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# Fuerteventura island volume

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This volume carries 10 papers related to the intra-disciplinary workshop

**Geometry at Large,**

which took place on Fuerteventura island (Spain) in December 2018. This workshop, supported by the EPSRC Programme Grant *Symmetries and Correspondences*, involved leading researchers working in adelic, affine, algebraic, anabelian, arithmetic, birational, complex, derived, enumerative, local, non-commutative, motivic, model theoretical, $p$-adic, real, symplectic, and tropical geometries. During the workshop, fifteen colloquium style talks were delivered by

- Caucher Birkar (University of Cambridge),
- Fedor Bogomolov (Courant Institute),
- Olivia Caramello (University of Como),
- Ivan Cheltsov (University of Edinburgh),
- Christopher Deninger (University of Münster),
- Ivan Fesenko (University of Nottingham),
- Ehud Hrushovski (University of Oxford),
- Ilia Itenberg (Sorbonne University, Paris),
- Yujiro Kawamata (University of Tokyo),
- Laurent Lafforgue (IHES, Paris),
- Grigory Mikhalkin (University of Genève),
- Fabien Morel (University of München),
- Wojciech Porowski (University of Nottingham),
- Michael Spieß (University of Bielefeld),
- Boris Zilber (University of Oxford).

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The speakers presented very recent results and potential new developments, which helped to notice many intra-disciplinary analogies. A lot of sunshine and excellent weather have positively contributed to the success of the workshop – participants fruitfully interacted with each other during the lectures, between the lectures, and after the lectures. Even when participants climbed the tallest local mountain, they were discussing mathematics. Climbing mathematical picks and enjoying the views has various analogies elsewhere. As British philosopher William Hamilton said

> *The mathematical process in a symbolical (algebraic) method is like running a rail-road through a tunnelled mountain; that in the ostensive (geometrical) like crossing mountain on foot. The former carries us, by a short and easy transit, to our destined point, but in miasma, darkness and torpidity, whereas the latter allows us to reach it only after time and trouble, but feasting us at each turn with glances of the earth and of the heavens, while we inhale health in the pleasant breeze, and gather new strength at every effort we put forth.*

This volume contains papers written by participants of the workshop, as well as contribution from other geometers, whose research influenced the participants. We now briefly describe the ten papers of the volume.

The paper *Generalised pairs in birational geometry* by Caucher Birkar presents the recent theory of generalised pairs over algebraically closed fields of characteristic zero and its role in birational geometry of higher dimensional varieties. This theory has already produced important contributions to birational geometry discussed in the paper. One of them is its indirect role in the solution of the famous Borisov–Alexeev–Borisov conjecture by Birkar. The last section of the paper discusses open problems regarding generalised pairs.

Gavin Brown, Miles Reid and Jan Stevens' paper *Tutorial on Tom and Jerry: the two smoothings of the anticanonical cone over* $\mathbb{P}(1,2,3)$ is the first systematic introduction to the powerful unprojection methods, which has been developed over the last two decades. The paper is focused on a special type of unprojections, known as Tom and Jerry, which answer many practical questions about constructing codimension four Gorenstein subschemes in weighted projective spaces.

The problem of existence of cylinders in Fano varieties serves as a bridge between affine and birational geometries. The paper *Cylinders in Fano varieties* by Ivan Cheltsov, Jihun Park, Yuri Prokhorov and Mikhail Zaidenberg describes this bridge in details in the case of (possibly singular) del Pezzo surfaces, and surveys recent results about cylinders in Fano threefolds and Fano fourfolds. In addition, the authors present relevant results about cylinders in Mori fibre spaces, and overview some results beyond cylindricity including K-stability of some non-cylindrical Fano varieties.

