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Interpolation between H^p Spaces and non-commutative generalizations, II

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Abstract. We continue an investigation started in a preceding paper. We discuss the classical results of Carleson connecting Carleson measures with the $\bar{\partial}$-equation in a slightly more abstract framework than usual. We also consider a more recent result of Peter Jones which shows the existence of a solution of the $\bar{\partial}$-equation, which satisfies simultaneously a good L_∞ estimate and a good L_1 estimate. This appears as a special case of our main result which can be stated as follows: Let $(\Omega, \mathcal{A}, \mu)$ be any measure space. Consider a bounded operator $u : H^1 \to L_1(\mu)$. Assume that on one hand u admits an extension $u_1 : L^1 \to L_1(\mu)$ bounded with norm C_1, and on the other hand that u admits an extension $u_\infty : L^\infty \to L_\infty(\mu)$ bounded with norm C_∞. Then u admits an extension $\widetilde{u}$ which is bounded simultaneously from L^1 into $L_1(\mu)$ and from L^∞ into $L_\infty(\mu)$ and satisfies

$$\|\tilde{u}\colon L_\infty \to L_\infty(\mu)\| \le C\, C_\infty$$
$$\|\tilde{u}\colon L_1 \to L_1(\mu)\| \le C\, C_1$$

where C is a numerical constant.

Introduction.

We will denote by D the open unit disc of the complex plane, by $\mathbb{T}$ the unit circle and by m the normalized Lebesgue measure on $\mathbb{T}$. Let $0 < p \le \infty$. We will denote simply by L_p the space $L_p(\mathbb{T}, m)$ and by H^p the classical Hardy space of analytic functions on D. It is well known that H^p can be identified with a closed subspace of L_p, namely the closure in L_p (for $p = \infty$ we must take the weak*-closure) of the linear span of the functions $\{e^{int} : n \ge 0\}$. More generally, when B is a Banach space, we denote by $L_p(B)$ the usual space of Bochner-p-integrable B-valued functions on $(\mathbb{T}, m)$, so that when $p < \infty$, $L_p \otimes B$ is dense in $L_p(B)$. We denote by $H^p(B)$ (and simply H^p if B is one dimensional) the Hardy space of B-valued analytic functions f such that $\sup_{r<1}(\int \|f(rz)\|^p \, dm(z))^{1/p} < \infty$. We denote

$$\|f\|_{H^p(B)} = \sup_{r<1} \Big(\int \|f(rz)\|^p \, dm(z)\Big)^{1/p}.$$

We refer to [G] and [GR] for more information on H^p-spaces and to [BS] and [BL] for more on real and complex interpolation.

We recall that a finite positive measure μ on D is called a Carleson measure if there is a constant C such that for any $r > 0$ and any real number θ, we have

$$\mu(\{z \in D : \ |z| > 1 - r, \ |\arg(z) - \theta| < r\}) \le C\,r\,.$$

We will denote by $\|\mu\|_C$ the smallest constant C for which this holds. Carleson (see [G]) proved that, for each $0 < p < \infty$, this norm $\|\mu\|_C$ is equivalent to the smallest constant C' such that

$$\int |f|^p \, d\mu \le C' \, \|f\|_{H^p}^p\,, \qquad \text{for all } f \in H^p\,. \tag{0.1}$$

Moreover, he proved that, for any $p > 1$ there is a constant A_p such that any harmonic function v on D admitting radial limits in $L_p(\mathbb{T}, m)$ satisfies

$$\int_D |v|^p d\mu \le A_p \, \|\mu\|_C \int_{\mathbb{T}} |v|^p \, dm\,. \tag{0.2}$$

We observe in passing that a simple inner outer factorisation shows that if (0.1) holds for some $p > 0$ then it also holds for all $p > 0$ with the same constant.

It was observed a few years ago (by J. Bourgain [B], and also, I believe, by J. García-Cuerva) that Carleson's result extends to the Banach space valued case. More precisely, there is a numerical constant K such that, for any Banach space B, we have

$$\int \|f\|^p \, d\mu \leq K \, \|\mu\|_C \, \|f\|^p_{H^p(B)} \,, \tag{0.3}$$

for all $p > 0$ and $f \in H^p(B)$. Since any separable Banach space is isometric to a subspace of ℓ_∞, this reduces to the following fact. For any sequence $\{f_n : \ n \geq 1\}$ in H^p, we have

$$\int \sup_n |f_n|^p \, d\mu \leq K \, \|\mu\|_C \int \sup_n |f_n|^p \, dm \,. \tag{0.4}$$

This can also be deduced from the scalar case using a simple factorisation argument. Indeed, let F be the outer function such that $|F| = \sup_n |f_n|$ on the circle. Note that by the maximum principle we have $|F| \geq \sup_n |f_n|$ inside D, hence (0.1) implies

$$\int \sup_n |f_n|^p \, d\mu \leq \int |F|^p \, d\mu \leq C' \int |F|^p \, dm = \int \sup_n |f_n|^p \, dm \,.$$

This establishes (0.4) (and hence also (0.3)).

