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Functional Analysis — *Holomorphic mappings associated to composition ideals of polynomials*, by Richard Aron, Geraldo Botelho, Daniel Pellegrino and Pilar Rueda, communicated on 24 June 2010.

Abstract. — Many operator ideals $\mathcal{I}$ can be naturally associated to polynomial ideals $\mathcal{Q}$. In this paper we initiate a research program whose aim is to relate those holomorphic mappings f that admit factorizations $f = u \circ g$, where $u \in \mathcal{I}$ and g is holomorphic, with those f whose derivative belongs to the associated composition polynomial ideal $\mathcal{Q} = \mathcal{I} \circ \mathcal{P}$.

Key words: Holomorphic mappings, polynomial ideals.

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Introduction

The basic motivation for this work arises from the following example (see, e.g., [4]). Let $f : E \to F$ be a holomorphic (complex analytic) mapping between complex Banach spaces E and F. Then f is locally compact if and only if for every n, the n-homogeneous Taylor polynomial $\hat{d}^n f(0) : E \to F$ takes the unit ball of E to a relatively compact subset of F. The cogent fact here is that the non-linear behavior of f is reflected by the behavior of its associated set of Taylor "coefficients," and conversely. It is this type of situation that will be the focus of this article.

The study of holomorphic mappings associated to special classes of homogeneous polynomials goes back to Nachbin [24]. (See Section 1 for precise definitions of the term "associated" and related concepts.) The notion of ideals of homogeneous polynomials originated with Pietsch [28]. In recent years, several ideals of polynomials have proved to be suitable environments for the study of associated holomorphic mappings (see [6, 19, 20, 25, 26]), so a systematic investigation of holomorphic mappings associated to ideals of polynomials is in order.

All mappings considered will be between Banach spaces. We let $\mathcal{I}$ denote an operator ideal and $\mathcal{P}$ the class of continuous homogeneous polynomials. In this paper, we initiate the study of holomorphic mappings associated to the composition polynomial ideal $\mathcal{I} \circ \mathcal{P}$ (see Definitions 1.2 and 1.3). The main question is whether or not a holomorphic function f is associated to $\mathcal{I} \circ \mathcal{P}$ if and only if it admits a factorization $f = u \circ g$ where g is holomorphic and u belongs to $\mathcal{I}$. The aim is to provide a number of results showing that advances in this direction are possible. We study the general problem, focusing first on entire mappings since this seems to be more accessible. We finish the paper doing some incursions for holomorphic (non necessarily entire) mappings.

In Proposition 2.1 we see that classical results of the first author and Schottenloher [4] and Ryan [29, 30] can be viewed as positive solutions to the above question for entire mappings and the ideals of compact and weakly compact operators. With these in mind, we get a positive answer for any closed surjective ideal $\mathcal{I}$. Concerning non-surjective closed ideals, some slight advances are also shown.

We also consider this question in the context of bounded holomorphic mappings. According to Mujica [22], every bounded holomorphic mapping $f : U \subseteq E \to F$ admits a factorization $f = u \circ g$ where $g : E \to G^\infty(U)$ is a bounded holomorphic mapping, $G^\infty(U)$ is a Banach space depending only on the open set U, and $u : G^\infty(U) \to F$ is a continuous linear operator. The question of whether or not a bounded holomorphic mapping f is associated to $\mathcal{I} \circ \mathcal{P}$ if and only if it admits a factorization $f = u \circ g$ with g bounded and $u \in \mathcal{I}$ arises naturally. By using the linearization of bounded holomorphic mappings we get a negative answer. That is, one cannot assure in general that the function g through which f factors is also bounded.

The paper is organized as follows. In Section 2 we show that if either f is a polynomial or $\mathcal{I}$ is a closed surjective ideal, then an entire mapping f is factorizable if and only if it is locally factorizable, if and only if it is associated to $\mathcal{I} \circ \mathcal{P}$. We also examine some of the most important non-surjective closed ideals. Specifically, we obtain conditions that allow us to reduce the case of spaces of entire mappings associated to the ideals of approximable, completely continuous, or strictly singular operators to the compact case. Section 3 deals with bounded holomorphic mappings. We prove that a bounded holomorphic mapping f factors as $f = u \circ g$ with g bounded and holomorphic and $u \in \mathcal{I}$ if and only if its linearization belongs to $\mathcal{I}$. By using this result we get an example in the setting of compact operators that solves the aforementioned bounded problem in the negative. Section 4 is devoted to the study of the factorization of holomorphic, non necessarily entire, mappings associated to $\mathcal{I} \circ \mathcal{P}$, where $\mathcal{I}$ is the ideal of compact operators. We get that in this particular case any holomorphic mapping defined on a separable Banach space or an absolutely convex open set of an arbitrary Banach space, is factorizable. If we remove the above conditions and consider general domains (that is, arbitrary open sets in arbitrary Banach spaces) then we get that any holomorphic mapping of bounded type associated to $\mathcal{I} \circ \mathcal{P}$, where $\mathcal{I}$ is the ideal of compact or weakly compact operators, is factorizable.

1. Background and notation

Throughout this paper E, F, G will stand for complex Banach spaces and n will be a positive integer. By $\mathcal{P}(^nE; F)$ we denote the Banach spaces of all continuous n-homogeneous polynomials from E to F with the usual sup norm, and by $\mathcal{H}(U; F)$ the linear space of all holomorphic mappings from an open subset U of E to F. For technical reasons we define $\mathcal{P}(^0E; F) = F$. If $F = \mathbb{C}$ we simply write $\mathcal{P}(^nE)$ and $\mathcal{H}(U)$. By $\check{P}$ we mean the unique continuous symmetric n-linear mapping associated to the continuous n-homogeneous polynomial P. A mapping $P : E \to F$ is a *polynomial* if $P = P_0 + P_1 + \cdots + P_n$ where each $P_k \in \mathcal{P}(^kE; F)$.

