, Xu and Zhang [24].

(2) If $F = 0$, then (F-ICP) reduces to the implicit complementary problem (in short ICP) of finding $x^* \in K$ such that

$$\langle f(x^*), g(x^*)\rangle = 0 \quad \text{and} \quad \langle f(x^*), y\rangle \geq 0, \quad \forall y \in K.$$

which has been studied by Ahmad, Kazmi and Rehman [1] and Isac [15, 17].

(3) If g is the identity mapping on K and $F = 0$, then (F-ICP) reduces to the complementary problem (in short CP) of finding $x^* \in K$ such that

$$\langle f(x^*), x^*\rangle = 0 \quad \text{and} \quad \langle f(x^*), y\rangle \geq 0, \quad \forall y \in K.$$

which has been studied by many authors, see [15 - 20]. If $X = X^* = R^n$, then (CP) becomes the classical complementarity problem, which has been introduced and studied by Cottle [3].

We also introduce the following F-implicit variational inequality problem:

(F-IVIP): Find $x^* \in K$ such that

$$\langle f(x^*), y - g(x^*)\rangle \geq F(g(x^*)) - F(y), \quad \forall y \in K.$$

We first establish the equivalence between (F-ICP) and (F-IVIP).

Theorem 3.1. *It holds:*

(i) *If x^* solves (F-ICP), then x^* solves (F-IVIP).*

(ii) *If $F : K \to R$ is a positive homogeneous and convex function and x^* solves (F-IVIP), then x^* solves (F-ICP).*

Proof. (i) Let x^* be a solution of (F-ICP). Then, $x^* \in K$ such that

$$\langle f(x^*), g(x^*)\rangle + F(g(x^*)) = 0 \quad \text{and} \quad \langle f(x^*), y\rangle + F(y) \geq 0, \quad \forall y \in K.$$

It follows that

$$\langle f(x^*), y - g(x^*)\rangle = \langle f(x^*), y\rangle - \langle f(x^*), g(x^*)\rangle \geq F(g(x^*)) - F(y), \quad \forall y \in K.$$

Thus, x^* is a solution of (F-IVIP).

(ii) Let x^* be a solution of (F-IVIP). Then, $x^* \in K$ such that

$$\langle f(x^*), y - g(x^*)\rangle \geq F(g(x^*)) - F(y), \quad \forall y \in K. \tag{3.1}$$

Since $F : K \to R$ is a positive homogeneous and convex function, and K is a convex cone, then let $y = 2g(x^*)$ and $y = \frac{1}{2}g(x^*)$ in (3.1). Thus, we have

$$\langle f(x^*), g(x^*)\rangle + F(g(x^*)) \geq 0, \quad \text{and} \quad \langle f(x^*), g(x^*)\rangle + F(g(x^*)) \leq 0,$$

from which, we have

$$\langle f(x^*), g(x^*)\rangle + F(g(x^*)) = 0.$$

By using this equality and (3.1), we obtain

$$\langle f(x^*), y\rangle = \langle f(x^*), y - g(x^*)\rangle + \langle f(x^*), g(x^*)\rangle \geq -F(y), \quad \forall y \in K,$$

which shows that x^* solves (F-ICP). ∎

If g is the identity mapping on K, then we have the following results.

Corollary 3.1. ([24]) *It holds:*

(i) *If x^* solves (F-CP), then x^* solves (F-VIP).*

(ii) *If $F : K \to R$ is a positive homogeneous and convex function and x^* solves (F-VIP), then x^* solves (F-CP).*

Theorem 3.2. *Assume that*

(a) *$f : K \to X^*$ and $g : K \to K$ are continuous, $F : K \to R$ is a lower semicontinuous function;*

(b) *there exists a function $h : K \times K \to R$ such that*

  (i) $h(x, x) \geq 0 \quad \forall x \in K$

(ii) $h(x, y) - \langle f(x), y - g(x)\rangle \leq F(y) - F(g(x)) \quad \forall x, y \in K$

(iii) *the set* $\{y \in K : h(x, y) < 0\}$ *is convex for all* $x \in K$*;*

(c) *there exists a non-empty, compact, convex subset* C *of* K*, such that for all* $x \in K \backslash C$ *there exists a* $y \in C$ *such that*

$$\langle f(x), y - g(x)\rangle < F(g(x)) - F(y).$$

Then, (F-IVIP) *has a solution. Furthermore, the solution set of* (F-IVIP) *is compact.*

