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Dominated Bilinear Forms and 2-homogeneous Polynomials

by

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Abstract

The main goal of this note is to establish a connection between the cotype of the Banach space X and the parameters r for which every 2-homogeneous polynomial on X is r-dominated. Let $\cot X$ be the infimum of the cotypes assumed by X and $(\cot X)^*$ be its conjugate. The main result of this note asserts that if $\cot X > 2$, then for every $1 \leq r < (\cot X)^*$ there exists a non-r-dominated 2-homogeneous polynomial on X.

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§1. Introduction

The notion of p-dominated multilinear mappings and homogeneous polynomials between Banach spaces plays an important role in the nonlinear theory of absolutely summing operators. It was introduced by Pietsch [17] and has been investigated by several authors since then (see, e.g., [5, 6] and references therein).

Let X be a Banach space and m be a positive integer. A continuous m-linear form A on X^m is *r-dominated* if $(A(x_j^1, \dots, x_j^m))_{j=1}^\infty \in \ell_{r/m}$ whenever $(x_j^1)_{j=1}^\infty, \dots, (x_j^m)_{j=1}^\infty$ are weakly r-summable in X. In a similar way, a scalar-valued m-homogeneous polynomial P on X is r-dominated if $(P(x_j))_{j=1}^\infty \in \ell_{r/m}$ whenever $(x_j)_{j=1}^\infty$ is weakly r-summable in X.

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In [11, Lemma 5.4] it is proved that for every infinite-dimensional Banach space X, every $p \geq 1$ and every $m \geq 3$, there exists a continuous non-p-dominated m-linear form on X^m. For polynomials the question has recently been settled in [6], where it is proved that for every infinite-dimensional Banach space X, every $p \geq 1$ and every $m \geq 3$, there exists a continuous non-p-dominated scalar-valued m-homogeneous polynomial on X. So, coincidence situations can occur only for $m = 2$. Sometimes it happens that every continuous bilinear form on X^2 is 2-dominated, for example if X is either an $\mathcal{L}_\infty$-space, the disc algebra $\mathcal{A}$ or the Hardy space H^∞ (see [4, Proposition 2.1]). In this case every continuous bilinear form on X^2 and every continuous scalar-valued 2-homogeneous polynomial on X are r-dominated for every $r \geq 2$. But what about r-dominated bilinear forms and 2-homogeneous polynomials for $1 \leq r < 2$?

Those spaces X that enjoy the property that all bilinear forms on X^2 are 1-dominated are all of cotype 2 (Example 1). In Proposition 3.2 we see that having cotype $2 + \varepsilon$ for every $\varepsilon > 0$ is a necessary condition. So, for a space X such that $\cot X > 2$ it is natural to investigate how close r can be to 1 with the property that every bilinear form on X^2 (or 2-homogeneous polynomial on X) is r-dominated. For bilinear forms it is not difficult to see (Proposition 3.3) that $(\cot X)^*$, the conjugate of the number $\cot X$, is the closest r can be to 1. As usual, for polynomials the situation is more delicate. In the main result of this paper, Theorem 3.2, we prove that the estimate $(\cot X)^*$ holds for 2-homogeneous polynomials as well. We also point out that this result is in a sense sharp.

§2. Notation

Throughout this paper, n and m are positive integers, and X and Y will stand for Banach spaces over $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$. The Banach spaces of all continuous m-linear mappings $A\colon X^m \to Y$ and continuous m-homogeneous polynomials $P\colon X \to Y$, endowed with the usual sup norms, are denoted by $\mathcal{L}(^m X; Y)$ and $\mathcal{P}(^m X; Y)$, respectively ($\mathcal{L}(X; Y)$ if $m = 1$). When $m = 1$ and $Y = \mathbb{K}$ we write X^* to denote the topological dual of X. The closed unit ball of X is represented by B_X. The notation $\cot X$ denotes the infimum of the cotypes assumed by X. The identity operator on X is denoted by id_X. For details on the theory of multilinear mappings and homogeneous polynomials between Banach spaces we refer to [10, 14].

