History of Mathematics — *Newton's* Philosophiae Naturalis Principia Mathematica *"Jesuit" Edition: The Tenor of a Huge Work*, by Paolo Bussotti and Raffaele Pisano, communicated on 26 June 2014.

Abstract. — This paper has the aim to provide a general view of the so called *Jesuit Edition* (hereafter JE) of Newton's *Philosophiae Naturalis Principia Mathematica* (1739–1742). This edition was conceived to explain all Newton's methods through an apparatus of notes and commentaries. Every Newton's proposition is annotated. Because of this, the text—in four volumes—is one of the most important document to understand Newton's way of reasoning. This edition is well known, but systematic works on it are still missing. We are going to fill this gap by means of a project exposed in the final remarks of this paper. In this paper we will: A) expound the way in which the notes and the additions to the JE were conceived by the commentators; B) provide some pieces of information about the commentators; C) summarize the most important of their notes; D) examine closely their notes as to a particularly important question: the so called "inverse problem of the central forces".

1. Aims of the paper

The purpose of this article is twofold:

a) To highlight the features of the so called *Jesuit Edition* of Newton's *Principia*, published in the period 1739–1742 as a reprint of the third edition (1726) of Newton's masterpiece (Newton [1726, 1739–42, 1760], 1822) calling attention to:

  1) *The general structure of the edition*;
  2) *The personalities of the editors*;
  3) *The role of the JE among* Principia*'s editions and commentaries published from 1687—first original edition of Newton's work—to 1833*.

For, in 1833, the mathematician John Martin Frederick Wright wrote *A commentary on Newton's Principia* (Wright, 1833), which is particularly significant in this context. The tradition of the commentaries to Newton's *Principia* is still living nowadays. The text of Subrahmanyan Chandrasekhar (1910–1995) *Newton's*
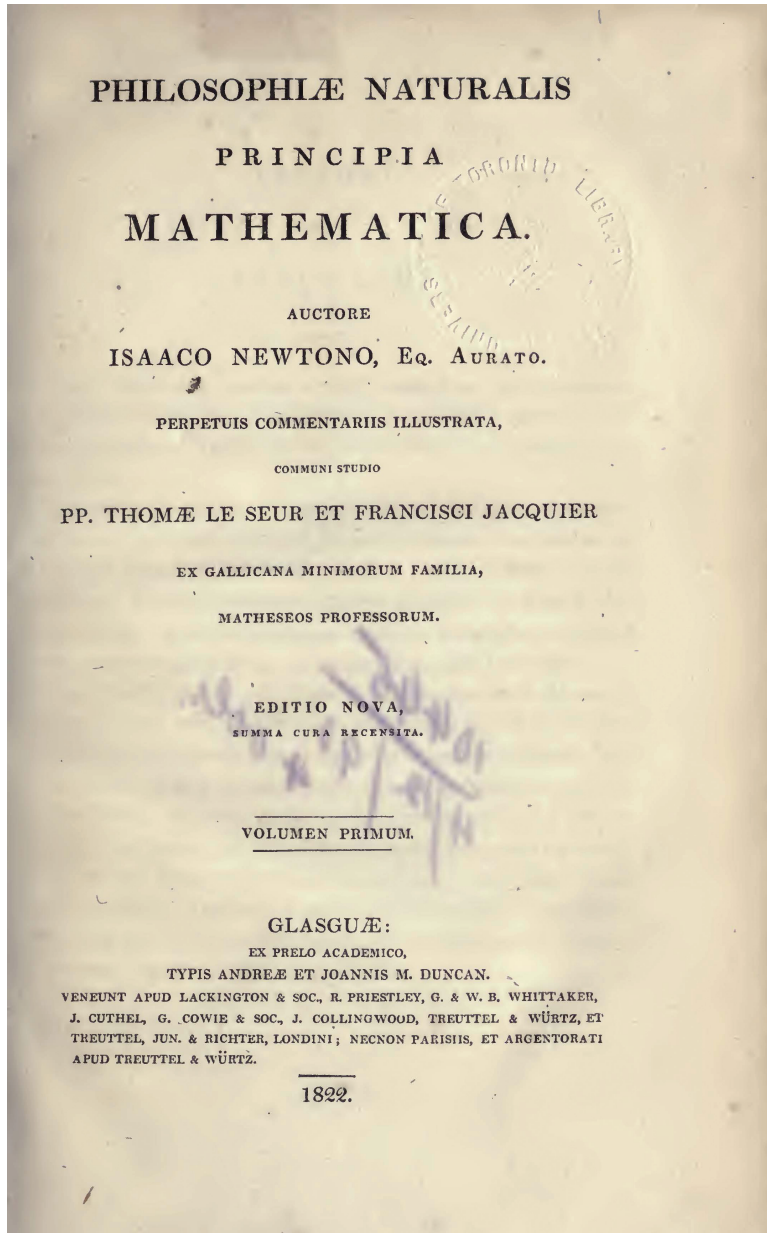
PHILOSOPHIÆ NATURALIS

PRINCIPIA

MATHEMATICA.

AUCTORE

ISAACO NEWTONO, Eq. Aurato.

PERPETUIS COMMENTARIIS ILLUSTRATA,

COMMUNI STUDIO

PP. THOMÆ LE SEUR ET FRANCISCI JACQUIER

EX GALLICANA MINIMORUM FAMILIA,

MATHESEOS PROFESSORUM.

EDITIO NOVA,
SUMMA CURA RECENSITA.

VOLUMEN PRIMUM.

GLASGUÆ:

EX PRELO ACADEMICO,

TYPIS ANDREÆ ET JOANNIS M. DUNCAN.

VENEUNT APUD LACKINGTON & SOC, R. PRIESTLEY, G. & W. B. WHITTAKER,
J. CUTHEL, G. COWIE & SOC, J. COLLINGWOOD, TREUTTEL & WÜRTZ, ET
TREUTTEL, JUN. & RICHTER, LONDINI; NECNON PARISIIS, ET ARGENTORATI
APUD TREUTTEL & WÜRTZ.

1822.

Figure 1. Frontispiece of the Glasgow Jesuit edition (Newton [1726] [1739–1742], 1822)

Principia for the common reader (Chandrasekhar, 1995) is the last example of such a tradition[1].

4) *The historical and conceptual meaning of the JE.* The third edition of the *Principia* had been published only 13 years before the first volume of the JE. An English version of the second edition of the *Principia* (1713), translated by Motte, appeared in 1729 (Newton 1713, 1729). *Why was a further edition published then?* An answer to this question depends on the profound changes occurred in mathematics, in physics and in the application of mathematics to physics between the end of the 17th century and the beginning of the 18th century. The question is also connected with Newton's personal and difficult geometrical methods, their reception and their replacement with more analytical methods.

b) An editorial project in five volumes concerning the JE is in start-up phase. We aim at providing a *critical translation into English of the four Latin volumes (1822), and an added* introductory *volume.*

Since several but not structured works on the JE are present in the literature, we feel the need to deeply explain the importance of such a crucial commentary, as the JE. An extensive rework of this edition JE is necessary to clarify the context in which Newtonian science was developed. This can be a useful means for historians, philosophers and scientist busy with Newton's and Newtonian studies. Under this perspective, this article and a previous one (Bussotti and Pisano, 2014) aim at familiarizing and introducing to the tenor of a huge work.

1.1. The Structure of the Paper

This paper is composed of the following paragraphs: 2) A brief history of *Principia*'s editions until Wright's commentary; 3) The JE and its main features; 4) An Example of how the editors of the JE explain Newton's procedures and results; 5) Conclusion.

2. A brief history of *Principia*'s editions till Wright's commentary

The first book of the *Philosophiae Naturalis Principia Matematica* was presented to the Royal Society in April 1686. At the end of June, the Royal Society decided the publication. Because of this positive decision, Newton sent Edmond Halley (1656–1687) the second and the third books of the *Principia* respectively in March and in April 1687. On 5th July 1687, the *Principia* were published. The run was limited because only between 200 and 400 copies of Newton's work were probably published. Four years after the publication, it was almost impossible to find a copy of the book (Munby, 1952). The work is divided into four main parts:

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[1] In Bussotti-Pisano 2014, we have dedicated the note 4, p. 35 to the most significant commentaries to Newton's *Principia*. See also the note 13 in this paper. Here we wish to remind the reader two important commented edition of Newton's text: Newton 1972, edited by Koyré-Cohen-Whitman; Newton 1999, translated into German and edited by Volkmar Schüller.