We wish to make a connection between Carleson measures and the following result due to Mireille Lévy [L]

Theorem 0.1. *Let S be any subspace of L_1 and let $u : S \to L_1(\mu)$ be an operator. Let C be a fixed constant. Then the following are equivalent:*

i) *For any sequence $\{f_n : \ n \geq 1\}$ in S, we have*

$$\int \sup_n |u(f_n)| \, d\mu \leq C \int \sup_n |f_n| \, dm \,.$$

ii) *The operator u admits an extension $\widetilde{u} : L_1 \to L_1(\mu)$ such that $\|\widetilde{u}\| \leq C$.*

PROOF. This theorem is a consequence of the Hahn Banach theorem in the same style as in the proof of Theorem 1 below. We merely sketch the proof of (i) implies (ii). Assume (i). Let $V \subset L_\infty(\mu)$ be the linear span of the simple functions (*i.e.* a function in V is a linear combination of disjointly supported indicators). Consider the space $S \otimes V$ equipped with the norm induced by the space $L_1(m; L_\infty(\mu))$. Let $w = \sum_1^n \varphi_i \otimes f_i$ with $\varphi_i \in V$, $f_i \in S$. We will write

$$\langle u, w\rangle = \sum \langle \varphi_i, u f_i\rangle .$$

Then (i) equivalently means that for all such w

$$|\langle u, w\rangle| \le C\, \|w\|_{L_1(m;L_\infty(\mu))} .$$

By the Hahn Banach Theorem, the linear form $w \to \langle u, w\rangle$ admits an extension of norm $\le C$ on the whole of $L_1(m; L_\infty(\mu))$. This yields an extension of u from L_1 to $L_\infty(\mu)^* = L_1(\mu)^{**}$, with norm $\le C$. Finally composing with the classical norm one projection from $L_1(\mu)^{**}$ to $L_1(\mu)$, we obtain (ii).

In particular, we obtain as a consequence the following (known) fact which we wish to emphasize for later use.

Proposition 0.2. *Let μ be a Carleson measure on D, then there is a bounded operator $T : L_1 \to L_1(\mu)$ such that $T(e^{int}) = z^n$ for all $n \ge 0$, or equivalently such that T induces the identity on H^1.*

PROOF. We simply apply Lévy's Theorem to H^1 viewed as a subspace of L_1, and to the operator $u : H^1 \to L_1(\mu)$ defined by $u(f) = f$. By (0.1) we have $\|u\| \le K\, \|\mu\|_C$, but moreover by (0.4) and Lévy's Theorem there is an operator $T : L_1 \to L_1(\mu)$ extending u and with $\|T\| \le K\, \|\mu\|_C$. This proves the proposition.

Allthough we have not seen this proposition stated explicitly, it is undoubtedly known to specialists (see the remarks below on the operator T^*). Of course, for $p > 1$ there is no problem, since in that case the inequality (0.2) shows that the operator of harmonic extension (given by the Poisson integral) is bounded from L_p into $L_p(\mu)$ and of course it induces the identity on H^p. However this same operator is well known to be unbounded if $p = 1$. The adjoint of the operator T appearing in Proposition 0.2 solves the $\bar\partial$-equation in the sense that for any φ in $L_\infty(\mu)$ the function $G = T^*(\varphi)$ satisfies $\|G\|_{L_\infty(m)} \le \|T\|\ \|\varphi\|_\infty$ together with

$$\int fG\, dm = \int f\varphi\, d\mu , \qquad \text{for all } f \in H^1 ,$$

and by well known ideas of Hörmander [H] this means equivalently that $G\,dm$ is the boundary value (in the sense of [H]) of a distribution g on $\bar{D}$ such that $\bar{\partial} g = \varphi\mu$. In conclusion, we have

$$\bar{\partial} g = \varphi\mu \qquad \text{and} \qquad \|G\|_{L_\infty(m)} \le K\,\|\mu\|_C\,\|\varphi\|_\infty\,.$$