Proof. Define

$$G(y) = \{x \in C | \langle f(x), y - g(x)\rangle \geq F(g(x)) - F(y)\} \quad \forall y \in K.$$

From assumption (a), we have that for any $y \in K$, $G(y)$ is closed in C. Since every element $x^* \in \cap_{y \in K} G(y)$ is a solution of (F-IVIP), we have to show that $\cap_{y \in K} G(y) \neq \emptyset$. Since C is compact, it is sufficient to prove that the family $\{G(y)\}_{y \in K}$ has the finite intersection property. Let $\{y_1, y_2, \ldots, y_n\}$ be a finite subset of K and set $B = \overline{\text{conv}}(C \cup \{y_1, \ldots, y_n\})$. Then B is a compact and convex subset of K.

We define two point-to-set mappings $F_1, F_2 : B \to 2^B$ as follows:

$$\begin{aligned} F_1(y) &= \{x \in B | \langle f(x), y - g(x)\rangle \geq F(g(x)) - F(y)\} \quad \forall y \in B \\ F_2(y) &= \{x \in B | h(x, y) \geq 0\} \quad \forall y \in B. \end{aligned}$$

From assumptions (i) and (ii) of (b), we have $h(y, y) \geq 0$ and

$$h(y, y) - \langle f(y), y - g(y) \leq F(y) - F(g(y)).$$

Then, we have

$$\langle f(y), y - g(y) \geq F(g(y)) - F(y)$$

and so $F_1(y)$ is non-empty. As above, we can prove that for any $y \in K$, $F_1(y)$ is closed. Since $F_1(y)$ is a closed subset of a compact set B, we know that $F_1(y)$ is compact. Next, we prove that F_2 is a KKM-mapping. Suppose that there exists a finite subset $\{u_1, u_2, \ldots, u_n\}$ of B and $\lambda_i \geq 0, i = 1, 2, \ldots, n$, with $\sum_{i=1}^n \lambda_i = 1$, such that

$$u = \sum_{i=1}^n \lambda_i u_i \notin \bigcup_{j=1}^n F_2(u_j).$$

Then

$$h(u, u_j) < 0, \quad \text{for } j = 1, 2, \ldots, n.$$

From the assumption (b)(iii), we have $h(u,u) < 0$ which contradicts to assumption (b)(i). Hence, F_2 is a KKM-mapping. From assumption (b)(ii), we have $F_2(y) \subseteq F_1(y), \forall y \in B$. In fact, $x \in F_2(y)$ implies that $h(x,y) \geq 0$, and by assumption (b)(ii), we have

$$h(x,y) - \langle f(x), y - g(x) \rangle \leq F(y) - F(g(x)).$$

It follows that

$$\langle f(x), y - g(x) \rangle \geq F(g(x)) - F(y),$$

i.e. $x \in F_1(y)$. Thus, F_1 is also a KKM-mapping. From Lemma 2.1, there exists $x^* \in B$ such that $x^* \in F_1(y)$ for all $y \in B$. Thus, there exists $x^* \in B$ such that $\langle f(x^*), y - g(x^*) \rangle \geq F(g(x^*)) - F(y)$ for all $y \in B$. By assumption (c), we get $x^* \in C$ and moreover $x^* \in G(y_i)$ for $i = 1, 2, \ldots, n$. Hence, $\{G(y)\}_{y \in K}$ has the finite intersection property.

Since $f : K \to X^*$ and $g : K \to K$ are continuous, $F : K \to R$ is a lower semicontinuous function, then it is easy to see that the solutions set of (F-IVIP) is closed. From the assumption (c), any elements outside the set C cannot be a solution of (F-IVIP). Therefore, the solutions set of (F-IVIP) must be constained in C. Since C is compact we know that the solutions set of (F-IVIP) is compact. ∎