For $f \in \mathscr{H}(U;F)$, $a \in U$ and $k \in \mathbb{N}$, $\hat{d}^k f(a)$ is the k-th differential polynomial of f at a. For the general theory of homogeneous polynomials and holomorphic mappings we refer to S. Dineen [11] and J. Mujica [21].

By $\hat{\otimes}_\pi^{n,s} E$ we mean the n-fold completed projective symmetric tensor product of E and by σ_n the canonical n-homogeneous polynomial from E to $\hat{\otimes}_\pi^{n,s} E$ ($\sigma_n(x) = \otimes_n x$). Given $P \in \mathscr{P}(^n E; F)$, P_L denotes the linearization of P, that is $P_L : \hat{\otimes}_\pi^{n,s} E \to F$ is the bounded linear operator satisfying $P = P_L \circ \sigma_n$.

DEFINITION 1.1 (*Polynomial ideals*). An *ideal of homogeneous polynomials* (or *polynomial ideal*) $\mathscr{Q}$ is a subclass of the class of all continuous homogeneous polynomials between Banach spaces such that for all $n \in \mathbb{N}$ and Banach spaces E and F, the components $\mathscr{Q}(^n E, F) = \mathscr{P}(^n E, F) \cap \mathscr{Q}$ satisfy the following two conditions:

(i) $\mathscr{Q}(^n E, F)$ is a linear subspace of $\mathscr{P}(^n E, F)$ which contains the n-homogeneous polynomials of finite type.
(ii) (*The ideal property*): If $u \in \mathscr{L}(G, E)$, $P \in \mathscr{Q}(^n E, F)$ and $t \in \mathscr{L}(F, H)$, then the composition $t \circ P \circ u$ is in $\mathscr{Q}(^n G, H)$.

Suppose that $\|\cdot\|_{\mathscr{Q}} : \mathscr{Q} \to \mathbb{R}^+$ satisfies the following three properties:

(i′) $(\mathscr{Q}(^n E; F), \|\cdot\|_{\mathscr{Q}})$ is a normed (Banach) space for all Banach spaces E and F and all n,
(ii′) $\|P^n\|_{\mathscr{Q}} = 1$, where $P^n : \mathbb{K} \to \mathbb{K}$ is given by $P^n(x) = x^n$ for all n,
(iii′) If $u \in \mathscr{L}(G, E)$, $P \in \mathscr{Q}(^n E, F)$ and $t \in \mathscr{L}(F, H)$, then $\|t \circ P \circ u\|_{\mathscr{Q}} \leq \|t\| \, \|P\|_{\mathscr{Q}} \|u\|^n$.

Then $(\mathscr{Q}, \|\cdot\|_{\mathscr{Q}})$ is called a *normed* (*Banach*) *polynomial ideal*.
It is understood that $\mathscr{Q}(^0 E; F) = F$.

The case $n = 1$ recovers the classical theory of operator ideals, for which the reader is referred to [10]. An operator ideal $\mathscr{I}$ is *surjective* if for every surjective operator $u \in \mathscr{L}(G; E)$ the following holds: if $v \in \mathscr{L}(E; F)$ and $v \circ u \in \mathscr{I}(G; F)$, then $v \in \mathscr{I}(E; F)$. Besides compact and weakly compact operators, the most usual examples of surjective closed operator ideals are those formed by operators $T : X \to Y$ that are Rosenthal (any sequence in $T(B_X)$ has a weakly Cauchy subsequence), Banach-Saks (every bounded sequence in X has a subsequence $(x_n)_n$ such that the sequence $(T(x_1 + \cdots + x_n)/n)_n$ converges), separable ($T(X)$ is separable), strictly cosingular ([27, 1.10.2]), limited (for each weak*-null sequence (x_n^*) in Y^*, one has $\lim_n \sup_{a \in T(B_X)} |x_n^*(a)| = 0$), Grothendieck ($(T^*(y_n^*))_n \subset X^*$ is weak null whenever $(y_n^*)_n$ is a weak* null sequence) or Asplund (T factors through an Asplund space). Lists of surjective operator ideals with references can be found in [10] and [16]. Some abstract procedures to generate polynomial ideals from operator ideals have been developed. Next we describe one of these procedures, which is a particular case of the technique known as composition ideals (see [14, 7.3]) and was investigated in [7]:

DEFINITION 1.2 (*Composition polynomial ideals*). Given a Banach operator ideal $\mathcal{I}$, the *composition ideal* of polynomials $\mathcal{I} \circ \mathcal{P}$ consists of all homogeneous polynomials P between Banach spaces that can be factored as $P = u \circ Q$ where Q is an homogeneous polynomial and u is linear operator belonging to $\mathcal{I}$. $\mathcal{I} \circ \mathcal{P}$ becomes a Banach polynomial ideal (see [7]) with the usual composition norm $\| \cdot \|_{\mathcal{I} \circ \mathcal{P}}$ given by

$$\|P\|_{\mathcal{I} \circ \mathcal{P}} := \inf\{\|u\|_{\mathcal{I}} \|Q\| : P = u \circ Q, Q \in \mathcal{P}(^nE; G), u \in \mathcal{I}(G; F)\}.$$

DEFINITION 1.3 (*Associated holomorphic mappings*). A holomorphic mapping f from an open subset U of a Banach space E to a Banach space F is said to be *associated* to $\mathcal{I} \circ \mathcal{P}$, written $f \in \mathcal{H}_{\mathcal{I} \circ \mathcal{P}}(U; F)$, if its derivatives $\hat{d}^k f(a)$ belong to $\mathcal{I} \circ \mathcal{P}$ for all $k \in \mathbb{N}$ and all $a \in U$, and for every $a \in U$ there are $C, c \geq 0$ such that $\left\| \frac{1}{k!} \hat{d}^k f(a) \right\|_{\mathcal{I} \circ \mathcal{P}} \leq C \cdot c^k$ for all $k \in \mathbb{N}$.