This is precisely the basic L_∞-estimate for the $\bar{\partial}$-equation proved by Carleson to solve the corona problem, (*cf.* [G, Theorem 8.1.1, p. 320]). More recently, P. Jones [J] proved a refinement of this result by producing an explicit kernel which plays the role of the operator T^* in the above. He proved that one can produce a solution g of the equation $\bar{\partial} g = \varphi\mu$ which depends linearly on φ with a boundary value G satisfying simultaneously

$$\|G\|_{L_\infty(m)} \le K\,\|\mu\|_C\,\|\varphi\|_\infty \qquad \text{and} \qquad \|G\|_{L_1(m)} \le K \int |\varphi|\,d\mu\,,$$

where K is a numerical constant. (Jones [J] mentions that A. Uchiyama found a different proof of this. A similar proof, using weights, was later found by S. Semmes.) Taking into account the previous remarks, our Theorem 1 below gives at the same time a different proof and a generalization of this theorem of Jones.

Our previous paper [P] contains simple direct proofs of several consequences of Jones' result for interpolation spaces between H^p-spaces. We will use similar ideas in this paper.

Let us recall here the definition of the K_t functional which is fundamental for the real interpolation method. Let A_0, A_1 be a compatible couple of Banach (or quasi-Banach) spaces. For all $x \in A_0 + A_1$ and for all $t > 0$, we let

$$K_t(x, A_0, A_1) = \inf\left\{\|x_0\|_{A_0} + t\|x_1\|_{A_1} : x = x_0 + x_1, x_0 \in A_0, x_1 \in A_1\right\}.$$

Let $S_0 \subset A_0$, $S_1 \subset A_1$ be closed subspaces. As in [P], we will say that the couple (S_0, S_1) is K-closed (relative to (A_0, A_1)) if there is a constant C such that for all $t > 0$ and $x \in S_0 + S_1$,

$$K_t(x, S_0, S_1) \le C\,K_t(x, A_0, A_1)\,.$$

Main results.

Theorem 1. *Let $(\Omega, \mathcal{A}, \mu)$ be an arbitrary measure space. Let $u : H^\infty \to L_\infty(\mu)$ be a bounded operator with norm $\|u\| = C_\infty$. Assume that u is also bounded as an operator from H^1 into $L_1(\mu)$, moreover assume that there is a constant C_1 such that for all finite sequences $x_1, \ldots, x_n$ in H^1 we have*

$$\int \sup_i |u(x_i)| \, d\mu \le C_1 \int \sup |x_i| \, dm .$$

Then there is an operator $\tilde{u} : L^\infty \to L_\infty(\mu)$ which is also bounded from L^1 into $L_1(\mu)$ such that

$$\|\tilde{u} : L_\infty \to L_\infty(\mu)\| \le C\, C_\infty$$
$$\|\tilde{u} : L_1 \to L_1(\mu)\| \le C\, C_1$$

where C is a numerical constant.

PROOF. Let w be arbitrary in $L_\infty(\mu) \otimes H^\infty$. We introduce on $L_\infty(\mu) \otimes L^\infty(m)$ the following two norms for all w in $L_\infty(\mu) \otimes L^\infty(m)$

$$\|w\|_0 = \int \|w(\omega, \cdot)\|_{L^\infty(dm)} \, d\mu(\omega) ,$$
$$\|w\|_1 = \int \|w(\cdot, t)\|_{L^\infty(d\mu)} \, dm(t) .$$

Let A_0 and A_1 be the completions of $L_\infty(\mu) \otimes L^\infty(m)$ for these two norms. (Note that A_0 and A_1 are nothing but respectively $L_1(d\mu; L_\infty(dm))$ and $L_1(dm; L_\infty(d\mu))$.) Let S_0 and S_1 be the closures of $L_\infty(\mu) \otimes H^\infty$ in A_0 and A_1 respectively.