1) An introductory part in which Newton expounds the definitions and the axioms;

2) The *Book I*, that begins with the famous first section *De metodo rationum primarum et ultimatum, cujus ope sequentia demonstrantur* in which the elements of infinitesimal calculus used in the course of the book are introduced, concerns rational mechanics. It is impressive. It is divided into 14 sections and contains 98 propositions (distinguished in problems and theorems), 29 lemmas and 18 *scholia*. The propositions and the lemmas are numbered, the *scholia* are not. Many propositions have corollaries. Here Newton develops many parts of modern rational mechanics. As a matter of fact, he invents the very concept of rational mechanics. As, in the first book no phenomenon is described, rather, the general relations between physical quantities, treated from a mathematical point of view, are faced. Newton was aware of the distinction between rational mechanics and physics, for we read in the *Definitions*:

> I likewise call Attractions and Impulses, in the same sense, Accelerative, and Motive; and use the words Attraction, Impulse or Propensity of any sort towards a centre, promiscuously, and indifferently, one for another; considering those forces not Physically but Mathematically: Wherefore, the reader is not to immagine, that by those words, I any where take upon me define the kind, or the manner of any Action, the causes or the physical reason thereof, or that I attribute forces, in a true and Physical sense, to certain centres (which are only Mathematical points); when at any time I happen to speak of centres as attracting or as endued with attractive powers.[2]

In the *Scholium* which concludes the 11th section—where Newton dealt with some questions regarding the three-body problem—, he wrote:

> In the same general sense I use the word impulse, not defining in this treatise the species of physical qualities of forces, but investigating the quantities and mathematical proportions of them; as I observed before in the definitions. In mathematics we are to investigate the quantity of forces with their proportions consequent upon any conditions supposed; then when we enter upon physics, we compare those proportions with the phaenomena of Nature; that we may know what conditions of those forces answer to the several kinds of attractive bodies. And this preparation being made, we argue more safely concerning the physical species, causes, and proportions of forces.[3]

This is one of the most innovative aspects of the *Principia*. As well known, the concept whose characteristics are analysed is that of (dynamic) *force*, the very foundation of Newtonian rational mechanics and physics.

3) The second book, which contains 53 propositions (13 problems and 40 theorems), 4 lemmas and 11 *scholia*. In the second and third edition, the propositions are 53, too, even if there is not a perfect correspondence among the propositions in the various editions. The lemmas are 7 and there are also some differences as to the *scholia*. The book deals with fluid mechanics.

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[2] Newton [1726] [1739–1742], 1822, I, p. 8. Translation drawn from Newton 1729, I, pp. 8–9. This passage is unmodified in the three editions of Newton's work.

[3] *Ivi*, p. 356. Translation drawn from Newton 1729, p. 262. This passage is unmodified in the three editions of Newton's work.

4) The third book. This is Newton's famous system of the world, where the general propositions of the first book and—even if in a minor manner—some of the second book are applied to celestial mechanics. The book contains 9 hypotheses, 42 propositions (21 theorems and 21 problems) and 11 lemmas. This book got important changes in the following two editions because the part of the hypotheses was enlarged and divided into two parts: 1) *Regulae philosophandi*; 2) *Phaenomena*. The propositions are still 42, but with partially relevant differences, for example the problems are 22. This means that the first book did not get important modifications in the two following *Principia*'s editions (1713 and 1726), while the second, and above all, the third ones, did.

Before the publication of the *Principia*, the only concrete results in physics were due, excluding Galileo Galilei (1564–1642), to Christiaan Huygens (1629–1695), who is in fact, together with Galileo, the only physicist quoted by Newton. The results obtained before Newton were not comparable with those achieved in the *Principia*. Newton's book inspired a series of researches between the end of the 17th century and the beginning of the 18th century. In this sense, too, the *Principia* represented the birth of modern physics. Scientists as Pierre Varignon (1654–1722), David Gregory (1659–1708), Guillaume François Antoine de Sainte Mesme marquis de l'Hôpital[4] (1661–1704), Johann Bernoulli (1667–1748), Abraham de Moivre (1667–1754), John Keill (1671–1721), Jacob Hermann (1678–1733), William Jacob 's Gravesande (1688–1742) were probably the most famous ones who gave fundamental contributions to physics, in great part by developing results expounded in the *Principia*. This happened shortly after the publication of Newton's masterpiece. This means that the scientific environment was ready to receive Newton's doctrines. It is known that many criticisms were raised at Newton. In particular, someone rejected the idea that the forces had an immediate action at a distance (for example Huygens himself and de l'Hôpital): this kind of action was deemed not realistic and it was considered as a residue of *vitalistic* and prescientific conceptions. For example, Leibniz moved severe, and in many aspects perspicuous, critics to the concepts of absolute space and absolute time[5], explained by Newton in the long and very important *Scholium*

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[4] Sometime spelled as "l'Hospital", and sometimes—because of French spelling—the silent "s" is removed and replaced with a circumflex. Even if the former spelling is still used in English where there is no circumflex, due historical methodological reasons, we prefer let his original name.

[5] For the concept of absolute space and absolute time Newton (1666?) is fundamental. Almost every general publication on Newton has a section concerning the concepts of absolute space and absolute time: Arthur 1995, Carriero 1990, Di Salle 2006, Bussotti and Pisano 2014, in particular see pp. 107–108, p. 122. With regard to Huygens' conception of gravity see, for example, Huygens [1690] 1944, in particular the *Addition* are fundamental because of the comparison, made by Huygens himself, between his own conception of gravity and Newton's. See also the letter to de l'Hôpital 1692 (Huygens, 1905, p. 354). Huygens greatly appreciated Newton's results, but not his methodology. See Bertoloni Meli, 2006; Harper 2011, pp. 206–207. A partially out-of-chorus publication, which is in favour of the action at a distance is in Assis 1999.

to the Definitions. Many scientists continued to believe in Cartesian physics, even if it was clear, from Newton's results, that the vortices could not be stable[6]. Leibniz, who was so acute with regard to the concept of absolute space and absolute time, thought that the area law and the elliptical orbits, in which the centre of the forces is in one of the two foci, could be obtained developing vortices theory through harmonic circulation (*circulatio harmonica*)[7]. Criticism connected to those exposed, but slightly different, were addressed to Newton's use of gravity: he was accused—for example by Leibniz himself—to speak of a force whose causes were unknown. A similar polemic had been carried out by Robert Hooke (1635–1703) and Robert Boyle (1627–1691) against Newton's conception of light and colours (Guicciardini 2011, chapter 4, pp. 65–90, chapter 8, pp. 218–219). These criticisms notwithstanding, Newton's work and results became an unavoidable basis for advanced physics.

The second edition of the *Principia* was published in 1713 (Newton 1713). It contains the remarkable foreword of the editor, Roger Cotes (1682–1716), who explains the characteristics of Newton's *philosophy* in a perspicuous manner. In this edition, Newton provided a more accurate examination of the theory of the fluids resistance (second book), a more complete theory of the moon, of the equinoxes and of the comets (third book). The general *Scholium* appeared at the end of the work. The second edition was reprinted in Amsterdam one year later (Newton 1714) with the correction of some mistakes occurred in the 1713 edition. In 1723, in Amsterdam, the second edition was printed once again with the addition of the mathematical paper *Enumeratio linearum tertii ordinis* (Newton, 1723).

In 1726, the third edition was published (Newton, 1726). Newton added (second book) some experiments concerning the resistance met by the heavy bodies when they fall in the air. Further specifications on the moon theory and on the

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[6] The explicit proof of the vortices instability is given by Newton at the beginning of the general *Scholium*, that appears for the first time in the second edition of the *Principia* (1713). However, this brief proof is entirely based on properties already demonstrated in the first edition. As to this problem, Newton is also clear in the proposition LIII of the second book and in the subsequent *Scholium*. This proposition and the *Scholium* are present in the first edition, too.

[7] This aspect of Leibniz's production is interesting from a historical point of view. With regard to this conception by Leibniz, after his reading of the *Principia*, the most important works are Leibniz ([1689] [1860] 1962, pp. 144–161) and Leibniz (Ms. [1860] 1962), pp. 161–187 almost surely written immediately afterwards. Leibniz expressed the same concepts in several letters, for example: 1) to Henry Justel, 10/20 October 1690 (Leibniz LSB, I, 6, p. 265); 2) to Erhard Weigl, September 1690, (Leibniz LSB, II, 2 p. 347; 3) to Huygens, 1/11 April 1692, (Leibniz LSB, II, 5, p. 288; 4) to Huygens, 10/20 March 1693 (*Ivi*, p. 517). Because of Newton's, Gregory's and Keill's critics, Leibniz specified some aspects of his planetary theory in two manuscripts written around 1706, see Leibniz (Ms. [1880] 1971, pp. 254–276). The most profound studies on Leibniz's planetary theory are those by Aiton; see Aiton 1960; Aiton 1962; Aiton 1964; Aiton 1965; Aiton 1972. A Fundamental text is also Bertoloni Meli 1993.

comets were added in the third book, in particular a clearer explanation that gravity is responsible for the moon to gravitate around the earth. If the results by Newton were generally accepted and physics was developing on a Newtonian basis starting from the beginning of the 18th century, there was an aspect of his work that was refused and modified by the physicists of the generation immediately after Newton: his mathematical methods[8]. Which were the characteristics of these methods that made them obsolete only few years after the publication of the *Principia*? The thing is that all Newton's argumentations expounded in the *Principia* are based on geometrical reasoning. He develops his argumentations according to synthetic geometry, he resorts to difficult arguments in order to obtain geometrical properties of the figures, and finally he transforms them into relations between the physical quantities with which he is dealing[9]. These figures are not certainly only lines and circles: in most cases they are conic sections, but in many cases they are also curves of higher degree or transcendental curves[10], therefore the procedures are fascinating and complicated at the same time. There are various hypotheses as to why Newton adopted this technique[11], but in our context what is important is that he adopted it. Hence no proposition is transcribed into analytical terms. The infinitesimal concepts are used because Newton needs instantaneous quantities and hence the limit of the ratio of two quantities when both of them tend to 0 is essential in almost all reasoning, but these differential relations are not transcribed into explicitly written differential equations, rather—when they do not represent the final step of a reasoning— they are used to obtain further properties of the figures, so the argumentation goes on. It is only natural that the physicists, immediately after the publications of the *Principia*, began to use analytical methods, by resorting explicitly to the differential concepts and the differential equations and limiting the geometrical methods as much as possible. Their interest consisted in obtaining a general mathematical means that allowed them to treat the largest possible number of physical problems through uniform methods. Newton's geometrical approach was not suitable for this. An extensive use of the calculus was.