REMARK 1.4. The above definition rests heavily on the concept of *holomorphy type* of Nachbin [24], as generalized in [6].

2. FACTORIZATION THEOREMS

By $\mathcal{K}$ and $\mathcal{W}$ we mean the ideals of compact and weakly compact linear operators, respectively. The following particular case of our forthcoming main theorem shows that classical results of [4, 29, 30] can be rewritten as factorizations of functions in $\mathcal{H}_{\mathcal{K} \circ \mathcal{P}}(E; F)$ and $\mathcal{H}_{\mathcal{W} \circ \mathcal{P}}(E; F)$. Recall that a mapping $f : U \to F$, $U \subseteq E$, is compact (weakly compact) if every $a \in U$ admits a neighborhood V_a such that $f(V_a)$ is relatively compact (relatively weakly compact) in F.

PROPOSITION 2.1. *Let* $f \in \mathcal{H}(E; F)$.

(a) $f \in \mathcal{H}_{\mathcal{K} \circ \mathcal{P}}(E; F)$ *if and only if* $f = u \circ g$, *where* G *is a Banach space,* $g \in \mathcal{H}(E; G)$ *and* $u \in \mathcal{K}(G; F)$.
(b) $f \in \mathcal{H}_{\mathcal{W} \circ \mathcal{P}}(E; F)$ *if and only if* $f = u \circ g$, *where* G *is a Banach space,* $g \in \mathcal{H}(E; G)$ *and* $u \in \mathcal{W}(G; F)$.

PROOF. $f \in \mathcal{H}_{\mathcal{K} \circ \mathcal{P}}(E; F)$

$\overset{(1)}{\Leftrightarrow}$ $\hat{d}^n f(a) \in \mathcal{K} \circ \mathcal{P}(^nE; F)$ for any $a \in E$ and $n = 0, 1, 2, \ldots$
$\overset{(2)}{\Leftrightarrow}$ $\hat{d}^n f(a)$ is compact for any $a \in E$ and $n = 0, 1, 2, \ldots$
$\overset{(3)}{\Leftrightarrow}$ f is compact
$\overset{(4)}{\Leftrightarrow}$ $f = u \circ g$, where G is a Banach space, $g \in \mathcal{H}(E; G)$ and $u \in \mathcal{K}(G; F)$.

These equivalences are explained by the following facts:

(1) Since $\mathcal{K}$ is a closed operator ideal, $\| \cdot \|_{\mathcal{K} \circ \mathcal{P}}$ coincides with the usual sup norm (see [7, Corollary 2.8]).
(2) [29, Lemma 4.1] (see also [5, Proposition 37(b)] and [22, Proposition 3.4(a)]).

(3) [4, Proposition 3.4].
(4) [4, Proposition 3.5].

The proof for the weakly compact case is analogous using [30, Theorems 3.2 and 3.7]. □

We are concerned with possible extensions of the above Proposition to arbitrary operator ideals as well as with the relationship between global and local factorizations.

THEOREM 2.2. *Let $\mathscr{I}$ be a closed and surjective Banach operator ideal and $f \in \mathscr{H}(E;F)$. Then the following conditions are equivalent:*

(a) *$f = u \circ g$, where G is a Banach space, $g \in \mathscr{H}(E;G)$ and $u \in \mathscr{I}(G;F)$.*
(b) *Every $a \in E$ admits an open neighborhood V_a such that $f|_{V_a} = u_a \circ g_a$, where G_a is a Banach space, $g_a \in \mathscr{H}(V_a;G_a)$ and $u_a \in \mathscr{I}(G_a;F)$.*
(c) *There is an open neighborhood V of 0 such that $f|_V = u \circ g$, where G is a Banach space, $g \in \mathscr{H}(V;G)$ and $u \in \mathscr{I}(G;F)$.*
(d) *$f \in \mathscr{H}_{\mathscr{I}\circ\mathscr{P}}(E;F)$.*

PROOF. (a) ⇒ (b) is obvious. Let us prove (b) ⇒ (d): Given $a \in E$, let V_a be an open neighborhood of a such that $f|_{V_a} = u_a \circ g_a$, where G_a is a Banach space, $g_a \in \mathscr{H}(V_a;G_a)$ and $u_a \in \mathscr{I}(G_a;F)$. Since f and g_a are holomorphic, there is $r > 0$ such that

$$f(x) = \sum_{n=0}^{\infty} \frac{1}{n!}\hat{d}^n f(a)(x-a) \quad \text{and} \quad g_a(x) = \sum_{n=0}^{\infty} \frac{1}{n!}\hat{d}^n g_a(a)(x-a)$$

uniformly on $B(a;r) \subseteq V_a$. Since u_a is linear and continuous,

$$\sum_{n=0}^{\infty} \frac{1}{n!}\hat{d}^n f(a)(x-a) = f(x) = u_a(g_a(x)) = \sum_{n=0}^{\infty} \frac{1}{n!}(u_a \circ \hat{d}^n g_a(a))(x-a)$$

uniformly on $B(a;r)$. By [24, Proposition 4.2] it follows that $\hat{d}^n f(a) = u_a \circ \hat{d}^n g_a(a)$ for every n. So, $\hat{d}^n f(a) \in \mathscr{I} \circ \mathscr{P}(^n E;F)$ for every n. Let C, c be such that $\left\|\frac{1}{n!}\hat{d}^n g_a(a)\right\| \le Cc^n$ for every n. Thus,

$$\left\|\frac{1}{n!}\hat{d}^n f(a)\right\|_{\mathscr{I}\circ\mathscr{P}} \le \|u_a\|_{\mathscr{I}} \left\|\frac{1}{n!}\hat{d}^n g_a(a)\right\| \le \|u_a\|_{\mathscr{I}} Cc^n$$

for every n.