The completion of the proof is an easy aplication (via the Hahn-Banach theorem) of the following result which is proved further below

Lemma 2. *(S_0, S_1) is K-closed.*

Indeed, assuming the lemma proved for the moment, fix $t > 0$, and consider w in $L_\infty(\mu) \otimes H^\infty$, we have (for some numerical constant C)

$$K_t(w, S_0, S_1) \le C\, K_t(w, A_0, A_1) , \qquad \text{for all } t > 0 .$$

Recall that we denote by $V \subset L_\infty(\mu)$ the dense subspace of functions taking only finitely many values. Let $w = \sum_1^n \varphi_i \otimes f_i$ with $\varphi_i \in V$, $f_i \in H^\infty$. We will write, for every operator $u : H^1 \to L_1(\mu)$,

$$\langle u, w \rangle = \sum \langle \varphi_i, u f_i \rangle .$$

Clearly

$$\left\| \sum \varphi_i \otimes u(f_i) \right\|_{L^1_\mu(L^\infty_m)} \le C_\infty \, \|w\|_0 \tag{1}$$

and

$$\left\| \sum \varphi_i \otimes u(f_i) \right\|_{L^1_m(L^\infty_\mu)} \le C_1 \, \|w\|_1 \tag{2}$$

Moreover, by completion, we can extend (1) (respectively (2)) to the case when w is in S_0 (respectively, S_1). Hence, if $w = w_0 + w_1$ with $w_0 \in S_0$, $w_1 \in S_1$ we have by (1) and (2)

$$\begin{aligned} |\langle u, w \rangle| = \left| \sum \langle \varphi_i, u(f_i) \rangle \right| &\le C_\infty \, \|w_0\|_0 + C_1 \, \|w_1\|_1 \\ &\le C_\infty \, K_s(w, S_0, S_1) \\ &\le C \, C_\infty \, K_s(w, A_0, A_1) \end{aligned}$$

where $s = C_1 (C_\infty)^{-1}$. By Hahn-Banach, there is a linear form ξ on $A_0 + A_1$ such that

$$\xi(w) = \langle u, w \rangle \qquad \text{for all } w \in S_0 + S_1 \tag{3}$$

and

$$|\xi(w)| \le C \, C_\infty \, K_s(w, A_0, A_1) \qquad \text{for all } w \in A_0 + A_1 \, .$$

Clearly this implies

$$|\xi(w)| \le C \, C_\infty \, \|w\|_0 \, , \qquad \text{for all } \omega \in A_0 \, , \tag{4}$$

$$\begin{aligned} |\xi(w)| &\le C \, C_\infty \, s \, \|w\|_1 \, , \qquad \text{for all } \omega \in A_1 \, , \\ &\le C \, C_1 \, \|w\|_1 \, . \end{aligned} \tag{5}$$

Now (4) implies for all $\varphi \in L_\infty(\mu)$ and for all $f \in L_\infty(dm)$

$$|\langle \xi, \varphi \otimes f \rangle| \le C \, C_\infty \, \|\varphi\|_1 \, \|f\|_\infty \, . \tag{6}$$

Define $\tilde{u} : L_\infty \to L_1(\mu)^* = L_\infty(\mu)$ as $\langle \tilde{u}(f), \varphi\rangle = \langle \xi, \varphi \otimes f\rangle$. Then, (6) implies

$$\|\tilde{u}(f)\|_{L_\infty(\mu)} \le C\, C_\infty \, \|f\|_\infty \,,$$

while (5) implies

$$|\langle \xi, \varphi \otimes f\rangle| \le C\, C_1 \, \|\varphi\|_\infty \, \|f\|_1 \,,$$

hence $\|\tilde{u}(f)\|_1 \le C\,C_1\,\|f\|_1$. Finally (3) implies that for all $f \in H^\infty$ and for all $\varphi \in L^\infty(\mu)$

$$\langle \tilde{u}(f), \varphi\rangle = \langle \varphi, u(f)\rangle$$

so that $\tilde{u}|_{H^\infty} = u$.

PROOF OF LEMMA 2. We start by reducing this lemma to the case when Ω is a finite set or equivalently, in case the σ-algebra $\mathcal{A}$ is generated by finitely many atoms, with a fixed constant independent of the number of atoms. Indeed, let V be the union of all spaces $L_\infty(\Omega, \mathcal{B}, \mu)$ over all the subalgebras $\mathcal{B} \subset \mathcal{A}$ which are generated by finitely many atoms. Assume the lemma known in that case with a fixed constant C independent of the number of atoms. It follows that for any w in $H^\infty \otimes V$ we have

$$K_t(w, S_0, S_1) \le C\, K_t(w, A_0, A_1)\,, \qquad \text{for all } t > 0\,.$$

Since $H^\infty \otimes V$ is dense in $S_0 + S_1$, this is enough to imply Lemma 2.

