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Erratum to "Non-Hypoellipticity for Degenerate Elliptic Operators"

By

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In page 28 line 3 up from the bottom of [1], it was claimed without the proof that $w(x, y) \notin C^\infty$. However, it seemes to be hard to see this. Hence, the definition of $w(x, y)$ in page 28 line 4 up from the bottom should be replaced by the following :

$$w(x, y)=\sum_{k=1}^{\infty} k^{-4} \exp(iy\cdot k^2) v_0(x, k^2).$$

Then $w(x, y) \notin C^\infty$ in $(-1, 1)\times \boldsymbol{R}_y^1$. In fact, if $\varphi(y)$ is a C_0^∞ -function such that $\hat{\varphi}(0)=1$ and if $F(x, \eta)$ denotes the Fourier transform of φw with respect to y then $F(x, \eta)=\sum_{k=1}^{\infty} k^{-4} v_0(x, k^2)\hat{\varphi}(\eta-k^2)$. Since $\lim_{k\to\infty} \| v_0(x ; k)\|_{L^2(I_{1/2})}=1$ by the observation due to Hoshiro [2 ;(2.3)], we have for a large integer $l>0$

$$\begin{aligned} \| F(x, l^2)\|_{L^2(I_{1/2})} &\geq l^{-4}/2-\sum_{k\neq l}^{\infty} k^{-4} | \hat{\varphi}(l^2-k^2)| \\ &\geq l^{-4}/2-\text{Const. } l^{-5} \end{aligned}$$

because $\hat{\varphi}\in \mathscr{S}$ and $| l^2-k^2 | = | l-k | \; | l+k | \geq l$ if $l\neq k$. Hence $w(x, y)\notin C^\infty$.

It follows from the above change of w that $W=(-1, 1)\times \boldsymbol{R}_y$ in pages 27–29 of [1] should be replaced by $W=(-1, 1)^2$. The estimate (12) in page 29 should be also replaced by

$$\begin{aligned} \| A(x, D_x, D_y)^N w(x, y)\|_{L^2(W)} &= \| \sum_{k=1}^{\infty} k^{-4} \lambda_0(1, k^2)^N v_0(x, k^2) \exp(iy\cdot k^2)\|_{L^2(W)} \\ &\leq 2\sum_{k=1}^{\infty} k^{-4}\lambda_0(1, k^2)^N \leq 2C_1^N \sum_{k=1}^{\infty} k^{-4} (\log k)^{2N} \\ &\leq C_2^N (2N\,!)\sum_{k=1}^{\infty} k^{-3} \leq C_2^{N+1}(2N\,!). \end{aligned}$$

Now the proof of Theorem 1 goes through when $g(x)$ satisfies the condition (5) in

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page 26.

In the case where $g(x)$ satisfies the condition (5)', it suffices to exchange the above w into the following;

$$w(x, y)=\sum_{k=1}^{\infty} k^{-3}\eta_k^{-1}\exp(iy\cdot\eta_k)v_0(x, \eta_k)$$

with $\eta_k=\exp(\delta_1/2a_k)$. Here $\{a_j\}_{j=1}^{\infty}$ is a sequence in page 29. By taking a subsequence of $\{a_j\}$, if necessary, we may assume that $|\eta_k-\eta_l|\geq|\eta_l|^{1/2}$ for $k\neq l$. After those corrections, the subsequent parts become complete and the results of [1] remain unchanged.

References

[1] Morimoto Y., Non-hypoellipticity for degenerate elliptic operators, *Publ. RIMS, Kyoto Univ.*, **22** (1986), 25–30.

[2] Hoshiro T., Hypoellipticity for infinitely degenerate elliptic and parabolic operators of second order, *J. Math. Kyoto Univ.*, **28** (1988), 615–632.