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[8] We do not have rooms to deal with the relationships between physical and mathematical methods before Newton. I.e. one could see Leonardo da Vinci, Tartaglia, Galilei, Kepler, Descartes (Pisano, 2007; Pisano, 2011; Pisano and Bussotti, 2012, 2013, Pisano, 2013, Bussotti and Pisano, 2013, Pisano and Capecchi, 2014).

[9] In order to clarify the situation with an example, see the fourth section of this paper.

[10] See, for example, Book I, section VI, Proposition XXXI, where he uses the trochoid; or Book I, section X, where, dealing with the motion of the pendulum, the cycloid plays a fundamental role.

[11] With regard to the reasons why Newton adopted this technique and their connections with the difference between the development of calculus in England and on the continent we refer to (without any pretension to be exhaustive): De Gandt 1995; Fellmann 1988; Force 1983; Force, Hutton 2004; Guerlac 1981; Guicciardini 1989, Guicciardini 1998, Guicciardini 1999, Guicciardini 2002; King-Hele and Rupert Hall A, 1988; Pulte and Mandelbrot 2011; Purrington and Durham 1990; Rupert Hall 1999; Westfall, 1971.

Hence, Newtonian physics was developed without Newtonian methods. The difficulties of Newtonian methods induced a series of commentaries or partial explanations of Newton's procedures starting from the beginning of the 18th century[12]. The situation in 1739—this means only 13 years after the publication of the third edition of Newton's work—was such that a group of scholars[13] felt the necessity to publish an edition of the *Principia* in which all the methods and the reasoning used by Newton were explicitly and clearly explained. This is the origin of that huge enterprise, the JE, and no sentence can be more explicit than the following ones written by the editors of the JE in the incipit of the *Monitum*. For, we read:

> All who had heard even only the name of the very famous author knew how secret and at the same time useful the doctrines exposed in the *Philosophiae Naturalis Principia Mathematica* are. The dignity and the sharpness of the subject, the more than geometric brevity of the reasoning are so conspicuous that the magnificent work seems to be written only for a very limited number of expert geometers[14].

Therefore, the first aim of the editors is to explain Newtonian methods whose mysteries are open only to the most expert geometers. However, the editors make far more than this:

a) They inform the reader of all the most recent discoveries in physics;
b) They often translate Newton's results into analytical terms, also mentioning the physicists who dealt with this operation;
c) They add entire treatises, of which they themselves are the authors, which can be useful to understand Newton's presuppositions and methods;
d) They insert treatises of other physicists who developed some particularly important aspects of Newton's physics.

The result of this work are four huge books where: in the first one, the first book of the third edition of the *Principia* is referred and commented; in the second one, the second book of the third edition; in the third one the initial 24 propositions of the third book; in the fourth one, the other propositions of the third book.

The edition was published in Geneva between 1739 and 1742 (1739 first volume; 1740 second volume; 1742 third and fourth volumes).

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[12] With regard to commentaries to Newton's text, or to commentated translations (we mention only that by Madame du Chatelet because of its known importance), or to text strictly connected to Newton's, not limiting to the beginning of the 18th century, but, once again without any pretence to be exhaustive, we mention: Chandrasekhar, 1995; Châtelet, 1759; Clarke [1730], 1972; Desaguilers, 1717; Gregory, 1702; Keill, 1701; McLaurin [1748], 1971; Pemberton, 1728; Rouse Ball [1893], 1972; Whiston, 1707; Wright 1833. In the context of the first commentaries on the *Principia*, the role of David Gregory was very important. With regard to this, we refer to the pioneering, but still important: Hiscock 1937; Wightman 1957. We also mention Eagles 1977.

[13] For the details concerning the editors and the edition, see the next paragraph.

[14] Newton [1726] [1739–1742], 1822, I, p. VII, lines 1–6. (The translation is ours).

A second edition of the JE was printed in Colonia Allobrogum (Geneva) in 1760. This edition corrects some typos of the previous one and is in three volumes.

Finally in 1822 the third edition of the JE was published in Glasgow by Duncan and Duncan (Newton [1726] [1739–1742], 1822). This edition is in four volumes, as the first one. Several mistakes of the previous editions are corrected. The mathematician John Martin Frederick Wright accomplished this hard work, providing us with precious and error-free source for the researches on Newton and on history of physics and mathematics.

Our story has *A commentary on Newton's Principia* (Wright, 1833), written by Wright himself as the final step. Here Wright provided a systematic transcription into modern analytical terms of Newton's most important results and added many explanations to clarify Newton's work. He also added some results connected to the *Principia*, but obtained after Newton. Finally, Wright remarked some cases—they are not many—in which the editors of the JE did not understand completely Newton's reasoning.

3. The Jesuit Edition and its main features

This section is divided into two parts: in the former, we are going to present the commentators of the JE and in the latter, we are going to provide the reader a general view on the nature of the notes and interventions they carried out.

3.1. *The commentators*

The notes in the JE are due to three scholars: two French mathematicians, Thomas Le Seur (1703–1770) and François Jacquier (1711–1788) and Swiss scientist, Jean Louis Calandrini (1703–1758).[15] Actually, in the title-page of the edition only the names of the two French scholars appear. Nevertheless, at the end of the *Monitum* to the first book written by Le Seur and Jacquier we read:

> We do not want to omit to express publicly our gratitude to J. L. Calandrini, Professor at the Geneva Academy and a very expert in mathematics, who took the care to adorn this edition of ours of Newton's *Principia* according to that very elegant edition, that, enriched by many additions, appeared in London in 1726. For, that extremely learned man not only assumed carefully on himself the duty to control that the figures were engraved, posed in the appropriate place and that the typographical mistakes were corrected, but he also wrote those elements of conic sections that we have already praised. Sometimes he illustrated those parts of our explanation, which did not seem perspicuous, with his own annotations.[16]

Calandrini is also mentioned in the *Monitum* to the second volume for the care with which he followed the publication (Newton [1726] [1739–1742], 1822, II,

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[15] Recently, an interesting work was published on the personalities of the three commentators and, above all of Calandrini (Guicciardini, 2014).

[16] Newton [1726] [1739–1742], 1822, I, p. VIII. (The translation is ours).

Monitum, page without number, lines 16–20) as well as in the *Monitum* to the third volume (*Ivi*, III, page without number, lines 9–13) for his precious contributions to the edition. This indicates that the work was carried out by the three scholars. Indeed, the number of notes and interventions by Calandrini is huge. They can be recognized because they are indicated by an asterisk. Although Calandrini wrote, in the *Editoris Monitum* at the beginning of the third volume (*Ivi*, *Editoris Monitum*, page without number, lines 11–15), that the use of the asterisk would have been avoided in the fourth volume, this was not the case because this mark continues to be present and allows us to recognize Calandrini's interventions.

As we read in the title-page of the JE, Thomas Le Seur (1703–1770) and François Jacquier (1711–1788) were not Jesuits, but belonged to the "Gallicana minimorum familia", this means they were Minim Frias, because of this the name "Jesuit Edition" is not correct. Probably this erroneous denomination derives from what the typographers of the Glasgow edition wrote. For we read in the initial page of this 1822 third JE edition:

> Our intention was to publish the edition of Le Seur and Jacquier, belonging to the Jesuit Society, in an integral form, with their commentaries, correcting the mistakes that here and there could have occurred.[17]

Le Seur and Jacquier are basically known because of the JE, but they also wrote other essays, e.g., *Elémens du calcul integral* (Le Seur and Jacquier 1768), *Riflessioni de' Padri Tommaso Le Seur, Francesco Jacquier de el' Ordine de' Minimi, e Ruggiero Giuseppe Boscovich della Compagnia di Gesù sopra alcune difficoltà spettanti i danni, e risarcimenti della cupola di S. Pietro* (Le Seur, Jacquier and Boscovich 1743) and the *Elementi di perspettiva, secondo i principii di Brook Taylor con varie aggiunte spettanti all'ottica e alla geometria* (Jacquier 1755).