Given $r \in [0, \infty)$, let $\ell_r(X)$ be the Banach (r-Banach if $0 < r < 1$) space of all absolutely r-summable sequences $(x_j)_{j=1}^\infty$ in X with the norm $\|(x_j)_{j=1}^\infty\|_r = (\sum_{j=1}^\infty \|x_j\|^r)^{1/r}$. We denote by $\ell_r^w(X)$ the Banach (r-Banach if $0 < r < 1$) space of all weakly r-summable sequences $(x_j)_{j=1}^\infty$ in X with the norm $\|(x_j)_{j=1}^\infty\|_{w,r} = \sup_{\varphi \in B_{X^*}} \|(\varphi(x_j))_{j=1}^\infty\|_r$.

Let $p, q > 0$. An m-linear mapping $A \in \mathcal{L}(^m X; Y)$ is *absolutely* $(p; q)$*-summing* if $(A(x_j^1, \dots, x_j^m))_{j=1}^\infty \in \ell_p(Y)$ whenever $(x_j^1)_{j=1}^\infty, \dots, (x_j^m)_{j=1}^\infty \in \ell_q^w(X)$. It is well-known that A is absolutely $(p; q)$-summing if and only if there is a constant $C \geq 0$ such that

$$\Big(\sum_{j=1}^n \|A(x_j^1, \dots, x_j^m)\|^p\Big)^{1/p} \leq C \prod_{k=1}^m \|(x_j^k)_{j=1}^n\|_{w,\,q}$$

for every positive integer n and every $x_1^k, \dots, x_n^k \in X$, $k = 1, \dots, m$. The infimum of such C is denoted by $\|A\|_{\mathrm{as}(p;q)}$. The space of all absolutely $(p; q)$-summing m-linear mappings from X^m to Y is denoted by $\mathcal{L}_{\mathrm{as}(p;q)}(^m X; Y)$, and $\|\cdot\|_{\mathrm{as}(p;q)}$ is a complete norm (p-norm if $p < 1$) on $\mathcal{L}_{\mathrm{as}(p;q)}(^m X; Y)$.

An m-homogeneous polynomial $P \in \mathcal{P}(^m X; Y)$ is *absolutely* $(p; q)$*-summing* if the symmetric m-linear mapping associated to P is absolutely $(p; q)$-summing, or, equivalently, if $(P(x_j))_{j=1}^\infty \in \ell_p(Y)$ whenever $(x_j)_{j=1}^\infty \in \ell_q^w(X)$. It is well-known that P is absolutely $(p; q)$-summing if and only if there is a constant $C \geq 0$ such that

$$\Big(\sum_{j=1}^n \|P(x_j)\|^p\Big)^{1/p} \leq C(\|(x_j)_{j=1}^n\|_{w,\,q})^m$$

for every positive integer n and every $x_1, \dots, x_n \in X$. The infimum of such C is denoted by $\|P\|_{\mathrm{as}(p;q)}$. The space of all absolutely $(p; q)$-summing m-homogeneous polynomials from X to Y is denoted by $\mathcal{P}_{\mathrm{as}(p;q)}(^m X; Y)$, and $\|\cdot\|_{\mathrm{as}(p;q)}$ is a complete norm (p-norm if $p < 1$) on $\mathcal{P}_{\mathrm{as}(p;q)}(^m X; Y)$.

An m-homogeneous polynomial $P \in \mathcal{P}(^m X; Y)$ is said to be r*-dominated* if it is absolutely $(r/m; r)$-summing. In this case we write $\mathcal{P}_{\mathrm{d},r}(^m X; Y)$ and $\|\cdot\|_{\mathrm{d},r}$ instead of $\mathcal{P}_{\mathrm{as}(r/m;r)}(^m X; Y)$ and $\|\cdot\|_{\mathrm{as}(r/m;r)}$. As usual we write $\mathcal{P}_{\mathrm{d},r}(^m X)$ and $\mathcal{P}(^m X)$ when $Y = \mathbb{K}$. The definition (and notation) for r-dominated multilinear mappings is analogous (for the notation just replace $\mathcal{P}$ by $\mathcal{L}$). For details we refer to [2, 4, 11].

§3. Results

First we establish the existence of Banach spaces on which every bilinear form (hence every scalar-valued 2-homogeneous polynomial) is 1-dominated. By $X \tilde{\otimes}_\pi X$ and $X \tilde{\otimes}_\varepsilon X$ we mean the completions of the tensor product $X \otimes X$ with respect to the projective norm π and the injective norm ε, respectively. For the basics on tensor norms we refer to [8, 19].