(d) ⇒ (a) Let $\mathscr{I}$ be a closed surjective operator ideal. Let $f \in \mathscr{H}_{\mathscr{I}\circ\mathscr{P}}(E;F)$ and $n \in \mathbb{N}$. By definition, $\hat{d}^n f(0) \in \mathscr{I} \circ \mathscr{P}(^n E;F)$. Of course we can write $\hat{d}^n f(0) = u_n \circ Q_n$ with $Q_n \in \mathscr{P}(^n E;G_n)$, $u_n \in \mathscr{I}(G_n;F)$ and $\|Q_n\| = 1$. So $\hat{d}^n f(0)(B_E) = u_n(Q_n(B_E)) \subseteq u_n(B_{G_n})$. By [16, Proposition 5], to each $x \in E$ corresponds a neighborhood V_x such that $f(V_x) \in \{A \subseteq E : A \subseteq u(B_Z)$ for some Banach space Z and some operator $u \in \mathscr{I}(Z;E)\}$. Now (a) follows from [16, Theorem 6].

(b) $\Rightarrow$ (c) is clear. (c) $\Rightarrow$ (a) Assume that there is an open neighborhood V of 0 such that $f|_V = u \circ g$, where G is a Banach space, $g \in \mathscr{H}(V;G)$ and $u \in \mathscr{I}(G;F)$. Then, by the uniqueness of the Taylor polynomials and the Identity Theorem

$$\hat{d}^n f(0) = u \circ \hat{d}^n g(0).$$

We can assume that $\|\hat{d}^n g(0)\| = 1$. Then $\hat{d}^n f(0)(B_E) = u \circ \hat{d}^n g(0)(B_E) \subset u(B_G)$ and the conclusion follows as in (d) $\Rightarrow$ (a). □

Remark 2.3. The condition of $\mathscr{I}$ being closed and surjective is only needed in the proof of (d) $\Rightarrow$ (a) and (c) $\Rightarrow$ (a). Therefore (a) $\Rightarrow$ (b) $\Rightarrow$ (d) are true for any arbitrary Banach operator ideal $\mathscr{I}$.

Theorem 2.4. *Let $\mathscr{I}$ be a Banach operator ideal and $P : E \to F$ a continuous polynomial. Then the following conditions are equivalent:*

(a) *$P = u \circ Q$, where G is a Banach space, $Q : E \to G$ is a continuous polynomial and $u \in \mathscr{I}(G;F)$.*
(b) *$P \in \mathscr{H}_{\mathscr{I}\circ\mathscr{P}}(E;F)$.*

Proof. Only (b) $\Rightarrow$ (a) requires proof. Let $P = P_0 + P_1 + \cdots + P_n$, $P_j \in \mathscr{P}(^j E;F)$, $j = 0, 1, \ldots, n$. Since we are assuming that $P \in \mathscr{H}_{\mathscr{I}\circ\mathscr{P}}(E;F)$, $P_k = \frac{1}{k!}\hat{d}^k P(0) \in \mathscr{I} \circ \mathscr{P}(^k E;F)$ for $k = 0, 1, \ldots, n$. So, there are Banach spaces $G_0, G_1, \ldots, G_n$, homogeneous polynomials $Q_k \in \mathscr{P}(^k E;G_k)$ and operators $u_k \in \mathscr{I}(G_k;F)$ such that $P_k = u_k \circ Q_k$ for $k = 0, 1, \ldots, n$. Define $G = G_0 \times \cdots \times G_n$, $Q : E \to G$ and $u : G \to F$ by

$$Q(x) = (Q_0(x), \ldots, Q_n(x)), \quad u(y_0, \ldots, y_n) = u_0(y_0) + \cdots + u_n(y_n).$$

It is immediate that $P = u \circ Q$. In order to see that u belongs to $\mathscr{I}$, use that each u_k belongs to $\mathscr{I}$ and observe that $u = u_0 \circ \pi_0 + \cdots + u_n \circ \pi_n$, where $\pi_k(y_0, \ldots, y_n) = y_k$. For $k = 0, 1, \ldots, n$, define $R_k \in \mathscr{P}(^k E;G)$ by $R_k(x) = (0, \ldots, 0, Q_k(x), 0, \ldots, 0)$. It follows that $Q = R_0 + R_1 + \cdots + R_n$, so Q is a polynomial. □

We do not know if the conditions of Theorem 2.2 are equivalent for arbitrary Banach operator ideals. Let us examine holomorphic mappings associated to the compositions ideals of polynomials generated by three of the most usual closed non-surjective ideals.

By $\mathscr{A}$ we mean the ideal of approximable operators (sup-norm limits of finite rank operators), by $\mathscr{CC}$ the ideal of completely continuous operators (weakly convergent sequences are sent to norm convergent sequences) and by $\mathscr{SS}$ the ideal of strictly singular operators (restrictions to infinite-dimensional subspaces are never isomorphisms). We let τ_ω denote the Nachbin ported topology on $\mathscr{H}(E)$ [24]. A Banach space E is *polynomially reflexive* if $\mathscr{P}(^n E)$ is reflexive for every $n \in \mathbb{N}$.