Example 3.1. Let $X = Y = R^2, K = R^2_+ = [0,\infty) \times [0,\infty)$, $C = [0,1] \times [0,1]$. Let

$$g(x) = \left(\frac{x_2}{2}, \frac{x_1}{2}\right), \quad F(x) = x_1, \quad f(x) \equiv f$$

and $\langle f(x), z \rangle = f(z) = z_1 + z_2$ for any $x, z \in K$ with $x = (x_1, x_2)$ and $z = (z_1, z_2)$. Then,

$$\langle f(x), y - g(x) \rangle = (y_1 + y_2) - \frac{x_1 + x_2}{2}$$

for any $x, y \in K$ with $x = (x_1, x_2)$ and $y = (y_1, y_2)$. If we set

$$h(x,y) = (2y_1 + y_2) - \left(\frac{x_1}{2} + x_2\right)$$

for any $x, y \in K$ with $x = (x_1, x_2)$ and $y = (y_1, y_2)$, then all assumptions in Theorem 3.2 hold. It is easy to see that $(0,0) \in K$ is a unique solution of (F-IVIP).

If g is the identity mapping on K, then from Theorem 3.2, we obtain the existence theorems for (F-VIP).

Corollary 3.2. *Assume that:*

(a) $f : K \to X^*$ *is continuous,* $F : K \to R$ *is a lower semicontinuous function;*

(b) *there exists a function* $h : K \times K \to R$ *such that*

- **(i)** $h(x,x) \geq 0$, $\forall x \in K$;
- **(ii)** $h(x,y) - \langle f(x), y-x\rangle \leq F(y) - F(x)$, $\forall x, y \in K$;
- **(iii)** *the set* $\{y \in K : h(x,y) < 0\}$ *is convex,* $\forall x \in K$;

(c) *there exists a non-empty, compact, convex subset* C *of* K, *such that* $\forall x \in K \backslash C$, $\exists\ y \in C$ *such that*

$$\langle f(x), y-x\rangle < F(x) - F(y).$$

Then, (F-VIP) *has a solution. Furthermore, the solution set of* (F-VIP) *is compact.*

Theorem 3.3. *Assume that* $f : K \to X^*$ *and* $g : K \to K$ *are continuous,* $F : K \to R$ *is a positive homogeneous, lower semicontinuous and convex function. If assumptions* (b) *and* (c) *in Theorem* 3.2 *hold, then* (F-ICP) *has a solution. Furthermore, the solution set of* (F-ICP) *is compact.*

Proof. It follows from Theorems 3.1 and 3.2 that the conclusion holds. ∎

Remark 3.1. It is easy to see that assumptions (i) and (ii) of (b) in Theorem 3.2 imply that $\langle f(x), x - g(x)\rangle \geq F(g(x)) - F(x)$, $\forall x \in K$. If replacing the assumption (iii) of (b) in Theorem 3.2 by the convexity of F and applying Lemma 2.2, then we can also prove the existence of solutions of (F-IVIP).

Theorem 3.4. *Assume that* $f : K \to X^*$ *and* $g : K \to K$ *are continuous,* $F : K \to R$ *is lower semicontinuous and convex function. And assume that* $\langle f(x), x - g(x)\rangle \geq F(g(x)) - F(x)$, *for all* $x \in K$. *Furthermore, if there exists a non-empty, compact, convex subset* C *of* K, *such that for all* $x \in K\backslash C$ *there exists a* $y \in C$ *such that*

$$\langle f(x), y - g(x)\rangle < F(g(x)) - F(y),$$

then (F-IVIP) *has a solution. Furthermore, the solution set of* (F-IVIP) *is compact.*

Proof. Set $A = \{(x,y) \in K \times K | \langle f(x), y - g(x)\rangle \geq F(g(x)) - F(y)\}$. The proof of the Theorem consists of four steps.

`Step 1:` For each $x \in K$ we have $(x,x) \in A$ since $\langle f(x), x - g(x)\rangle \geq F(g(x)) - F(x)$ for all $x \in K$.

`Step 2:` Since $f : K \to X^*$ and $g : K \to K$ are continuous, $F : K \to R$ is lower semicontinuous, then $A_y = \{x \in K | (x,y) \in A\}$ is closed in K for all $y \in K$.