By Π_r we denote the ideal of absolutely r-summing linear operators. The following well-known factorization theorem (see, e.g., [17, Theorem 14] or [2, Proposition 46(a)]) will be useful a couple of times.

Lemma 3.1. *$\mathcal{L}_{\mathrm{d},r}(^mX;Y) = \mathcal{L} \circ (\Pi_r, \overset{(m)}{\ldots}, \Pi_r)(^mX;Y)$ and $\mathcal{P}_{\mathrm{d},r}(^mX;Y) = \mathcal{P} \circ \Pi_r(^mX;Y)$ for all positive integers m and Banach spaces X and Y.*

Proposition 3.1. *Let X be a cotype 2 space. Then $X \tilde{\otimes}_\pi X = X \tilde{\otimes}_\varepsilon X$ if and only if $\mathcal{L}_{\mathrm{d},1}(^2X) = \mathcal{L}(^2X)$.*

Proof. This result is contained, in essence, in [11]. We give the details for the sake of completeness.

Assume that $X \tilde{\otimes}_\pi X = X \tilde{\otimes}_\varepsilon X$ and let $A \in \mathcal{L}(^2X)$. Denoting the linearization of A by A_L we have $A_L \in (X \tilde{\otimes}_\pi X)' = (X \tilde{\otimes}_\varepsilon X)'$. Regarding X as a subspace of $C(B_{X'})$ and using that ε respects the formation of subspaces, A_L admits a continuous extension to $C(B_{X'}) \tilde{\otimes}_\varepsilon C(B_{X'})$, hence to $C(B_{X'}) \tilde{\otimes}_\pi C(B_{X'})$ because $\varepsilon \leq \pi$. As bilinear forms on $C(K)$-spaces are 2-dominated, the bilinear form associated to this extension is 2-dominated. But restrictions of 2-dominated bilinear forms are 2-dominated as well, so A is 2-dominated. Since 2-summing operators on cotype 2 spaces are 1-summing [9, Corollary 11.16(a)], it follows that $\Pi_1(X;Y) = \Pi_2(X;Y)$ for every Y, so by Lemma 3.1 we have

$$\mathcal{L}_{\mathrm{d},2}(^2X) = \mathcal{L} \circ (\Pi_2, \Pi_2)(^2X) = \mathcal{L} \circ (\Pi_1, \Pi_1)(^2X) = \mathcal{L}_{\mathrm{d},1}(^2X).$$

It follows that A is 1-dominated.

Conversely, assume that $\mathcal{L}_{\mathrm{d},1}(^2X) = \mathcal{L}(^2X)$ and let $A \in \mathcal{L}(^2X)$. Since 1-dominated bilinear forms are 2-dominated, it follows that A is 2-dominated, hence extendible by [13, Theorem 23]. Adapting the proof of [7, Proposition 1.1] to bilinear forms we conclude that A is integral. Now apply [8, Ex. 4.12] to get $X \tilde{\otimes}_\pi X = X \tilde{\otimes}_\varepsilon X$. □

Example 1. Pisier [18] proved that every cotype 2 space E embeds isometrically in a cotype 2 space X such that $X \tilde{\otimes}_\pi X = X \tilde{\otimes}_\varepsilon X$. So for every such space X we have $\mathcal{L}_{\mathrm{d},1}(^2X) = \mathcal{L}(^2X)$.

It is easy to see that $\cot X = 2$ is a necessary condition for every bilinear form on X to be 1-dominated:

Proposition 3.2. *If $\mathcal{L}_{\mathrm{d},1}(^2X) = \mathcal{L}(^2X)$, then $\cot X = 2$.*

Proof. By [1, Lemma 3.4] every bounded linear operator from X to X' is 1-summing. So, from [12, Proposition 8.1(2)] we conclude that the identity operator on X is (2;1)-summing. It follows that $\cot X = 2$ by [9, Theorem 14.5]. □

Let X be such that $\cot X > 2$. Since we cannot have $\mathcal{L}_{\mathrm{d},1}(^2X) = \mathcal{L}(^2X)$, for which $r > 1$ is it possible to have $\mathcal{L}_{\mathrm{d},r}(^2X) = \mathcal{L}(^2X)$? Or, at least, $\mathcal{P}_{\mathrm{d},r}(^2X) =$

$\mathcal{P}(^2X)$? In other words, we seek estimates for the numbers

$$\mathcal{L}_X := \inf\{r : \mathcal{L}_{\mathrm{d},r}(^2X) = \mathcal{L}(^2X)\} \quad \text{and} \quad \mathcal{P}_X := \inf\{r : \mathcal{P}_{\mathrm{d},r}(^2X) = \mathcal{P}(^2X)\}.$$

It is not difficult to give a lower bound for $\mathcal{L}_X$. By q^* we denote the conjugate exponent of $q > 1$.