PROPOSITION 2.5. *Let E and F be Banach spaces.*

(1) *If either* $(\mathscr{H}(E), \tau_\omega)$ *or F has the approximation property, then* $\mathscr{H}_{\mathscr{A}\circ\mathscr{P}}(E;F) = \mathscr{H}_{\mathscr{K}\circ\mathscr{P}}(E;F)$.
(2) *If E is polynomially reflexive, then* $\mathscr{H}_{\mathscr{CC}\circ\mathscr{P}}(E;F) = \mathscr{H}_{\mathscr{K}\circ\mathscr{P}}(E;F)$ *for every Banach space F.*
(3) $\mathscr{H}_{\mathscr{SS}\circ\mathscr{P}}(\ell_1;\ell_1) = \mathscr{H}_{\mathscr{K}\circ\mathscr{P}}(\ell_1;\ell_1)$.

PROOF. (1) Let $f \in \mathscr{H}_{\mathscr{K}\circ\mathscr{P}}(E;F)$, $n \in \mathbb{N}$ and $a \in E$ be given. By definition, $\hat{d}^n f(a) \in \mathscr{K} \circ \mathscr{P}(^nE;F)$, so $\hat{d}^n f(a) = u_n \circ Q_n$ where $Q_n \in \mathscr{P}(^nE;G_n)$ and $u_n \in \mathscr{K}(G_n;F)$. Assume that F has the approximation property. Since every compact F-valued operator is approximable, we have $u_n \in \mathscr{A}(G_n;F)$. We get $\hat{d}^n f(a) \in \mathscr{A} \circ \mathscr{P}(^nE;F)$. Suppose now that $(\mathscr{H}(E), \tau_\omega)$ has the approximation property. By virtue of [4, Proposition 4.2] we find that $(\hat{\otimes}_\pi^{n,s}E)' = \mathscr{P}(^nE)$ has the approximation property. The linear operator $(\hat{d}^n f(a))_L$ is compact by [7, Proposition 2.2(b)], so it is approximable. The factorization $\hat{d}^n f(a) = (\hat{d}^n f(a))_L \circ \sigma_n$ gives $\hat{d}^n f(a) \in \mathscr{A} \circ \mathscr{P}(^nE;F)$ in this case as well. The conclusion $f \in \mathscr{H}_{\mathscr{A}\circ\mathscr{P}}(E;F)$ follows because $\mathscr{A}$ is closed, so $\mathscr{H}_{\mathscr{K}\circ\mathscr{P}}(E;F) \subseteq \mathscr{H}_{\mathscr{A}\circ\mathscr{P}}(E;F)$. The converse inclusion is obvious as $\mathscr{A} \subseteq \mathscr{K}$.

(2) The proof here is similar to that of part (1). For any n, $\hat{\otimes}_\pi^{n,s}E$ is reflexive as $(\hat{\otimes}_\pi^{n,s}E)' = \mathscr{P}(^nE)$ is reflexive by assumption. Let $f \in \mathscr{H}_{\mathscr{CC}\circ\mathscr{P}}(E;F)$, $n \in \mathbb{N}$ and $a \in E$ be given. By definition, $\hat{d}^n f(a) \in \mathscr{CC} \circ \mathscr{P}(^nE;F)$, so [7, Proposition 2.2(b)] and [17, Proposition 17.1.10] give that $(\hat{d}^n f(a))_L$ is a completely continuous, hence compact, operator on the reflexive space $\hat{\otimes}_\pi^{n,s}E$. The factorization $\hat{d}^n f(a) = (\hat{d}^n f(a))_L \circ \sigma_n$ implies that $\hat{d}^n f(a) \in \mathscr{K} \circ \mathscr{P}(^nE;F)$. The conclusion $f \in \mathscr{H}_{\mathscr{K}\circ\mathscr{P}}(E;F)$ follows because $\mathscr{K}$ is closed, so $\mathscr{H}_{\mathscr{CC}\circ\mathscr{P}}(E;F) \subseteq \mathscr{H}_{\mathscr{K}\circ\mathscr{P}}(E;F)$. The converse inclusion is obvious as $\mathscr{K} \subseteq \mathscr{CC}$.

(3) Repeat the same reasoning using $\mathscr{K} \subseteq \mathscr{SS}$ [27, Proposition 1.11.9], $\hat{\otimes}_\pi^{n,s}\ell_1 = \ell_1$ and $\mathscr{K}(\ell_1;\ell_1) = \mathscr{SS}(\ell_1;\ell_1)$ [18, p. 62]. □

One example of a polynomially reflexive space is Tsirelson's original space T^* [2]. The reader is referred to [2, 3, 5, 13] for further information about polynomially reflexive spaces. The approximation property on spaces of holomorphic functions has been widely studied (see, e.g., [4, 8, 9, 12]). For instance, $(\mathscr{H}(\ell_1), \tau_\omega)$ [4, Proposition 5.1], $(\mathscr{H}(c_0), \tau_\omega)$ ([4, Proposition 4.2] and [29, Corollary 5.1]) and $(\mathscr{H}(T^*), \tau_\omega)$ ([4, Proposition 4.2] and [1, Theorem 8]) have the approximation property, while $(\mathscr{H}(\ell_2), \tau_\omega)$ does not have the approximation property (otherwise its complemented subspace $\mathscr{P}(^2\ell_2) = \mathscr{L}(\ell_2;\ell_2)$ would have the approximation property [11, p. 467], [31]).

COROLLARY 2.6. *Let E and F be Banach spaces and* $f \in \mathscr{H}(E;F)$.

(1) *If F has the approximation property, then the conditions of Theorem* 2.2 *are equivalent for the ideal* $\mathscr{A}$.
(2) *If E is polynomially reflexive, then the conditions* (a), (b) *and* (d) *of Theorem* 2.2 *are equivalent for the ideal* $\mathscr{CC}$.

