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Erratum: A Parabolic Quasilinear Problem for Linear Growth Functionals

(Rev. Mat. Iberoamericana **18** (2002), no. 1, 135–185)

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Abstract

We give the correct proof of Lemma 3.6 of the paper *A Parabolic Quasilinear Problem for Linear Growth Functionals* (Rev. Mat. Iberoamericana **18** (2002), no. 1, 135-185).

Following [2], let

$$X(\Omega) = \{z \in L^\infty(\Omega, \mathbb{R}^N) : div(z) \in L^1(\Omega)\}. \tag{1.1}$$

In [2], the weak trace on $\partial\Omega$ of the normal component of $z \in X(\Omega)$ is defined. More precisely, it is proved that there exists a linear operator $\gamma : X(\Omega) \to L^\infty(\partial\Omega)$ such that

$$\|\gamma(z)\|_\infty \leq \|z\|_\infty$$

$$\gamma(z)(x) = z(x) \cdot \nu(x) \quad \text{for all } x \in \partial\Omega \ \text{ if } \ z \in C^1(\overline{\Omega}, \mathbb{R}^N).$$

We shall denote $\gamma(z)(x)$ by $[z, \nu](x)$.

Assuming that $\partial\Omega$ is of class C^1, it is proved in [3] that if $x_0 \in \partial\Omega$ is a Lebesgue point of the function $[z, \nu]$, then

$$[z, \nu](x_0) = \lim_{\rho \to 0} \lim_{r \to 0} \frac{1}{2rw_{N-1}\rho^{N-1}} \int_{C_{r,\rho}(x_0, \nu(x_0))} z(y) \cdot \nu(x_0)\, dy, \tag{1.2}$$

$C_{r,\rho}(x, \alpha)$ being the cylinder defined by

$$C_{r,\rho}(x, \alpha) = \left\{ y \in \mathbb{R}^N : |(y - x) \cdot \alpha| < r, |(y - x) - [(y - x) \cdot \alpha] \cdot \alpha| < \rho \right\}.$$

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Our purpose is to give the correct proof of the following Lemma.

Lemma 1.1 ([1, Lemma 3.6])

i) Let $u_n \in BV(\Omega) \cap L^2(\Omega)$ and $z \in X(\Omega)$. Suppose that

$$\mathbf{a}(x, \nabla u_n) \rightharpoonup z \qquad \textit{weakly}^* \textit{ in } \; L^\infty(\Omega, \mathbb{R}^N) \tag{1.3}$$

and

$$\textit{div}\,(\mathbf{a}(x, \nabla u_n)) \rightharpoonup \textit{div}\, z \qquad \textit{weakly in } \; L^2(\Omega). \tag{1.4}$$

Then

$$[\mathbf{a}(x, \nabla u_n), \nu(x)] \to [z, \nu(x)] \quad \textit{weakly in } L^2(\partial\Omega) \textit{ and} \tag{1.5}$$

$$|z(x) \cdot \nu(x)| \leq f^0(x, \nu(x)) \quad \textit{a.e. in } \partial\Omega. \tag{1.6}$$

ii) Let $u_n \in W^{1,2}(\Omega)$. Let $\mathbf{a}_n(x, \xi) = \mathbf{a}(x, \xi) + \frac{1}{n}\xi$. Suppose that

$$\|u_n\|_2 \quad \textit{is bounded in } L^2(\Omega), \tag{1.7}$$

$$\frac{1}{n}|\nabla u_n| \to 0 \quad \textit{in } L^2(\Omega), \tag{1.8}$$

$$\mathbf{a}_n(x, \nabla u_n) \rightharpoonup z \qquad \textit{weakly in } \; L^2(\Omega, \mathbb{R}^N) \tag{1.9}$$

and

$$\textit{div}\,(\mathbf{a}_n(x, \nabla u_n)) \rightharpoonup \textit{div}\, z \qquad \textit{weakly in } \; L^2(\Omega). \tag{1.10}$$

Then

$$[\mathbf{a}_n(x, \nabla u_n), \nu(x)] \rightharpoonup [z, \nu(x)] \quad \textit{weakly in } W^{1/2,2}(\partial\Omega)^* \textit{ and} \tag{1.11}$$

$$|[z(x), \nu(x)]| \leq f^0(x, \nu(x)) \quad \textit{a.e. in } \partial\Omega. \tag{1.12}$$

To prove it, let us recall the following result, which corresponds to Lemma 3.7 in [1]. We notice that its proof is independent of Lemma 3.6.

Lemma 1.2 ([1, Lemma 3.7]) *Suppose that any of the assumptions of Lemma* 3.6 *hold. Moreover we assume that*

$$u_n \to u \text{ in } L^2(\Omega) \text{ and } \|u_n\|_{BV} \text{ is bounded,} \tag{1.13}$$

Then

$$z(x) = \mathbf{a}(x, \nabla u(x)) \qquad \text{a.e. } x \in \Omega. \tag{1.14}$$

Proof of Lemma 1.1. Since both proofs are based on similar arguments, we shall only prove *ii*). For the proof of (1.11) we refer to [1]. Let us prove (1.12). By (1.2), we have

(1.15)
$$[z,\nu](x) = \lim_{\rho\to 0}\lim_{r\to 0}\frac{1}{2rw_{N-1}\rho^{N-1}}\int_{C_{r,\rho}(x,\nu(x))} z(y)\cdot\nu(x)\,dy \quad \mathcal{H}^{N-1}\text{-a.e. on } \partial\Omega.$$

On the other hand, by assumption (H_5),

$$\mathbf{a}(x,\xi)\cdot\eta \le f^0(x,\eta)$$

for all $\xi, \eta \in \mathbb{R}^N$, and all $x \in \overline{\Omega}$. Then, by 1.2, we have

$$z(y)\cdot\nu(x) = \mathbf{a}(y, \nabla u(y))\cdot\nu(x) \le f^0(y, \nu(x)). \tag{1.16}$$

Finally, since $f^0(\cdot,\xi)$ is continuous in $\overline{\Omega}$ for all $\xi \in \mathbb{R}^N$, using (1.15), (1.16), we get

$$\begin{aligned}|[z,\nu](x)| &\le \lim_{\rho\to 0}\lim_{r\to 0}\frac{1}{2rw_{N-1}\rho^{N-1}}\int_{C_{r,\rho}(x,\nu(x))} |z(y)\cdot\nu(x)|\,dy \\ &\le \lim_{\rho\to 0}\lim_{r\to 0}\frac{1}{2rw_{N-1}\rho^{N-1}}\int_{C_{r,\rho}(x,\nu(x))} f^0(y,\nu(x))\,dy = f^0(x,\nu(x)), \\ &\qquad\qquad \mathcal{H}^{N-1} - \text{a.e. on } \quad \partial\Omega.\end{aligned}$$

■

References

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