Proposition 3.3. *If* $\cot X > 2$, *then* $\mathcal{L}_X \geq (\cot X)^*$.

Proof. By Proposition 3.2 we know that $\mathcal{L}_{\mathrm{d},1}(^2X) \neq \mathcal{L}(^2X)$. Using the equality $\Pi_1(X;Y) = \Pi_r(X;Y)$ whenever $1 \leq r < (\cot X)^*$ [9, Corollary 11.16(b)] and Lemma 3.1, we find that

$$\mathcal{L}_{\mathrm{d},r}(^2X) = \mathcal{L} \circ (\Pi_r, \Pi_r)(^2X) = \mathcal{L} \circ (\Pi_1, \Pi_1)(^2X) = \mathcal{L}_{\mathrm{d},1}(^2X) \neq \mathcal{L}(^2X)$$

for every $1 \leq r < (\cot X)^*$, so the result follows. □

It is not clear at once that the same holds for polynomials. Here the situation is usually more delicate: for instance, in [6] one can find a non-r-dominated bilinear form whose associated 2-homogeneous polynomial happens to be r-dominated. However, we shall prove in Theorem 3.2 that again $\mathcal{P}_X \geq (\cot X)^*$.

The following proof extends an argument which was first used in this context in [15].

Theorem 3.1. *Let m be an even positive integer and X be an infinite-dimensional real Banach space. If $q < 1$ and $\mathcal{P}_{\mathrm{as}(q;r)}(^mX) = \mathcal{P}(^mX)$, then id_X is absolutely $(\frac{mq}{1-q}, r)$-summing.*

Proof. The open mapping theorem gives us a constant $K > 0$ such that $\|Q\|_{\mathrm{as}(q;r)} \leq K\|Q\|$ for all continuous m-homogeneous polynomials $Q\colon X \to Y$.

Let $n \in \mathbb{N}$ and $x_1, \dots, x_n \in X$ be given. Consider $x_1^*, \dots, x_n^* \in B_{X^*}$ such that $x_j^*(x_j) = \|x_j\|$ for every $j = 1, \dots, n$. Let $\mu_1, \dots, \mu_n$ be real numbers with $\sum_{j=1}^n |\mu_j|^s = 1$, where $s = 1/q$. Define $P\colon X \to \mathbb{R}$ by

$$P(x) = \sum_{j=1}^n |\mu_j|^{1/q} x_j^*(x)^m \qquad \text{for every } x \in X.$$

Since m is even and $\mathbb{K} = \mathbb{R}$, it follows that $P(x) \geq 0$ for every $x \in X$. Also, $|P(x)| = P(x) \geq |\mu_k|^{1/q} x_k^*(x)^m$ for every $x \in X$ and every $k = 1, \dots, n$. From

$$|P(x)| = \Big|\sum_{j=1}^n |\mu_j|^{1/q} x_j^*(x)^m\Big| \leq \|x\|^m \sum_{j=1}^n |\mu_j|^{1/q} = \|x\|^m$$

we conclude that $\|P\|_{\mathrm{as}(q;r)} \leq K\|P\| \leq K$. So

$$\Big(\sum_{j=1}^{n} \|x_j\|^{mq}|\mu_j|\Big)^{1/q} = \Big(\sum_{j=1}^{n}(\|x_j\|^{m}|\mu_j|^{1/q})^{q}\Big)^{1/q} \leq \Big(\sum_{j=1}^{n}|P(x_j)|^{q}\Big)^{1/q}$$
$$\leq \|P\|_{\mathrm{as}(q;r)}(\|(x_j)_{j=1}^{n}\|_{w,r})^{m}.$$