(3) *If* $E = F = \ell_1$, *then the conditions* (a), (b) *and* (d) *of Theorem* 2.2 *are equivalent for the ideal* $\mathscr{S}\mathscr{S}$.

Proof. (1) By Theorem 2.2 we know that the four conditions are equivalent for $\mathscr{K}$. Since F has the approximation property, $\mathscr{K}(G;F)$ can be replaced by $\mathscr{A}(G;F)$ and $\mathscr{H}_{\mathscr{K}\circ\mathscr{P}}(E;F)$ by $\mathscr{H}_{\mathscr{A}\circ\mathscr{P}}(E;F)$ [Proposition 2.5(1)].

(2) Suppose $f \in \mathscr{H}_{\mathscr{C}\mathscr{C}\circ\mathscr{P}}(E;F)$. By Proposition 2.5(2), $f \in \mathscr{H}_{\mathscr{K}\circ\mathscr{P}}(E;F)$, so applying Theorem 2.2 for $\mathscr{K}$ we obtain $f = u \circ g$ with $g \in \mathscr{H}(E;G)$ and $u \in \mathscr{K}(G;F)$. The proof is complete as $\mathscr{K} \subseteq \mathscr{C}\mathscr{C}$.

(3) Repeat the reasoning of (2) using Proposition 2.5(3) and $\mathscr{K} \subseteq \mathscr{S}\mathscr{S}$. □

Open problem. We conjecture that the conditions of Theorem 2.2 are not equivalent for arbitrary Banach operator ideals.

3. Bounded holomorphic mappings

Holomorphic mappings $f \in \mathscr{H}(U;F)$ have canonical linearizations (through a space depending only on U) of the form $f = u \circ g$, where g is holomorphic and u is a continuous linear operator. However this universal space is not a Banach space. More precisely, let $G(U)$ be the inductive predual of $\mathscr{H}(U)$ endowed with the usual τ_δ topology and $\delta_U \in \mathscr{H}(U;G(U))$ be the evaluation map (see [11, p. 183, 184]). According to [11, Proposition 3.27], to every $f \in \mathscr{H}(U;F)$ corresponds a unique operator $T_f \in \mathscr{L}(G(U);F)$ such that $f = T_f \circ \delta_U$. Unfortunately $G(U)$ is not a Banach space, but only a complete locally convex space. Thanks to the following result due to J. Mujica, bounded holomorphic mappings factor canonically through Banach spaces. Let $\mathscr{H}^\infty(U;F)$ denote the space of bounded holomorphic mappings from U to F.

Theorem 3.1 [22, Theorem 2.1]. *Let U be an open subset of E. There exists a Banach space $G^\infty(U)$ and a bounded holomorphic mapping $\delta_U \in \mathscr{H}^\infty(U;G^\infty(U))$ with the following property: to every $f \in \mathscr{H}^\infty(U;F)$ corresponds a unique linear operator $T_f \in \mathscr{L}(G^\infty(U);F)$ such that $f = T_f \circ \delta_U$.*

Once we know that bounded holomorphic mappings f admit a canonical factorization through a Banach space, of the form $f = u \circ g$ with u continuous and linear and g bounded and holomorphic, it is natural to wonder if a "bounded version" of Theorem 2.2 holds. More precisely: given $f \in \mathscr{H}^\infty(U;F)$ and an operator ideal $\mathscr{I}$, consider the conditions (a$'$) $f = u \circ g$, where G is a Banach space, $g \in \mathscr{H}^\infty(U;G)$ and $u \in \mathscr{I}(G;F)$, (d) $f \in \mathscr{H}_{\mathscr{I}\circ\mathscr{P}}(U;F)$.

Does (a$'$) $\Rightarrow$ (d) for every $\mathscr{I}$? Does (d) $\Rightarrow$ (a$'$) whenever $\mathscr{I}$ is closed and surjective?

It is easy to check that (a) $\Rightarrow$ (b) $\Rightarrow$ (d) of Theorem 2.2 holds for holomorphic mappings on open subsets of a Banach space. So, (a$'$) $\Rightarrow$ (d) for every $\mathscr{I}$. To answer the second question we need the following characterization of condition (a$'$).

THEOREM 3.2. *Let* $f \in \mathcal{H}^\infty(U;F)$ *and let* $\mathcal{I}$ *be an operator ideal. Then* $f = u \circ g$, *where* G *is a Banach space,* $g \in \mathcal{H}^\infty(U;G)$ *and* $u \in \mathcal{I}(G;F)$ *if and only if* $T_f \in \mathcal{I}(G^\infty(U);F)$.

PROOF. Assume that $f = u \circ g$, where G is a Banach space, $g \in \mathcal{H}^\infty(U;G)$ and $u \in \mathcal{I}(G;F)$. Considering the linearizations T_f and T_g from Theorem 3.1,

$$T_f(\delta_U(x)) = f(x) = u(g(x)) = u(T_g(\delta_U(x))) = u \circ T_g(\delta_U(x)),$$

for every $x \in U$. Since T_f, T_g and u are linear and continuous and the set $\{\delta_U(x) : x \in U\}$ generates a dense subspace of $G^\infty(U)$ [22, p. 870], it follows that $T_f = u \circ T_g$. Hence $T_f \in \mathcal{I}(G^\infty(U);F)$. The converse is obvious. □

The next example shows that there is no "bounded version" of Theorem 2.2.