`Step 3:` We now show that $A_x = \{y \in K | (x, y) \notin A\}$ is convex or empty for any given $x \in K$. Suppose that $A_x \neq \emptyset$ for some $x \in K$. We prove that A_x is convex. In fact, for any $y_1, y_2 \in K$ and $t \in [0, 1]$, set $y_t = ty_1 + (1 - t)y_2$. We know that $y_i \in A_x$ $(i = 1, 2)$ implies that

$$\langle f(x), y_i - g(x) \rangle < F(g(x)) - F(y_i) \quad \text{for } i = 1, 2.$$

Since $F : K \to R$ is convex, it holds

$$\begin{aligned}
\langle f(x), y_t - g(x) \rangle &= \langle f(x), (ty_1 + (1 - t)y_2) - g(x) \rangle \\
&= t\langle f(x), y_1 - g(x) \rangle + (1 - t)\langle f(x), y_2 - g(x) \rangle \\
&< F(g(x)) - (tF(y_1) + (1 - t)F(y_2)) \\
&\leq F(g(x)) - F(y_t),
\end{aligned}$$

that is, $y_t \in A_x$.

`Step 4:` Let $B = \{x \in K | (x, y) \in A \ \forall y \in C\}$. We show that B is compact in C. By assumption, for each $x \in K \backslash C$ there exists a point $y \in C$ such that $\langle f(x), y - g(x) \rangle < F(g(x)) - F(y)$, that is, $(x, y) \notin A$, so that $x \notin B$. Thus, we have $B \subseteq C$. Since $B = \cap_{y \in C} A_y$, A_y is closed, and C is compact it follows that B is a closed, compact subset of C.

From the above four steps and Lemma 2.2, there exists $x^* \in K$ such that $\{x^*\} \times K \subseteq A$, that is, $\langle f(x^*), y - g(x^*) \rangle \geq F(g(x^*)) - F(y)$ for all $y \in K$.

As in Step 2, we can show that the solutions set of (F-IVIP) is closed. From the assumption, any elements outside the set C can not be a solution of (F-IVIP). Therefore, the solutions set of (F-IVIP) must be constained in C. Since C is compact we know that the solutions set of (F-IVIP) is compact. ∎

Example 3.2. Let $X = Y = R^2, K = R^2_+ = [0, \infty) \times [0, \infty)$ and $C = [0, 1] \times [0, 1]$. Let

$$g(x) = \left(x_1 + \frac{x_2}{2}, \frac{x_2}{2}\right), \quad F(x) = -\frac{x_1 + x_2}{2}, \quad f(x) \equiv f$$

and $\langle f(x), z \rangle = f(z) = z_1 + z_2$ for any $x, z \in K$ with $x = (x_1, x_2)$ and $z = (z_1, z_2)$. Then, $\langle f(x), y - g(x) \rangle = (y_1 + y_2) - (x_1 + x_2)$ for any $x, y \in K$ with $x = (x_1, x_2)$ and $y = (y_1, y_2)$, and all assumptions in Theorem 3.4 hold. It is easy to see that $(0, 0) \in K$ is a unique solution of (F-IVIP).

If g is the identity mapping on K, then from Theorem 3.4, we also obtain the existence theorems for (F-VIP).

Corollary 3.3. *Assume that $f : K \to X^*$ is continuous, $F : K \to R$ is a lower semicontinuous and convex function. And assume that $\langle f(x), x - x \rangle \geq$*

$F(x) - F(x)$ for all $x \in K$. Furthermore, if there exists a non-empty, compact, convex subset C of K, such that for all $x \in K \backslash C$ there exists a $y \in C$ such that

$$\langle f(x), y - x \rangle < F(x) - F(y),$$

then (F-VIP) *has a solution. Furthermore, the solution set of* (F-VIP) *is compact.*

Theorem 3.5. *Assume that $f : K \to X^*$ and $g : K \to K$ are continuous, $F : K \to R$ is a positive homogeneous, lower semicontinuous and convex function. If all assumptions in Theorem* 3.4 *hold, then* (F-ICP) *has a solution. Furthermore, the solutions set of* (F-ICP) *is compact.*

Proof. It follows from the Theorems 3.1 and 3.4 that the conclusion holds. ∎

Remark 3.2. In [24], Yin, Xu and Zhang proved the existence theorems for (F-VIP) under some assumptions with the F-pseudo-monotonicity. But here, we derive some existence theorems of solutions for (F-IVIP) and (F-ICP) under some different assumptions without the F-pseudo-monotonicity.

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