Observing that this last inequality holds whenever $\sum_{j=1}^{n}|\mu_j|^{s}=1$ and that $\frac{1}{s}+\frac{1}{s/(s-1)}=1$ we have

$$\Big(\sum_{j=1}^{n}\|x_j\|^{\frac{s}{s-1}mq}\Big)^{1/\frac{s}{s-1}} = \sup\Big\{\Big|\sum_{j=1}^{n}\mu_j\|x_j\|^{mq}\Big| : \sum_{j=1}^{n}|\mu_j|^{s}=1\Big\}$$
$$\leq \sup\Big\{\sum_{j=1}^{n}|\mu_j|\,\|x_j\|^{mq} : \sum_{j=1}^{n}|\mu_j|^{s}=1\Big\}$$
$$\leq \|P\|_{\mathrm{as}(q;r)}^{q}(\|(x_j)_{j=1}^{n}\|_{w,r})^{mq} \leq K^{q}(\|(x_j)_{j=1}^{n}\|_{w,r})^{mq}.$$

It follows that

$$\Big(\sum_{j=1}^{n}\|x_j\|^{\frac{s}{s-1}mq}\Big)^{1/\frac{s}{s-1}mq} \leq K^{1/m}\|(x_j)_{j=1}^{n}\|_{w,r}.$$

Since $\frac{s}{s-1}mq=\frac{mq}{1-q}$, n and $x_1,\dots,x_n \in X$ are arbitrary, we conclude that id_X is $(\frac{mq}{1-q};r)$-summing. ☐

The following theorem holds for spaces over $\mathbb{K}=\mathbb{R}$ or $\mathbb{C}$:

Theorem 3.2. *If* $\cot X = q > 2$, *then* $\mathcal{P}_{\mathrm{d},r}(^2X) \neq \mathcal{P}(^2X)$ *for* $1 \leq r < q^*$, *where* q^* *is the conjugate of* q. *In other words,* $\mathcal{P}_X \geq q^*$.

Proof. Real case: Let $1 \leq r < q^*$. Combining Lemma 3.1 and [9, Corollary 11.16(b)] it is immediate that $\mathcal{P}_{\mathrm{d},r}(^2X)=\mathcal{P}_{\mathrm{d},1}(^2X)$. If $\mathcal{P}_{\mathrm{d},r}(^2X)=\mathcal{P}_{\mathrm{d},1}(^2X)=\mathcal{P}(^2X)$, from Theorem 3.1 we could conclude that id_X is $(2;1)$-summing, but this is impossible because $\cot X > 2$.

Complex case: If X is a complex Banach space, $\cot X = q > 2$ and $1 \leq r < q^*$, then by [3, Lemma 3.1] we know that $\cot X_{\mathbb{R}} = q > 2$, so there is a non-$r$-dominated polynomial $P \in \mathcal{P}(^2X_{\mathbb{R}})$. Denoting by $\widetilde{P}$ the complexification of P we see that $\widetilde{P} \in \mathcal{P}(^2X)$ and following the lines of [16, Proposition 4.30] it is not difficult to prove that $\widetilde{P}$ fails to be r-dominated either. ☐

Remark. Let X be any of the spaces constructed by Pisier [18]. By Example 1 we know that $\mathcal{P}_{\mathrm{d},1}(^2X)=\mathcal{P}(^2X)$, which makes it clear that Theorem 3.2 is sharp in the sense that it is not valid for cotype 2 spaces.

Conjecture. We conjecture that if X is infinite-dimensional and $\mathcal{L}_{\mathrm{d},1}(^2X) = \mathcal{L}(^2X)$, then $X \tilde{\otimes}_\pi X = X \tilde{\otimes}_\varepsilon X$. Observe that for an infinite-dimensional space X with $\mathcal{L}_{\mathrm{d},1}(^2X) = \mathcal{L}(^2X)$ and $X \tilde{\otimes}_\pi X \neq X \tilde{\otimes}_\varepsilon X$, if any, we should have:

- X has no unconditional basis [4, Theorem 3.2];
- X has cotype $2 + \varepsilon$ for every $\varepsilon > 0$ (Proposition 3.2);
- X does not have cotype 2 (Proposition 3.1);
- X' is a GT space [11, Theorem 3.4];
- every linear operator from X to X' is absolutely 1-summing (by [1, Lemma 3.4] this is a consequence of $\mathcal{L}_{\mathrm{d},1}(^2X) = \mathcal{L}(^2X)$), in particular X is Arens-regular;
- not every linear operator from X to X' is integral (this is a consequence of $X \tilde{\otimes}_\pi X \neq X \tilde{\otimes}_\varepsilon X$).

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