Calandrini was a Newtonian since his youth, for in 1722 he presented a thesis on the colours to the Academy of Geneva in which he developed his argumentations following Newton's ideas and methods (see Calandrini 1722). As a mathematician, he was interested in plane and spherical trigonometry, the theory of differentials, the problems of quadrature and the infinite series. He also dealt with botanic, meteorology and with the problem of the aurora borealis (see Calandrini-Serres 1727). A work of his on trigonometry was not published. Calandrini was named Professor for mathematics at the Academy of Geneva in 1724, but for three years he preferred to take a cultural journey to the Low Countries, England and France. From 1734 to 1750 he was Professor of Philosophy. He had important political roles, too.

The three editors can therefore be considered good scientists and mathematicians, learned in this subject, but not prominent or particularly original scholars. This was maybe a luck for the edition because probably, since they did not have

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[17] *Ivi*, I, first page. This page is not numbered. (The translation is ours).

to follow a conspicuous train of thought of their own, they were completely involved in the edition and they made really a good job.

3.2. *Notes and other Interventions*

From a typographical point of view, there are four kinds of interventions:

1) The notes which are directly referred to passages by Newton are indicated in Newton's running text by a letter with a parenthesis, like this (^{a}). In the notes, there is the same letter. It is followed by a number if the subject of the note is remarkably different from that of the previous note, otherwise the series of letters continues and it can also happen that more letters are included inside the same number. Therefore, the letters represent a subdivision of a greater note indicated by a number. The letters always follow an alphabetic order and, in general, go from (^{a}) to (^{z}), after that they begin once again with (^{a}). However, if two consecutive references in the running text are far one from the other this last rule is not necessarily respected and some series of letters can end with a letter different from (^{z}). It can of course happen that a number begins with a letter different from (^{a}). Only as an example, the note 121 (p. 53) of the first volume begins with (^{z}). Sometimes Calandrini adds this symbol (*) in Newton's running text. It has the same function as a letter, but the content and the style of the note is something he judges different from the normal content and style of the notes indicated by letters. Furthermore under the (*) several numbered notes can be included, while this is not the case with letters.

2) The notes which are not interpreted by the commentators as direct explanations of Newton's text, but which represent either clarifications, or additions of the commentators are not indicated by a letter, but only by a number. The series of numbers of the notes of kind 1) and of kind 2) is continuous, that is the distinction among them depends only on the presence of the letter, rather than on two different numerations.

3) Treatises inserted by the commentators to introduce the general problems dealt with in Newton's text. For example: a) at the beginning of the third volume, the commentators add a treatise on the essential elements of positional astronomy (Newton [1726] [1739–1742], 1822, III, pp. IX–XXXVI); b) at the beginning of the fourth volume a brief treatise (*Ivi*, pp. III–VI) is added as an introduction to Newton's lunar theory.

4) Treatises of other authors who developed and made more perfect theories whose first detailed explanation was due to Newton. These treatises are three and concern the tides theory. They are inserted in the final part of the fourth volume. They are: a) Daniel Bernoulli: *Sur le Flux et Reflux de la Mer*, 1740 (Bernoulli, 1740, IV, pp. 101–207); b) Colin McLaurin: *De causa physica fluxus et refluxus maris*, 1740 (*Ivi*, pp. 209–245); c) Leonhard Euler: *Inquisitio physica in causam fluxus ac refluxus maris*, 1740 (*Ivi*, pp. 247–341). These works had won the praise of the *Académie Royale des Sciences*.

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utraque feretur corpus illud eodem tempore in diagonali ab A ad D. Nam
(b) quoniam vis N agit secundum lineam A C ipsi B D parallelam, hæc
vis per Legem II. nihil mutabit velocitatem accedendi ad lineam illam B D
a vi alterâ genitam. Accedet igitur corpus eodem tempore ad lineam B D,
sive vis N imprimatur, sive non; (c) atque ideo in fine illius temporis re-

ambiguitas, nihil aliud per hanc legem intellectum volumus, nisi æquales fieri in corpore agente et patiente status mutationes; cùm enim nulla possit esse actio corporis in aliud corpus, quin mutua fiat horumce corporum collisio (8), mutatio status æqualiter in utroque corpore recipi debet; undè licet actioni æquali semper sit et contraria reactio, non idcircò tamen inter corpus agens et patiens fieri debet æquilibrium, idque Newtoniano exemplo manifestum est; si equi lapidem trahentis conatus seu vis activa major sit vi quâ lapis per gravitatem suam, plani scabritiem, mediúque resistentiam, equo trahenti reluctatur, equus lapidem trahet cum eâ totius suæ vis parte, quæ post superatam lapidis gravitatem, plani scabritiem, medíque resistentiam, ipsi residua est; si autem totus trahentis equi conatus hisce tribus resistentiis minor sit, vel si ipsis sit æqualis, equus lapidem non movebit. Quare totus ac integer lapidis renixus qui componitur ex ipsius gravitate, plani scabritie, resistentiâ medii et inertiâ quæ lapidi etiam omnibus aliis viribus destituto inest, actioni equi lapidem trahentis est semper æqualis.

(b) 33. Quoniam vis *N*, agit secundùm lineam A C ipsi B D, parallelam, hæc vis, (per Leg 2.) nihil nisi velocitatem secundùm lineam ipsi B D, parallelam producet, ac proindè non mutabit velocitatem accedendi ad lineam illam B D, a vi alterâ genitam; cum corpus iners duabus hisce viribus ac directionibus simul obsequi possit, et (per leg. 1.), debeat, atque hîc supponatur vires *M*, et *N*, in mobile eodem modo simul agere ac si singulæ seorsim in illud quiescens imprimerentur.

(c) 34. Idcircò cum in fine ejusdem temporis, corpus quod hîc tanquam punctum consideratur, simul esse debeat in utraque lineâ C D, et B D, in utriusque lineæ concursu D reperiatur, necesse est; quia autem initio et fine temporis dati corpus reperitur in rectâ A D, nempè primùm in A, et deindè in D, toto tempore dato motum fuit per lineam A D, nam ex duobus punctis A, et D, datis, recta, A D, positione data est; et corpus quibuslibet viribus impulsum, cessante virium actione, movetur uniformiter in directum secundùm ultimam directionem ex viribus impressis resultantem, (per Leg. 1. et 9.)

35. Motus compositus per diagonalem A D, motibus per latera A B, AC, disjunctis non est æqualis, sed tantùm æquipollet. Nam cum eadem sit corporis massa, motûs quantitates per diagonalem et per latera sunt ut velocitates uniformes (6) seu ut spatia A D, A B, A C, eodem tempore percursa (5); est autem summa laterum A B + A C, major diagonali A D; ergo summa quantitatum motûs per latera, major est quanti-

tate motus per diagonalem. Verùm quia idem est motus, sive mobile per diagonalem A D, celeritate æquabili ut A D, ex vi unicâ impressâ feratur, sive viribus conjunctis per latera A B, A C, impellatur, liquet motum per diagonalem, motibus per latera disjunctis æquivalere.

Si mobile a pluribus quàm duabus viribus in loco A, simul impressis impellatur, inveniri semper poterit unica directio et velocitas ex omnibus separatis composita ipsisque æquipollens, quæ *media directio* dicitur; duarum enim virium media directio reperiatur (per coroll. 1. Newt.); deinde diagonalis illa tanquam spatium vi unicâ percursum consideretur, et cum spatio tertiâ vi descripto pari ratione componatur, sicque vires omnes ad unicam reducentur.

37. Motus omnis in quotcumque alios laterales ipsi æquipollentes resolvi potest; nam motus per A D, æquabilis; facto triangulo quocumque A B D, resolvitur in motus per latera A B, AC, motui per diagonalem A D, æquipollentes (35). Eâdem ratione motus per A B, in duos quoscumque alios, descripto circà latus A B triangulo resolvitur, idemque de motu per A C, et de aliis quibuscumque motibus dici debet.

38. Si corpus aliquod A, duplici vi per A C, et per A F, ità urgeatur, ut motus in eâdem ratione acceleretur vel retardetur, quo quod idem est, si spatia A B, et A D, AC, et A F, iisdem temporibus percursa, semper sint in constanti ratione, motu composito parallelogrammi diagonalem A G, describet . . . .
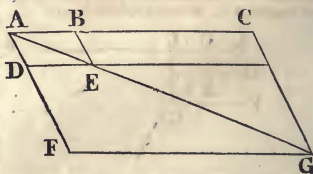
Dem. . . . Ductis D E ad A B, et B E ad A D, parallelis, corpus conjunctis viribus motum, reperiri debet simul in utráque lineâ D E, et E B, (34) adeóque in earum intersectione E; similiter ductis F G, ad A C, et C G, ad A F, parallelis, patet corpus motu composito eodem tempore reperiri in G, quo motibus disjunctis attingeret puncta C, et F; cum igitur (ex hyp.) sit A D, ad A B, seu D E, ut A F, ad A C, seu F G, recta A E, producta transit per punctum G; ergò corpus per diagonalem rectam A G, incedet. Q. e. d.