Now, if $(\Omega, \mathcal{B}, \mu)$ is finitely atomic as above we argue exactly as in Section 1 in [P] using the simple (so-called) "square/dual/square" argument, as formalized in Lemma 3.2 in [P]. We want to treat by the same argument the pair

$$H^1(L_\infty(\mu)) \subset L^1(L_\infty(\mu))\,,$$
$$L_1(\mu; H^\infty) \subset L_1(\mu; L^\infty)\,.$$

Taking square roots, the problem reduces to prove the following couple is K-closed

$$H^2(L_\infty(\mu)) \subset L^2(L_\infty(\mu))\,,$$
$$L_2(\mu; H^\infty) \subset L_2(\mu; L_\infty)\,,$$

provided we can check that

$$(7) \qquad H^2(L_\infty(\mu)) \cdot L_2(\mu; H^\infty) \subset (H^1(L_\infty(\mu)), L_1(\mu; H^\infty))_{\frac{1}{2}\infty} \,.$$

We will check this auxiliary fact below. By duality and by Proposition 0.1 in [P] , we can reduce to checking the K-closedness for the couple

$$\begin{aligned} H^2(L_1(\mu)) &\subset L^2(L_1(\mu)) \,, \\ L_2(\mu; H^1) &\subset L_2(\mu; L_1) \,. \end{aligned}$$

Taking square roots one more time this reduces to prove that the following couple is K-closed

$$\begin{cases} H^4(L_2(\mu)) \subset L^4(L_2(\mu)) \,, \\ L_4(\mu; H^2) \subset L_4(\mu; L_2) \,, \end{cases}$$

provided we have

$$(8) \qquad H^4(L_2(\mu)) \cdot L_4(\mu; H^2) \subset (H^2(L_1(\mu)), L_2(\mu; H^1))_{\frac{1}{2}\infty} \,.$$

But this last couple is trivially K-closed (with a fixed constant independent of $(\Omega, \mathcal{B}, \mu)$) because, by Marcel Riesz' Theorem, there is a simultaneously bounded projection

$$\begin{aligned} L_4(L_2(\mu)) &\to H^4(L_2(\mu)) \,, \\ L_4(\mu; L_2) &\to L_4(\mu; H^2) \,. \end{aligned}$$

It remains to check the inclusions (7) and (8). We first check (7). By Jones' Theorem (see the beginning of Section 3 and Remark 1.12 in [P])

$$(9) \qquad H^2(L_\infty(\mu)) = (H^1(L_\infty(\mu)), H^\infty(L_\infty(\mu)))_{\frac{1}{2}2}$$

also by an entirely classical result (*cf.* [BL, p. 109])

$$(10) \qquad L_2(\mu; H^\infty) = (L_\infty(\mu; H^\infty), L_1(\mu; H^\infty))_{\frac{1}{2}2} \,.$$

By the bilinear interpolation theorem (*cf.* [BL, p. 76]) the two obvious inclusions

$$\begin{aligned} H^1(L_\infty(\mu)) \cdot L_\infty(\mu; H^\infty) &\subset H^1(L_\infty(\mu)) \,, \\ H^\infty(L_\infty(\mu)) \cdot L_1(\mu; H^\infty) &\subset L_1(\mu; H^\infty) \,, \end{aligned}$$

(note that $H^\infty(L_\infty(\mu)) = L_\infty(\mu; H^\infty)$), imply that

$$(H^1(L_\infty(\mu)), H^\infty(L_\infty(\mu)))_{\frac{1}{2}2} \cdot (L_\infty(\mu; H^\infty), L_1(\mu; H^\infty))_{\frac{1}{2}2} \subset (H^1(L_\infty(\mu)), L_1(\mu; H^\infty))_{\frac{1}{2}\infty} \,.$$

Therefore, by (9) and (10), this proves (7). We now check (8). We will first prove an analogous result but with the inverses of all indices translated by $1/r$. More precisely, let $2 < r < \infty$, let p, r' be defined by the relations $1/2 = 1/r + 1/p$ and $1 = 1/r + 1/r'$. We will first check

$$\text{(11)} \qquad H^{2p}(L_{2r'}(\mu)) \cdot L_{2p}(\mu; H^{2r'}) \subset (H^p(L_{r'}(\mu)), L_p(\mu; H^{r'}))_{\frac{1}{2}\infty} \,.$$