EXAMPLE 3.3. Let Δ denote the open unit disk in $\mathbb{C}$ and consider

$$f : \Delta \to c_0 : f(\lambda) = (\lambda^n)_{n=1}^\infty.$$

It is plain that $f \in \mathcal{H}^\infty(\Delta;c_0)$ is compact and that it fails to have a relatively compact range. As in the proof of Proposition 2.1 it follows that $f \in \mathcal{H}_{\mathcal{K}\circ\mathcal{P}}(\Delta;c_0)$. On the other hand, T_f fails to be compact by [22, Proposition 3.4]. So by Proposition 3.2 f does not admit a factorization $f = u \circ g$, where G is a Banach space, $g \in \mathcal{H}^\infty(\Delta;G)$ and $u \in \mathcal{K}(G;c_0)$.

4. HOLOMORPHIC MAPPINGS ON OPEN SETS

An obvious question concerns the extension of the previous results to holomorphic mappings on open subsets of a Banach space. In this section we obtain factorization theorems for mappings $f \in \mathcal{H}_{\mathcal{I}\circ\mathcal{P}}(U;F)$ where $\mathcal{I} = \mathcal{K}$ and $\mathcal{I} = \mathcal{W}$. The results we obtain generalize Proposition 2.1 to some classes of non-entire holomorphic mappings. A bounded subset A of an open set $U \subseteq E$ is said to be *U-bounded* if the distance from A to the boundary of U is strictly positive (E-bounded sets are just the ordinary bounded subsets of E). By $\mathcal{H}_b(U;F)$ we mean the space of holomorphic mappings of bounded type from U to F, that is holomorphic mappings that are bounded on U-bounded sets.

PROPOSITION 4.1. *Let* U *be an open subset of* E *and* $f \in \mathcal{H}_b(U;F)$. *Then the conditions* (a), (b) *and* (d) *of Theorem* 2.2 *are equivalent for the ideals* $\mathcal{K}$ *and* $\mathcal{W}$.

PROOF. The proof of (a) ⇒ (b) ⇒ (d) in Theorem 2.2 works for holomorphic mappings on open sets, so we just have to prove (c) ⇒ (a). Suppose that $f \in \mathcal{H}_{\mathcal{K}\circ\mathcal{P}}(U;F)$. Reasoning as in the proof of Proposition 2.1 and using that [4, Proposition 3.4 (b) ⇒ (a)] holds for holomorphic mappings on open sets, we find that f is compact. Since f is of bounded type, according to [23, Proposition 7.2] there are a complete locally convex space $G_b(U)$, a holomorphic mapping $\delta_U \in \mathcal{H}_b(U;G_b(U))$ and a (unique) linear operator $T_f \in \mathcal{L}(G_b(U);F)$ such that

$f = T_f \circ \delta_U$. By [15, Theorem 5.2] we conclude that T_f is compact. Calling on the Factorization Lemma [11, Lemma 1.13], we know that there is a continuous seminorm α on $G_b(U)$ and a bounded linear operator $\tilde{T}_f : G_b(U)_\alpha := (G_b(U), \alpha)/\alpha^{-1}(0) \to F$ such that $\tilde{T}_f \circ \pi_\alpha = T_f$, where $\pi_\alpha : G_b(U) \to (G_b(U), \alpha)/\alpha^{-1}(0)$ is the quotient map.

$$\begin{array}{ccc} U & \xrightarrow{\ f\ } & F \\ {\scriptstyle \delta_U}\downarrow & \nearrow^{T_f} & \uparrow{\scriptstyle \tilde{T}_f} \\ G_b(U) & \xrightarrow{\ \pi_\alpha\ } & G_b(U)_\alpha \end{array}$$

Since T_f is compact, π_α is surjective and $T_f = \tilde{T}_f \circ \pi_\alpha$, it follows that $\tilde{T}_f$ is compact. Still calling $\tilde{T}_f$ its extension to the completion of the normed space $G_b(U)_\alpha$, the factorization $f = \tilde{T}_f \circ (\pi_\alpha \circ \delta_U)$ gives (a).

The proof for $f \in \mathscr{H}_{\mathscr{W}\circ\mathscr{P}}(U; F)$ is analogous, using [30, Theorem 3.2 (iii) $\Rightarrow$ (i)]. □

We are able to extend the above proposition to all holomorphic mappings from U to F, but only with an additional restriction on either the set U or the space E.

Proposition 4.2. *Let U be an open subset of E and $f \in \mathscr{H}(U; F)$. If either E is separable or U is absolutely convex, then the conditions* (a), (b) *and* (d) *of Theorem* 2.2 *are equivalent for the ideal $\mathscr{K}$.*

Proof. As before, we just have to prove (d) $\Rightarrow$ (a). The one and only thing that prevents our argument following the lines of the proof of Proposition 2.1 is the fact that we do not know if for every compact holomorphic mapping $f : U \to F$ there is a compact, convex, balanced set $L \subset F$ with $f(U) \subset F_L$, where $F_L = (span\, L, \|.\|_L)$, and $f : U \to F_L$ holomorphic (see [4, Proposition 3.5 (a) $\Rightarrow$ (b)]). So it suffices to show that [4, Proposition 3.5 (a) $\Rightarrow$ (b)] holds for holomorphic mappings on either an arbitrary open subset of a separable Banach space or an absolutely convex open subset of an arbitrary Banach space. To accomplish this, suppose that $f \in \mathscr{H}(U; F)$ is a compact holomorphic mapping on U, where either (i) U is an absolutely convex open set in an arbitrary Banach space, or (ii) U is an arbitrary open subset of a separable Banach space E.