39. Si spatia secundùm unam directionem

Figure 2. Examples of notes (Newton, [1726] [1739–1742], 1822), I, p. 18).
These notes concern the corollary I of the motion laws regarding the decompositions of forces. See the indications (b) and (c) in Newton's text, to which the notes (b) 33 and (c) 34 correspond. The notes 35, 36, 37, 38, 39 are not preceded by a letter because they are not directly referred to Newton's text. We have chosen this page because it also gets an interesting particular: the note 36 is not indicated. The numeration passes from 35 to 37, but it is clear that such a note should begin at the first paragraph of the second column.

The numbered notes are 561 in the first volume; 333 in the second volume; 109 in the third volume; in the fourth volume, the numeration continues—as the two volumes present the third book of the *Principia*—and reaches the number 174. Therefore, the notes, which belong exclusively to the fourth volume, should be 65. However, because of a print mistake, from the note 117 we reach the note 122, thence the four notes 118–121 are missing. This reduces the number of notes to 61. This is the impressive typographical structure of the JE.

From a conceptual point of view, apart from the treatises in 3) and 4) which can be logically classified, the description and classification of the notes is still more difficult than from a typographical standpoint (Bussotti and Pisano, 2014).

In principle, the notes can be conceptually divided into three categories:

1) Notes that represent a direct explanation of Newton's text. These notes can have different length and importance for the understanding of Newton's work. There are some brief notes, which are not very useful because the commentators, in their desire to clarify every single step of Newton's reasoning, comment on passages, which are clear in Newton's version, too. Nevertheless, in general these notes are extremely useful because often they complete the difficult geometrical demonstrations of which Newton had not given every detail and, as a second step, often propose a version of the proofs transformed into analytical terms. Often works by other physicists are used in this second phase;

2) Notes, which expound mathematical or physical results, which the commentators judge necessary in order to understand the development of Newton's argumentations. In some cases these notes, which are always useful, have the size of treatises;

3) Notes where results deriving from Newton's work, but not existing in the *Principia*, are deduced. In these cases, too, there are some brief notes, which are simple corollaries to Newton's propositions, whereas others contain results, which are profound development of the physics expounded in the *Principia*. In many cases these results are not due to the commentators but to the most important physicists who continued and developed Newton's work. The commentators mention them. In the following, we report some main examples drawn from the four volumes.

4. EXAMPLE OF A SIGNIFICANT INTERVENTION OF THE COMMENTATORS

The detailed presentation of the most significant interventions by the commentators is the main reason that induced us to carry out the research whose bases will be explained in the conclusion of this paper. Because of the ingent number of notes in the JE, it is impossible to give a precise idea in a paper. We have previously expounded (Bussotti and Pisano, 2014) and commented three cases—belonging to different categories of notes—of particularly remarkable

Table 1. Volume I containing the first book of the *Principia*. Six example are given.

1) Notes to the *Scholion* of the *Axiomata sive leges motus* (Newton [1726] [1739–1742], 1822, I, pp. 33–35). The notes 83, 84 and 85 are extremely significant because the commentators develop concepts and ways of reasoning, which are fundamental in the *Principia* and apply them to results that are direct consequences of Newton's work. The remarkable aspect is that the commentators show in a clear manner which is the conceptual basis of Newton's infinitesimal geometry. In particular, they highlight how it is possible to reach the concept of (potential) infinitesimals of different orders in a geometrical manner. This is clear in the note 83 where it is proved that: ''If a body moves in an immobile curve, the force with which the body presses the curve, compared with the *finite* force that causes the movement of the body on the curve, is not bigger than an *infinitesimal quantity of the first order*; the force or the velocity that the body misses in every single point of the curve is not greater than an *infinitesimal quantity of the second order* [. . .]'' [out italics].

2) Notes 213–216, pp. 81–82. These notes, which are proposed as a *Scholium* to the initial six propositions of the second section of the first book, are important for one of the subject we have deal with in the paper: the transcription into analytical terms of Newton's geometrical procedures. For, the commentators write: ''Through the previous propositions, Newton opened the road towards the theory of central forces and provided elegant formulas in the propositions and in the corollaries. Afterwards, by means of the analysis and of the method of fluxions, many of the most illustrious geometers [at that time this word meant mathematicians], obtained a plurality of results''. [p. 81, beginning of note 213]. The analytical transcriptions of Newton's results and new results by Varignon, Johannes Bernoulli, Hermann and Keill are expounded.

3) Note 224, pp. 86–103. This is one of the most important and surprising notes of the whole edition because it is a real treatise on the properties of the conic sections used by Newton. This long insertion is fundamental to catch every detail of many Newton's demonstrations because Newton often used formulas as ''this follows from Conics''. For an expert reader, too, the comprehension of the property to which Newton is referring is often difficult as well as the discovery of Newton's specific source (in general propositions by Apollonius (ca. 262 BC–ca. 190 BC), sometimes Philippe de la Hire (1640–1718)). The methodological aspect of the note is as relevant as the conceptual one since the properties of the conics are demonstrated through geometrical reasoning, but the explicit resort to trigonometry and to the use of limits is not missing. Therefore, this is a precious document of a manner to develop the mathematical reasoning which was typical of the 17th and beginning of the 18th century. This treatise is accurate from the philological point of view, too, because the commentators mention every single proposition by Apollonius' they are demonstrating in a manner different from Apollonius'.

4) Note 268 to the proposition XIII, Cor. 1, pp. 123–125. It is important from a historical point of view because the *direct*, but more remarkably, the famous *inverse problem of the central forces* is dealt with and solved, that is: given the inverse square law and the initial velocity, prove that the trajectory is a conic section in which the centre of the forces is in one of the foci. Newton spoke of this problem in the Cor. 1 to proposition XIII, but giving unsatisfactory explanations.

Table 1 (Continued)

However, he solved the problem geometrically, in general terms, in the proposition I, XLI (pp. 245–255). The historiography debates whether Newton had an analytical demonstration of the inverse problem is still present nowadays. It is however almost sure that he had[18].

5) Notes 365–390, pp. 209–225. These notes concern the VI section, proposition XXXI and subsequent *Scholium*. Here Newton deals with a problem, which is fundamental in astronomy: given an elliptic orbit and the time, find the position of the body which moves on such ellipsis. Newton provides two solutions: in the proposition, he uses the trochoid and in the *Scholium*, given the difficulty to trace such a curve, he uses methods of approximations. The commentators explain these methods more clearly than Newton did, also resorting to and expounding results by de l'Hôpital (1661–1704), Seth Ward (1617–1689), Ismael Boulliau (1605–1694), Giovanni Domenico Cassini (1625–1712), David Gregory (1659–1708), John Keill (1671–1721).

6) Until now, we have expounded notes that are fundamental contributions to the understanding of the *Principia* and, more in general, to physical and mathematical methodology. Here we wish to present notes of a different kind, that in any case, have to highlighted to understand the entire character of the JE: the notes that are almost useless because they explain what is already clear in Newton's text. One of the most important and complex part of the *Principia* is represented by the proposition 66 and by its 22 corollaries, pp. 324–352. Here Newton deals with some aspects of the three bodies problem. The argumentations by Newton, almost completely lacking of any mathematical symbols, are fascinating and extremely difficult. The commentators add some quite useful notes. However, the note 511 begins with a lemma, p. 341 in which the commentators prove that in an

arithmetical progression of three quantities a, $a+b$, $a+2b$ it is $\dfrac{a+b}{a} > \dfrac{a+2b}{a+b}$.

This is obvious and no mathematician ignored this property. Nevertheless, the commentators prove it. The notes (^{b}) and (^{c}), pp. 344–345, written by Calandrini inside note 512, are referred to the corollary 15 and, for once, make more complicated what in Newton is difficult but clear.

interventions of the commentators[19]. Here we present a further one, since we are convinced that every work concerning a general presentation of a research has to be, at least in part, based upon concrete examples on how the authors intend to proceed in their work.

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[18] In a letter to David Gregory, Newton communicated the transcription of this problem into analytical terms (Guicciardini 2011, p. 160).

[19] The three-example concern: 1) The proposition VI of the I book (Newton [1726] [1739–1742], 1822, I, pp. 79–82); 2) a theorem on hyperbola, inside the commentators' treatise on conic sections (Newton [1726] [1739–1742], 1822, I, pp. 86–102); 3) Newton's lemma XXVIII of the I book (Newton [1726] [1739–1742], 1822, I, pp. 203–206).

Table 2. Volume II containing the second book of the *Principia*. Three examples are given.

1) The book begins with Calandrini's treatise (asterisk) concerning the general concepts on the motion in resistant means. The treatise, pp. 1–12, is divided into 49 sections, which include lemmas, theorems and corollaries. From a conceptual point of view, we can identify three parts: in the first one, titled *generales resistentiae notiones exponens*, Calandrini expounds introductory notions useful to understand Newton's procedures as well as results due to Varignon, Johannes Bernoulli, Hermann and Euler. This part forms the sections 1–31. The second part, sections 32–46, deals with a long series of properties of the logarithmic function, which are fundamental for the subjects dealt with in the II book of the *Principia*. The final sections 47–49 consists of a series of brief considerations on the maxima and minima.