Indeed, we have

$$\text{(12)} \qquad \begin{aligned} H^{2p}(L_{2r'}(\mu)) \cdot L_{2p}(\mu; H^{2r'}) &\subset L^{2p}(L_{2r'}(\mu)) \cdot L_{2p}(\mu; L^{2r'}) \\ &\subset (L^p(L_{r'}(\mu)), L_p(\mu; L^{r'}))_{\frac{1}{2}} \,. \end{aligned}$$

The last inclusion follows from a classical result on the complex interpolation of Banach lattices, (*cf.* [C, p. 125]). But now, since all indices appearing are between 1 and infinity, the orthogonal projection from L_2 onto H^2 defines an operator bounded simultaneously from $L^p(L_{r'}(\mu))$ into $H^p(L_{r'}(\mu))$ and from $L_p(\mu; L^{r'})$ into $L_p(\mu; H^{r'})$, hence also bounded from

$$(L^p(L_{r'}(\mu)), L_p(\mu; L^{r'}))_{\frac{1}{2}} \quad \text{into} \quad (H^p(L_{r'}(\mu)), L_p(\mu; H^{r'}))_{\frac{1}{2}} \,.$$

Since the latter space is included into $(H^p(L_{r'}(\mu)), L_p(\mu; H^{r'}))_{\frac{1}{2},\infty}$, (*cf.* [BL, p. 102]) we obtain the announced result (11).

Then, we use the easy fact that any element g in the unit ball of $H^4(L_2(\mu))$ (respectively, h in the unit ball of $L_4(\mu; H^2)$) can be written as $g = Gg_1$ (respectively, $h = Hh_1$) with G and H in the unit ball of $H^{2r}(L_{2r}(\mu)) = L_{2r}(\mu; H^{2r})$ and with g_1 (respectively, h_1) in the unit ball of $H^{2p}(L_{2r'}(\mu))$ (respectively, $L_{2p}(\mu; H^{2r'})$). Then, by (11), there is a constant C such that

$$\|g_1 h_1\|_{(H^p(L_{r'}(\mu)), L_p(\mu; H^{r'}))_{\frac{1}{2}\infty}} \leq C \,.$$

Now, the product $M = GH$ is in the unit ball of $H^r(L_r(\mu)) = L_r(\mu; H^r)$, therefore the operator of multiplication by M is of norm 1 both from $H^p(L_{r'}(\mu))$ into $H^2(L_1(\mu))$ and from $L_p(\mu; H^{r'})$ into $L_2(\mu; H^1)$. By interpolation, multiplication by M also has norm 1 from

$$(H^p(L_{r'}(\mu)), L_p(\mu; H^{r'}))_{\frac{1}{2}\infty} \quad \text{into} \quad (H^2(L_1(\mu)), L_2(\mu; H^1))_{\frac{1}{2}\infty} \,.$$

Hence, we conclude that $gh = Mg_1 h_1$ has norm at most C in the space $(H^2(L_1(\mu)), L_2(\mu; H^1))_{\frac{1}{2}\infty}$. This concludes the proof of (8).

References.

[BL] Bergh, J., Löfström, J., ***Interpolation spaces, an introduction.*** Springer-Verlag, 1976.

[BS] Bennett, C., Sharpley, R., ***Interpolation of operators.*** Academic Press, 1988.

[B] Bourgain, J., On the similarity problem for polynomially bounded operators on Hilbert space. *Israel J. Math.* **54** (1986), 227-241.

[C] Calderón, A., Intermediate spaces and interpolation. *Studia Math.* **24** (1964), 113-190.

[G] Garnett, J., ***Bounded Analytic Functions.*** Academic Press, 1981.

[GR] García-Cuerva, J., Rubio de Francia, J. L., ***Weighted norm inequalities and related topics.*** North Holland, 1985.

[H] Hörmander, L., Generators for some rings of analytic functions. *Bull. Amer. Math. Soc.* **73** (1967), 943-949.

[J] Jones, P., L^∞ estimates for the $\bar{\partial}$-problem in a half plane. *Acta Math.* **150** (1983), 137-152.

[L] Lévy. M., Prolongement d'un opérateur d'un sous-espace de $L^1(\mu)$ dans $L^1(\nu)$. Séminaire d'Analyse Fonctionnelle 1979-1980, exposé 5. École Polytechnique. Palaiseau.

[P] Pisier, G., Interpolation between H^p spaces and non-commutative generalizations, I. *Pacific J. Math.* **155** (1992), 341-368.

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