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Algebraic Geometry — *Erratum: Points order* 2 *on theta divisors*, by Valeria Ornella Marcucci and Gian Pietro Pirola, communicated on 13 November 2015.[1]

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Robert Frederick Auffarth pointed out to us that the proof the main result of [3], Theorem 1.1, is not correct. Here we provide a proof of the basic result, Proposition 1, that implies the principal part of it, that is Theorem 1.1 i) of [3]. This does not allows us to recover the result on the non-symmetric theta divisor (1.1 ii)) of [3]. We remark however that in the forthcoming paper [1] a better bound will be given.

Given an abelian variety V we let $V[2]$ be the subgroup of the points of order two of V.

Proposition 1. *Let* (A, Θ) *be a complex principally polarized abelian variety. The divisor* Θ *cannot contain any translate of a maximal isotropic space of* $A[2]$.

Proof. Let $M \subset A[2]$ be a maximal isotropic subgroup (see [2]). We assume by contradiction that Θ contains the translate $t_a(M) = a + M$, $a \in A$, of M. Consider the isogeny $\pi : A \to B = A/M$ and its dual $\pi' : B \to A$. By construction one has $\pi \circ \pi' = 2_B$ where 2_B is the multiplication by 2 on B. Moreover there is a principally polarized divisor Θ' on B such that $\pi'^{*}\Theta \equiv 2\Theta'$. Set $\mathscr{L}' = (\pi')^{*}\mathcal{O}_A(\Theta)$ and $\psi : B \to \mathbb{P}^N$, $N = 2^g - 1$, be the map induced by the global sections of $\mathscr{L}'$. In particular the divisor $\Sigma = \pi^{-1}\Theta_A$ defines the hyperplane H of $\mathbb{P}^N : \Sigma = \psi^{-1}(H)$. Fix $b \in B$ such that $\pi'(b) = a$. It follows that

$$B[2]_b = \{x \in B : x = b + y : y \in B[2]\}$$

is contained in Σ, and therefore $\psi(B[2]_b) \subset H \subset \mathbb{P}^N$. Let L be the linear span of $\psi(B[2]_b)$, by construction we have $L \subset H$. On the other hand since $B[2]_b$ is invariant under the translation action of $B[2]$ it follows that L is invariant under the projective action of $B[2]$. By the irreducibility of the theta group representation

[1] This note is the erratum of the paper [3] in the References.

(see for instance [2]) we know that there is no a proper linear subspace of $\mathbb{P}^N$ invariant under the action of $B[2]$. This gives a contradiction. □

References

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