2) In the proposition IV of section I and in the seven subsequent corollaries, pp. 21–31, Newton analyses the motion of a projectile under the action of gravity, in the hypothesis that the resistance of the air is proportional to the velocity. All the passages by means of which Newton constructs the figures in the text and develops his reasoning are punctually explained by the commentators (notes 53–66). However, the most important contribution concerns the notes to Newton's *Scholium* (notes 67–77, pp. 31–36). Here the commentators: a) clarify how, given the equation of a curve, it is possible to construct the curve exploiting the logarithmic function (notes 68–71) (the technique of Varignon and Hermann is referred as well as a letter by Newton to Oldenburg (1618–1677) in 1676, note 68); b) deal with the problem of the *angulum elevationis*, that is how the trajectory of a projectile varies in function of the angle with which the projectile is shot and of its initial velocity (notes 72–74); c) show how a regular curve passing through a series of given points can be described. This problem is difficult and, in general, if the kind of curve is not specified, it can be solved by approximation with convergent series. The commentators remind the reader (note 75) that Newton dealt with this problem in his *Arithmetica Universalis*, providing the method, but not the demonstrations. The long following notes (76, 77) concern this question, also exploiting some results due to Hermann, Craig (died 1620) and Stirling (1692–1770).

3) In the section IV, Newton deals with the circular motion in resistant means. In this context, the fundamental curve is the logarithmic spiral. Calandrini poses two pages where he describes all the properties of this curve necessary to understand Newton's reasoning. The commentator writes explicitly: "In this section, Newton supposes some properties which are typical of the logarithmic spiral, the care of our duty demands hence that we highlight something concerning that curve" (p. 110, lines 1–4. Our translation). Despite this contribution is far shorter than that on the conic sections in the first book, the aims are the same: to introduce the reader into the mathematical properties used, but non-explained, by Newton.

Table 3. Volume III, containing the first 24 propositions of the third book of the *Principia* and the three treatises on the tides by Daniel Bernoulli, McLaurin and Euler. Two examples are given.

1) The proposition VI of the third book is a fundamental step towards the universal gravitation law because here Newton proves that the gravity acts as the product of the masses. Literally the proposition sounds like this: ''That all bodies gravitate towards every planet; and that the Weights of the bodies towards any the same Planet, at equal distances from the centre of the Planet, are proportional to the quantity of matter which they severally contain''. (Translation drawn from Newton 1729, II–III, p. 220). In the fifth corollary Newton briefly clarifies the differences between gravity and magnetism. Calandrini, in the note 66, writes the four-pages subnote (^{b})* (pp. 29–32) where he refers a series of experiments and conclusions by Muschenbroek (1692–1761) and Winston (1667–1752) on the terrestrial magnetism. With regard to Muschenbroek, Calandrini refers to the work *Dissertatio de magnete*, where, after various experiments, the author concludes that the magnetic attraction between two bodies decreases with a proportion which is far inferior than the cube of the distance (end of p. 29—begin of p. 30). On the other hand, Winston in *De acus magneticae inclinatione* deduces, from his experiments, that the magnetic force decreases as the cube of the distance and Muschenbroek himself finally appearently concluded that the magnetic force decreases as the fourth power of the distance (p. 30, first column, beginning of the second paragraph). On the basis of these experiments and calculations, Calandrini formulated the conjecture that the magnetic force decreases almost as the cube of the distance. He wrote: ''From the previous [experiments and calculations], I think it is proved with a sufficient certainty that *the magnetic force decreases almost as the cube of the distance from the magnet, at least according to what can be ascertained from those rather rough observations*''.

2) Two interesting notes are the 72 and 73 (pp. 47 and 48): Newton, in the corollary two of proposition XIV, claimed that the parallax due to the annual motion of the earth is insensitive. Calandrini, in these notes dealt with the parallax–problem and expounded an attempt ideated by Huygens to calculate the ratios of the distances between the Sun and the fixed stars based on supposed parallaxes. In particular, Calandrini refers that Huygens considered Sirius, the brightest star after the Sun. He hypothesized that Sirius is as big as the Sun and supposed a given parallax. Under these conditions, Huygens calculated the distance Sun-Sirius and Earth—Sirius as a function of the distance Earth-Sun.

Table 4. Volume IV containing the propositions XXV–XLII of the third book of the *Principia* and the *General scholium*. Two examples are given.

1) Newton, in the proposition XXVI, poses and solves the problem to find the hour–increment of the area described by the moon in its revolution around the earth. The geometrical proofs given by Newton need a series of complicated calculations described in their general lines by Newton. Calandrini explains all calculations in details and adds a further problem: given the hypotheses of the proposition XXXVI, explain the reasoning according to which the *momenta* of the considered area are described. The problem is hence to find the infinitesimal variation of the areas. The methodology used by Calandrini is instructive because he develops his reasoning in a geometrical manner, namely in Newton's style, but at the same time, when he has to consider the infinitesimal quantities—that he calls, with a term which is used nowadays, too, *elements* (*elementa*)—he provides a transcription into analytical terms. In a sense this method is between the infinitesimal geometry by Newton and a merely analytical approach. The note in which all these questions are dealt with is the 110 indicated as 110 (^{a})* (here the number preceeds the letter and the asterisk) and it is extended from page 4 to page 10. As the note is a commentary to Newton's text, the subnotes are indicated with the series of letters (^{a})*–(^{x})*.

2) Maybe the most surprising insertion of the commentators is a 32 pages treatise which begins at p. 64 and finishes at p. 95. Here the concepts and the results known at that time with regard to the lunar theory are reported. The reader can get a complete understanding of Newton's methods and its transcription into analytical terms. The first part of the treatise (pp. 64–73) is titled: *De incremento motus medii Lunae, et eius aequatione annua, ex Solis actione pendentibus, primum hypothesi orbem Lunae esse circularem, postea in hypothesi orbem Lunae esse ellipticum. Denique in orbe lunari ad eclipticam inclinato.* (On the increment of the average motion of the moon, and on its annual equation, depending on sun action, at the beginning in the hypothesis that Lunar path is circular, then elliptic. Finally in the hypothesis that lunar path is inclined on the ecliptic). This part is composed of 2 lemmas, 7 theorems and 5 problems. The second part (pp. 73–83) is titled *De incremento motus medii Lunae, et eius aequatione ex Solis actione pendentibus, in hypothesi eum orbem esse ellipticum, methodo diversa ad quae in calculo precedente fuit adhibita* (On the increment of the average motion of the moon, its equation, depending on sun action, in the hypothesis that Lunar path is elliptic, proved through a different method than that used in the previous calculation). This part consists of 8 theorems, 2 lemmas and 6 problems. The third part (pp. 83–85) is titled *De aequatione motus lunari semestri secunda quae pendet ex positione lineae nodorum, respectu lineae syzygiarum* (On the equation of the six-monthly lunar motion as far as it depends on the position of nods-line with respect to the syzigies-line). It is composed of 3 problems. The fourth part (pp. 85–89) deals with a partially different subject, for it is titled *De motu apsidum* (On the motion of apses). It is composed of two problems and concerns the recent method to determine such a motion. While the fifth part (pp. 89–91), *De motu apsidorum secundum Newtoni methodo* (On the motion of the apses according to Newton's method), is, as the title itself confirms, an explanation of Newton's method. It is divided into 2 lemmas and 2 theorems. Finally the last part (pp. 91–95), *De excentricitate orbitae lunaris* (On the eccentricity of lunar orbit) is divided into one theorem and 2 problems.

In the first book, section VIII, proposition XLI Newton solves, in general and geometrical terms, the fundamental inverse problem of the central forces. The proposition sounds like this:

> Supposing a centripetal force of any kind, and granting the quadratures of curvilinear figures, it is requires to find, as well the trajectories in which bodies will move, as the times of their motions in the trajectories found.[20]

The method used by Newton is based on his typical resort to the infinitesimals inside geometry. The commentators add many details, which are important to understand Newton's procedure, in the notes 417–419 (pp. 245–248). In the notes 420–422 (Newton [1726] [1739–1742], 1822, I, pp. 248–249) they give a general idea of the way how Newton's method can be transcribed into analytical terms and extended[21]. Now we will follow Newton's demonstration and the interventions of the commentators, because this is a paradigmatic case of their approach.

Newton bases his argumentation on an *analytical method*. Here the word *analytical* has to be interpreted in the classical geometrical sense, that is, Newton supposes that the problem is solved, finds some relations among the figures, and at the end, shows, that, according to the found relations, the figure that one supposes as already constructed can be really constructed with the initial data of the problem. We will divide our discussion into two parts: in the former one, the analytical phase of Newton reasoning will be expounded, in the latter the consequent constructive phase. In both phases, we will divide our presentation into two subparts: NEWTON (what Newton did); COMMENTATORS (what commentators added). In the section NEWTON we will indicate like this (A), (B), (C),... the points in which the commentators make an intervention and we will repeat the same symbol in the section COMMENTATORS so the reader can easily follow. Some interventions of ours will be added if it is necessary to clarify the concepts and the procedures.