(i) U is an absolutely convex open set. Let us see that the proof of [4, Proposition 3.5 (a) $\Rightarrow$ (b)] can be refined to cover this case. For $m, k \in \mathbb{N}$ and $x \in U$, define

$$A_{m,k}(x) = \left\{ \lambda y : y \in B\left(x, \frac{1}{m}\right), \lambda \in \mathbb{C}, |\lambda| \leq 1 + \frac{1}{k} \right\}.$$

Consider the set

$$U_{m,k} = \bigcup \left\{ B\left(x, \frac{1}{m}\right) : \|x\| \leq m, A_{m,k}(x) \subseteq U, \|f\|_{A_{m,k}(x)} \leq m \right\}.$$

Let us prove that $U = \bigcup_{m,k} U_{m,k}$. Given $x_0 \in U$, the subset $C_{x_0}^k = \{\lambda x_0 : |\lambda| \leq 1 + 1/k\}$ of E is contained in U for all sufficiently large k. Since f is a compact mapping and $C_{x_0}^k$ is a compact set, f is bounded in some neighborhood of $C_{x_0}^k$. It follows that there is m so that $x_0 \in U_{m,k}$. Thus, $U \subseteq \bigcup_{m,k} U_{m,k}$ and the other inclusion follows as each $U_{m,k}$ is contained in U. Since $\|f\|_{U_{m,k}} \leq m < +\infty$, we can define

$$K = \{0\} \cup \bigcup_{m,k\in\mathbb{N}} \frac{\overline{f(U_{m,k})}}{mk\|f\|_{U_{m,k}}}.$$

Since f is compact, by [4, Proposition 3.4] each $\hat{d}^n f(0)$ is a compact n-homogeneous polynomial. From this it follows that each $f(U_{m,k})$ is relatively compact in F, and therefore K is compact. It is easy to see that $f(U) \subseteq \operatorname{span} K$. Let L be the closed absolutely convex hull of K and set $F_L := \operatorname{span} L$, normed by the Minkowski functional of L. Then L is compact and absolutely convex. All that is left to prove is that $f : U \to F_L$ is holomorphic. For $x \in U$, define

$$C_x = \{a \in E : x + \lambda a \in U \text{ for } |\lambda| \leq 1\}$$

and take m, k so that $x \in U_{m,k}$. We have that $B(x, 1/m) \subseteq U$ and $B(0, 1/m) \subseteq C_x$. The argument in [4, Proposition 3.4 (a) $\Rightarrow$ (b)] yields that $\{\hat{d}^n f(x)(a) : \|a\| < 1/m\} \subseteq \overline{\text{co}}(f(B(x, 1/m)))$ for every n. So, for $\|a\| < 1/m$, we have

$$\hat{d}^n f(x)(a) \in \overline{\text{co}}(f(B(x, 1/m))) \subseteq \overline{\text{co}}(f(U_{m,k})) \subseteq mk\|f\|_{U_{m,k}} \overline{\text{co}}\Big(\frac{f(U_{m,k})}{mk\|f\|_{U_{m,k}}}\Big).$$

Putting $M_{m,k} = mk\|f\|_{U_{m,k}}$ we find that $\{\hat{d}^n f(x)(a) : \|a\| < 1/m\} \subseteq M_{m,k} L$. For $\|a\| < 1/m$, $f(x + a) = \sum_{n=0}^{\infty} \hat{d}^n f(x)(a)$ converges in F. In particular, for $\|a\| < 1/2m$ and $j \in \mathbb{N}$ we have

$$f(x+a) - \sum_{n=0}^{j} \hat{d}^n f(x)(a) = \sum_{n=j+1}^{\infty} \hat{d}^n f(x)(a) = 2^{-j} \sum_{n=j+1}^{\infty} 2^{j-n} \hat{d}^n f(x)(2a),$$

so $f(x+a) - \sum_{n=0}^{j} \hat{d}^n f(x)(a) \in 2^{-j} M_{m,k} L$ since L is absolutely convex and compact. Denoting by $\|\cdot\|_L$ the norm on F_L given by the Minkowski functional of L we get

$$\Big\| f(x+a) - \sum_{n=0}^{j} \hat{d}^n f(x)(a) \Big\|_L \leq 2^{-j} M_{m,k} \xrightarrow{j\to\infty} 0$$

uniformly for $a \in B(0, 1/2m)$, proving that $f : U \to F_L$ is holomorphic.

(ii) U is an open subset of a separable Banach space E. U has a countable base of open sets $\mathscr{B} = \{V_n : n \in \mathbb{N}\}$. For each $x \in U$ there is a neighborhood U_x of x such that $f(U_x)$ is relatively compact in F. Pick $N_x \in \mathbb{N}$ so that $x \in V_{N_x} \subseteq U_x$.

Therefore each $f(V_{N_x})$ is relatively compact in F. For each subset A of U we define $\|f\|_A := \sup\{\|f(y)\| : y \in A\}$. Let

$$K = \{0\} \cup \bigcup_{x \in U} \frac{\overline{f(V_{N_x})}}{n_x \|f\|_{V_{n_x}}}.$$

It is clear that this is a countable union, so it follows that K is compact. The argument proceeds as above. □

Open problem. We conjecture that Proposition 4.2 is not valid for holomorphic mappings on arbitrary open subsets of arbitrary Banach spaces.

Open problem. Is Proposition 4.2 valid when considering the ideal of weakly compact operators? And for any closed and surjective ideal of operators?

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Richard Aron
Department of Mathematical Sciences
Kent State University
Kent, Ohio 44242, USA
aron@math.kent.edu

Geraldo Botelho
Faculdade de Matemática
Universidade Federal de Uberlândia
38.400-902—Uberlândia, Brazil
botelho@ufu.br

Daniel Pellegrino
Departamento de Matemática
Universidade Federal da Paraíba
58.051-900—João Pessoa, Brazil
dmpellegrino@gmail.com

Pilar Rueda
Departamento de Análisis Matemático
Universidad de Valencia
46.100 Burjasot—Valencia, Spain
pilar.rueda@uv.es