Rev. Mat. Iberoam. **33** (2017), no. 1, 375–376
DOI 10.4171/RMI/941

Erratum to "A geometric criterion for the finite generation of the Cox rings of projective surfaces"

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Abstract. We add a reasonable hypothesis in Theorem 1 in Rev. Mat. Iberoam. **31** (2015) no. 4, 1131–1140, in order to make it correct.

We adopt the terminology and notation in [1]. It seems that Theorem 1 in [1], as it is stated, is not correct. Indeed, let X be the smooth projective rational surface obtained by blowing-up the projective plane $\mathbb{P}^2_k$ at nine points such that there is only one smooth cubic passing through these points, and let L be a fixed line in $\mathbb{P}^2_k$. We consider three points p_1, p_2, p_3 of $-K_X$ such that $r(p_1+p_2+p_3)$ does not belong to $r\mathfrak{U}$ for any positive integer r, where $\mathfrak{U}$ is the divisor class determined on $-K_X$ by the cycle $\widetilde{L}-K_X$, see [4], p. 562. Here, $\widetilde{L}$ is the strict transform of L in X. Now, we blow-up X at p_1, p_2 and p_3 and denote the obtained surface by Y which is obviously, not only rational, but also anticanonical. On the other hand, there exists naturally a birational projective morphism π between Y and X.

The surface Y contains the numerically effective divisor $\Gamma + E$, where Γ is the proper transform by π of an irreducible curve in $|\widetilde{L}|$ which does not pass through p_1, p_2 and p_3, and E is the proper transform by π of $-K_X$. The effective divisor $\Gamma + E$ is nef, since the intersection numbers of $\Gamma + E$ with Γ and with E are nonnegative. Therefore, the fixed component of the complete linear system of $n(\Gamma + E)$ is equal to E, for any positive integer n. See [4], p. 563.

In our point of view, Theorem 1 in [1] should be stated as follows:

Theorem 1. *Let S be a smooth projective rational surface defined over an algebraically closed field k of arbitrary characteristic such that the invertible sheaf associated to the divisor $-K_S$ has a nonzero global section. Here K_S denotes a canonical divisor on S. If S satisfies the property that every nef divisor on S which is orthogonal to K_S is zero, then the following assertions are equivalents:*

1. $\mathrm{Cox}(S)$ *is finitely generated.*
2. $M(S)$ *is finitely generated.*

Mathematics Subject Classification (2010): 14C20, 14C22, 14C40.
Keywords: Cox rings, rational surfaces, effective monoid, nef monoid, extremal surfaces.

3. *The surface S has only a finite number of (-1)-curves and only a finite number of (-2)-curves.*

Proof. The condition imposed on S, that is every nef divisor on S which is orthogonal to K_S is zero, ensures that S is extremal. Hence, the result follows from Lemma 12 and Theorem 21 in [1]. To see that S is extremal, let D be a nef divisor on S; then the intersection number of $2D$ with $-K_S$ is greater than 1. Therefore, the complete linear system of $2D$ is base point free, see Theorem III.1 in [2]. □

The condition that we imposed in the last theorem, that is every nef divisor on S which is orthogonal to K_S is zero, is reasonable and natural, since there are a lot of families of smooth projective anticanonical rational surfaces that satisfies such condition. To mention a few, any smooth projective rational surface Z with $K_Z^2 > 0$ and the surfaces studied in [3].

The weakness in the formulation of Theorem 1 in [1] does not affect the rest of the published version.

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Received October 4, 2016.

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