4.1. Analytical Part of the Proof

NEWTON: Let *C* (see Fig. 3 and Fig. 4) be the centre of the forces and let us suppose (analytical hypothesis) that *VIKk* is the trajectory. Let us describe, with centre *C* and radius *CV* the circle *VR*. With the same centre and arbitrary radius, let the two circles *ID* and *KE* be described, which cut the trajectory in *I* and *K* and the straight line *CV* in *D* and *E*. Let the straight line *CNIX* be traced cutting

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[20] The proposition and relative notes are in volume I (Newton [1726] [1739–1742], 1822, pp. 245–247). Translation drawn from Newton [1713] 1729, I, p. 171. This proposition is the same in all three editions.

[21] The problem is that the step from the general and correct solution provided by Newton to the transcription of the data into analytical terms—so to get a differential equation—for central forces that act as a specified function of distance is complicated. From a geometrical point of view, to solve the problem in a specific case means to determine the kind of utilized curves, whereas in the general case, the kind is not specified.

Table 5. Book I, Section VIII, Proposition XLI.

Figure 3. Proposition XLI: beginning of the demonstration (Newton [1726] [1739–1742], 1822, I p. 245. Newton's text is written in a bigger character.

Figure 4. Prosecution of the proof.

Table 5 (Continued)

Figure 5. Prosecution of the proof. The analytical part finishes at line 7.

Figure 6. Commentaries to proposition XLI. Almost the whole page is written by the commentators. Newton's text is relegated to the first two lines.

the circles KE and VR in N and X, while the straight line CKY cuts the circle VR in Y. Let the points I and K be reciprocally quite near (*vicinissima* in Latin. This means that the line IK has to be thought as an infinitesimal evanescent, hence potentially infinite, length). Let A be the point from which another body has to fall, so that in D this body acquires the same velocity as the velocity of the first body in I. Then, according to proposition XXXIX[22] the lineola IK, described in a given time as little as possible, will be as the velocity and hence as the straight line whose square is equal to the surface $ABFD$. (A) The triangle ICK, proportional to time will be given. Thence KN will be inversely proportional to the altitude IC. This means that, given a quantity Q and indicated the altitude IC by A, KN is as $\dfrac{Q}{A}$. Let us pose $Z = \dfrac{Q}{A}$ and let us suppose that, in some cases

$$(1) \qquad\qquad \sqrt{ABFD} : Z = IK : KN$$

Then this property is valid in every case (B) (we underline this is an immediate application of the proposition XL). This means that

$$(2) \qquad\qquad ABFD : Z^2 = IK^2 : KN^2;$$

and, splitting the second member of $(IK^2 - KN^2) : IK^2$ and applying Pythagoras's theorem, we have $IK^2 - KN^2 = IN^2$, so that:

(C)

$$(3) \qquad\qquad (ABFD - Z^2) : Z^2 = IN^2 : KN^2$$

So that

$$(4) \qquad\qquad \sqrt{(ABFD - Z^2)} : Z\left(= \frac{Q}{A}\right) = IN : KN$$

Thus

$$(5) \qquad\qquad A \cdot KN = \frac{Q \cdot IN}{\sqrt{ABDF - Z^2}}$$

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[22] In the proposition XXXIX Newton solves the following problem: "Supposing a centripetal force of any kind, and granting the quadrature of curvilinear figures; it is required to find the velocity of a body, ascending or descending in a right line, in the several places through which it passes; as also the time in which it will arrive at any place; And vice versa". (Newton [1713] 1729, p. 163). If (see Fig. 3 and 4) $AVDEC$ is the trajectory and EG is a perpendicular proportional to the centripetal force in E and $BLFG$ the line to which G belongs, then Newton proves that the velocity of the body in any point F, constructed as G, is proportional to the line whose square is the curvilinear area $ABFD$. More simply: the speed is proportional to $\sqrt{ABFD}$. Therefore, it is maybe useful for the reader to add that the lines AB, VL, DF, EG are proportional to the intensity of the centripetal force in the points A, V, D, E. Afterwards, in this same proposition XXXIX Newton proves that the line IK, described as in the proposition XLI, is proportional to the velocity.

(D) Since

(6) $$YX \cdot XC : A \cdot KN = CX^2 : A^2$$

Finally it will be

(7) $$XY \cdot XC = \frac{Q \cdot IN \cdot CX^2}{A^2 \sqrt{(ABDF - Z^2)}}$$

Here the analytical phase of the reasoning, which depends of the *supposition* that *VIKk* is the trajectory, ends and the constructive phase begins where the fundamental role is played by the fact that some sizes, strictly connected to the quantity

$$\frac{Q \cdot IN \cdot CX^2}{A^2 \sqrt{(ABDF - Z^2)}},$$

whose expression has been obtained supposing the trajectory is known, can be constructed *independently* of the trajectory.

COMMENTATORS: In this first phase of Newton's demonstration there are five brief notes; four of them—as indicated in the previous section—are direct annotations to Newton's text, whereas the longest one, numbered as 418, provides a property useful to understand more clearly a part of the whole reasoning. Let us analyse them.

Note 417 (^{p})*, p. 246, see Fig. 3. Note that we have indicated by (A). This is a very brief annotation in which it is explained that the triangle *IKC* is proportional to the time both because of the Proposition 1 and because of the fact that in this infinitesimal triangle, *KN* can be considered the basis referred to the altitude *IC*. Since the very brief (infinitesimal) time is given, $KN \cdot IC$ is constant and hence, as Newton claims, *KN* is inversely proportional to *IC*.

Here we think a consideration—not developed by the commentators—is necessary: based on Newton's language, one could get the impression that he is referring to an actual infinitesimal time because he speaks of a *given time*. This is not the case: Newton imagines a finite given time so brief that—without detectable mistakes for the physical results he is looking for—the lines *IK* and *KN* can be considered as straight lines and *KN* can be considered perpendicular to *IC*. Certainly we could call this time as *potential infinitesimal time*, which is correct, but this general category can perhaps hide the fact that the time is given and that, for a given time, the quantity $KN \cdot IC$ is constant.

Note 417 (^{q})*, p. 246, see Fig. 3. Note that we have indicated by (B). This note is brief, but significant enough because the commentators explain why, if $\sqrt{ABFD} : Z = IK : KN$ in one case, this proportion is true in every case. This depends on two facts: 1) as explained by Newton, the lineola *IK* is always as $\sqrt{ABFD}$; 2) *KN* is always inversely proportional to $IC = A$, hence it is as $Z = \frac{Q}{A}$. From 1) and 2) the thesis follows immediately. These kinds of notes, although they do not add fundamental explanations, make it easier the comprehension of the text avoiding to interrupt the reading and to concentrate on

not difficult missing passages which, however, need a brief reasoning to be fully justified.

Note 418, pp. 246–247, see Fig. 4 and Fig. 5. This note is an integration to Newton's text because the commentators explain how the quantity Q can be determined.

The reasoning runs like this: let VL (Fig. 8) and EG be perpendicular to AC, QV be tangent to the trajectory in V and CV perpendicular to it in Q, qI be the tangent to the trajectory in I and Cq its perpendicular, then it holds:

$$(8) \qquad CQ \cdot \sqrt{ABLV} = Cq \cdot \sqrt{ABFD} = \text{Constant}$$

For, according to the Corollary 1, prop. I, given a central force and the trajectory of a body, the velocity of the body in every point P is inversely proportional to the perpendicular drawn to the tangent in P. In the point V—according to what seen in the proposition—the velocity is equal to $\sqrt{ABLV}$ and, for the above-mentioned corollary, it is inversely proportional to CQ. This means exactly that $CQ \cdot \sqrt{ABLV}$ is constant. The reasoning is independent of the chosen point. This proves the assertion. Once assumed $Q = CQ \cdot \sqrt{ABLV}$, the commentators easily prove that

$$(9) \qquad IK : KN = IC : Cq = \sqrt{ABFD} : Z$$

(see 2)).

Table 6. Book I, Section VIII, Proposition XLI. Particular useful to follow the expounded reasoning.
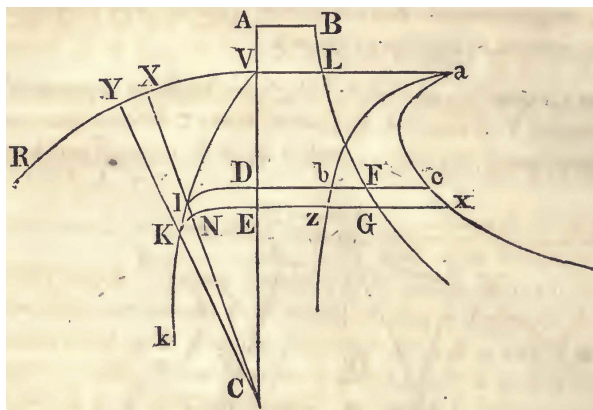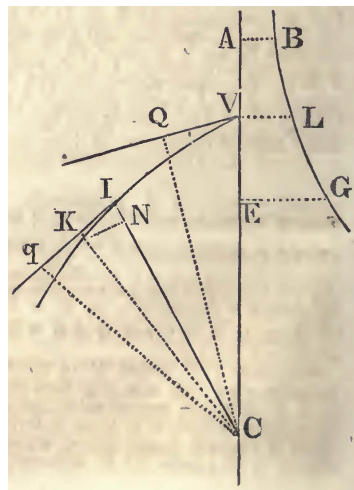

Figure 7. Newton's text: particular of p. 246

Figure 8. Note 418: particular of page 246

The notes 418 (^{r}) (we have referred as C)) and 418 (^{s})* (we have referred as D)) are very brief interventions that add nothing to Newton's text, therefore we will not consider them.