Class field theory is the crown achievement of algebraic number theory of the 20th century and several of its generalisations are essentially shaping modern research in number theory. The paper *Class field theory, its three main generalisations, and applications* by Ivan Fesenko is a panoramic overview of this fundamental part of algebraic number

theory and modern developments, including the most recent ones. The paper discusses two types of class field theory, special and general. These generalisations are higher class field theory using algebraic $K$-theory, anabelian geometry using étale fundamental groups and the Langlands correspondences (including the geometric Langlands correspondence) using representation theory. The survey outlines various connections between these directions and new developments. The text proposes several new fundamental problems, one of which is a unification of the generalisations.

The role of Adams operations in algebraic $K$-theory is well known. The paper *Connective $K$-theory and Adams operations* by Olivier Haution and Alexander Merkurjev illustrates the usefulness of the connective $K_0$-groups of an algebraic variety and Adams operations for the study of relations between $K$-theory and the Chow groups of the variety. Related torsion phenomena can be controlled by considering the action of the Adams operations. Discussed applications include computations of the connective $K_0$-theory of the variety of reduced norm one elements in a central division algebra of prime degree and of the classifying space of the split special orthogonal group of odd degree.

*Integrable systems and special Kähler metrics* by Nigel Hitchin discusses the interaction between the two objects and includes examples. Algebraically completely integrable Hamiltonian systems are algebraic varieties equipped with a symplectic form and a Lagrangian fibration, the generic fibre of which is an abelian variety. In recent years a large class of algebraically completely integrable systems is provided by the Hitchin system, thus there is an added interest in the special Kähler structure. The Hitchin system is the known integrable system associated to representations of the fundamental group of an algebraic curve. In particular, the hyperkähler structure of the moduli spaces provided the appropriate supersymmetric context for the physical interpretation of the geometric Langlands correspondence.

Ax's theorem on pseudo-finite fields is a classical result that played in important role in several directions in model theory. The paper *Ax's theorem with an additive character* by Ehud Hrushovski generalises Ax's theorem on pseudo-finite fields to a continuous-logic setting allowing for an additive character. Working with exponential functions is crucial for number theory applications; over finite fields this has direct connection with the generalised Riemann hypothesis. The paper allows the additive character the become part of the model-theoretic structure. The theory of all finite fields with an additive character is axiomatised unconditionally, it is proved that it admits quantifier-elimination to a geometric level, a definable measure and definable Fourier transform.

The paper *Area in real $K3$-surfaces* by Ilia Itenberg and Grigory Mikhalkin is a contribution to real algebraic geometry. The authors introduce areas of connected components of the real point set of such surfaces defined up to simultaneous multiplication by a positive real number, and proves several new results about them. Its main result of the paper is an inequality for the area of a non-spherical component: it is always greater than the area of any spherical component. Further study involves the use of simple Harnack curves on the surfaces.

The paper *On non-commutative formal deformations of coherent sheaves on an algebraic variety* by Yujiro Kawamata reviews the theory of non-commutative deformations of sheaves and versal non-commutative deformations using $A^\infty$-algebra formalism and injective resolutions. One third of the paper is dedicated to various non-trivial examples where the versal deformations are explicitly computed.

K-stability is an algebraic condition that characterises the existence of Kähler–Einstein metrics on Fano varieties. Recently, there has been substantial progress on K-stability ignited by the powerful valuative criterion discovered by Kento Fujita and Chi Li. Our volume is finalized by the survey *K-stability of Fano varieties: an algebro-geometric approach* by Chenyang Xu, which gives a comprehensive overview of the recent progress in K-stability of Fano varieties. This survey, based on the lecture series Xu gave at the Massachusetts Institute of Technology in Fall 2020, explains how techniques of birational geometry can be applied to prove K-stability results, and how K-stability can be used to construct well-behaved moduli spaces of Fano varieties.

Although Fuerteventura island does not have any research mathematical centers, we feel that the location is an ideal place for an international scientific meeting. This is confirmed by this volume, which illustrates how fruitful our workshop was. We hope that the reader will enjoy reading its contributions.

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