4.2. Constructive Part of the Proof

NEWTON: the constructive part (see Fig. 7) of the proof begins with the fundamental constructive step 10) because Newton claims that:

1) if the two quantities Db and Dc are constructed on the perpendicular DF so that:

(10)
$$Db = \frac{Q}{2\sqrt{(ABFD - Z^2)}};$$

$$Dc = \frac{Q \cdot CX^2}{2A^2\sqrt{(ABFD - Z^2)}}$$

2) If the curved lines ab, ac, to which the points b and c belong, are described;
3) If the perpendicular Va to AC is drawn from V, that cuts the curvilinear areas $Vdba$ and $Vdca$;
4) If the ordinates Ex and Ex are drawn,

one has:

I) The infinitesimal quantity $Db \cdot IN$, which is equal to $DbzE$, is equal to $\frac{1}{2}A \cdot KN$ or to the triangle ICK[23].

II) The infinitesimal quantity $Dc \cdot IN$, which is equal to $DcxE$, is equal to $\frac{1}{2}YX \cdot XC$ or to the triangle XCY (same reasoning as I).

Therefore, Newton continues, this means that the *nascent particles DbzE and ICK* of the areas $VDba$ and VIC are always equal and therefore the areas themselves are equal. Thus, since the area VIC is proportional to time, the area $VDba$ is proportional, too. Analogously, one concludes that the areas $VDca$ and VCX are equal because of the equality of their respective nascent particles $DcxE$ and XCY. Therefore the area $VDba$ is proportional to the time and represents the time, this means that (A) for any given time, it is possible to construct the area $VDba$. Therefore, the altitude of the body CI or CD is given, too, as well as the area $VDca$ and the sector VCX with its angle VCI; but, given the altitude and the angle, the point I of trajectory can be drawn. This concludes the reasoning.

   COMMENTATORS: The commentators make four interventions, the one we have indicated by (A), is the note 419 and it explains in details the final part of

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[23] We underline: 1) the quantity $DB \cdot IN$ is infinitesimal because of the infinitesimal lineola IN; 2) $Db \cdot IN = DbzE$ because the mixtilineal infinitesimal quadrilater $DbzE$ can be considered a rectangle with dimensions $Db = Ez$ and $DE = IN = bz$; 3) $Db \cdot IN = \frac{1}{2}A \cdot KN$ since $Db = \frac{Q}{2\sqrt{(ABFD - Z^2)}}$ by construction and in consequence of relation (5).

the proof from our (A) to the end. However, we think that the most significant note, in this context, is the note 420 where the commentators clearly explain that the fundamental constructive step (identity (10)) is legitimate, that is no datum deriving from the trajectory one has to construct is exploited. The notes 421 and 422 are an extension of Newton's result.

We will analyse in details the note 420 (p. 248) where the commentators clarify that Newton's constructive step is legitimate. They explain things resorting to algebra, so that, if (Fig. 7) one poses $IC = CD = x$ and given $VC = a$, it will be $VD = a - x$ and $Z = \dfrac{Q}{x}$. Given the quadrature of curvilinear figures, the area $ABDF$ can be expressed through the given quantities AV and VC and through the variable x. This means that the two quantities $Db = \dfrac{Q}{2\sqrt{(ABFD - Z^2)}}$; $Dc = \dfrac{Q \cdot CX^2}{2A^2\sqrt{(ABFD - Z^2)}}$ can also be expressed as functions of the constant a and of the variable x. Consequently, the equations of the curves ab and ac can be obtained as functions of constant quantities and of the variables CD and respectively Db or Dc. Therefore, the equations of the two curves can be written and the curves traced. Furthermore, since the sector VCX is equal to the area $VDca$, for the arch VX, it holds $VX = \dfrac{2VDca}{CV}$, so both the angle VCX and the point I of the trajectory can be found[24].

This brief explanation is fundamental and introduces us into a series of considerations concerning the links between synthetic geometry and analytical geometry which is rather interesting but to which we can only outline in this context: the reasoning by Newton is geometrical and the infinitesimal methods are used in the way we have more than once underlined. However the transcription of the geometrical properties into an algebraic form by posing a variable or a series of variables can—*not always*, but in many cases—guide to a geometrical construction far easier than a construction obtained only through synthetically means *a là Euclide* or *a là Apollonius*. This is the more truthful the more the curves are of high degree of irrational. On the other hand, it is implicit in the classical Greek analytical procedures that a certain quantity has to be considered as a variable, but treated as a known quantity. The development of analytical geometry is exactly the development of this technique, which was already present— *in nuce*—in the Greek geometry, especially in Apollonius.

In note 419, posed in the place we have indicated by (A), the commentators explain how, given the time, and knowing that the area $VDba$ is proportional to the time, it is possible to find the value of such an area. In substance, the proof is based on an application of Proposition I of the *Principia*.

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[24] For a very interesting transcription into complete analytical terms of Newton's proof by resorting to a modern use of calculus, see Wright 1833, pp. 224–226. The examples following the theoretical treatment are as significant as the treatment itself. Here we do not have room to enter into Wright's works.

Note 421 is a brief *Scholium* in which the commentators show how to describe the trajectory under conditions, which are slightly weaker than those posed by Newton.

Note 422 analyses when the trajectory is rational and when the area of the circular sector equal to *VDca* can be found through finite equations. At the end of this not long—but interesting—annotation, the commentators remind the reader that Hermann in the *Phoronomia*; I, Prop. XXV solved the elegant and difficult problem to find a general canon in order to determine the variable gravity for every algebraic curve *in infinitum* under the condition that the canon has to be expressed by means of finite quantities.

We have chosen this proposition by Newton and the relative notes because:

1) The importance of the proposition;
2) it is paradigmatic of Newton's way of reasoning and proceeding, based on his particular use of infinitesimal methods inside a geometrical argumentation;
3) The notes are paradigmatic of the commentators' way of working. They give explanations strictly tied to Newton's text, but also supply more general considerations connected to the development of physics;
4) The notes are significant for the history of mathematics. They are typical of a period in which calculus was more advanced than in Newton's time, but still far from a *modern* form. A comparison with the mentioned pages of Wright's commentary is meaningful;
5) The note 419 induces to reflect not only on the use—or lack of use—of the calculus in Newton, but also on the general concept of *analyticity* in Greek geometry, in analytical geometry, in Newton, in the commentators and, last but not the least, in the calculus.
6) Newton provided a general method and in the corollary 3 to this proposition, he considered the particular case of a force acting as the inverse of the cube-distance. However, he did not transcribe its results into analytical terms, which is, of course, important to solve specific problems. Based on what Newton wrote to Gregory we are convinced he knew such methods. To ask why he did not use them in the *Principia* is a fundamental historical question, but goes far beyond the purposes of this paper.

5. Final remarks

In this paper, the most important (Bussotti and Pisano 2014) characteristics of the JE have been traced and some examples of how the commentators carried out their huge work have been provided. We have also tried to highlight the meaning of this edition inside physics and mathematics in the late 17th and in the 18th centuries.

Finally we wish to propose a series of research-lines connected to the Jesuit Edition: we have expounded a conceptual map of the notes and of the interventions in the tables 1–4, revealing that many differences can be detected among these notes. A first line of research should concern the scientific personalities

of the authors in order to understand why they decided to organize the whole notes-apparatus in the manner we can read. A second line of research is related to the policy of science: who conceived and who financed such a huge enterprise? What were the reasons behind this choice? In what measure the trying to spread Newton's physics on the continent played a role? For, it is necessary to remember that Cartesian physics was still alive in the 30s of the 18th century. A third research-line concerns the way in which mathematics and physics changed in the first 40 years of the 18th century, till the point that Newtonian physics, written in Newtonian style, was becoming so difficult to be understood that a profound commentary was needed. It is worth underlining that the Jesuit Edition did not seem written with educative or popular aims: the notes are too specific, too long, too detailed to embrace such hypothesis. It seems written for specialists who wished to understand Newtonian physics by means of Newton's geometrical methods, which were almost disappearing. A fourth research-line is exactly connected to the previous one: who should have been the readers of the JE? As told, our hypothesis, our supposition is that they should be experts, but a research is necessary to reach the correct answer.

As all the significant scientific works, the JE open hence a series of questions. We are going to find—at least—some answers in our future researches on this edition of Newton's *Principia*.

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