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Corrigendum to “Mean convex properly embedded $[\varphi, \vec{e}_3]$-minimal surfaces in $\mathbb{R}^3$”

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Abstract. This corrigendum fixes wrong signs in two equations used in the proof of Theorem B of [Rev. Mat. Iberoam. 38 (2022), 1349–1370]. The statement of the theorem remains true, and here we present the corrected versions of both the equations and their consequences.

In the paper [1], we provide a Spruck–Xiao's type result, see Theorem B in [1], for the family of properly embedded $[\varphi, \vec{e}_3]$-minimal surface in $\mathbb{R}^3_\alpha = \{p \in \mathbb{R}^3 \mid \langle p, \vec{e}_3\rangle > \alpha\}$ with mean curvature $H \le 0$, locally bounded genus, and $\varphi\colon \mathbb{R} \to \mathbb{R}$ being a smooth function satisfying

$$\dot{\varphi} > 0, \quad \ddot{\varphi} \ge 0, \quad \dddot{\varphi} \le 0 \quad \text{on }]\alpha, \infty[, \tag{1}$$

and such that the function $z \mapsto \dot{\varphi}(z)/z$ is analytic at $+\infty$.

This correction concerns wrong signs found in two equations employed in the proof of Theorem B in [1]. The statement of the theorem remains true, and here we present the corrected versions of both the equations and their consequences.

The mistake was found in items (7) and (8) of Lemma 3.2 in [1], and the correct expressions are the following:

$$\nabla^2 H = -\eta\, \nabla^2\dot{\varphi} - (\nabla_{\nabla\varphi}\mathcal{S}) - H\mathcal{S}^{[2]} - \mathcal{B}, \tag{2}$$

$$\Delta\mathcal{S} + \nabla_{\nabla\varphi}\mathcal{S} + \eta\,\nabla^2\dot{\varphi} + |\mathcal{S}|^2\mathcal{S} + \mathcal{B} = 0, \tag{3}$$

where $\mathcal{S}^{[2]}$ and $\mathcal{B}$ are symmetric 2-tensors defined in Lemma 3.2 of [1]. As consequence, the formula of Lemma 5.3 in [1] is then given by

$$\Delta^\varphi k_i = -|\mathcal{S}|^2 k_i - \eta\,\nabla^2\dot{\varphi}(v_i, v_i) - \mathcal{B}(v_i, v_i) + 2(-1)^{i+1}\frac{Q^2}{k_1 - k_2} \quad \text{in } \Sigma\setminus\mathcal{U}, \tag{4}$$

where

$$Q^2 = h_{11,2}^2 + h_{22,1}^2 = \langle\nabla k_1, v_2\rangle^2 + \langle\nabla k_2, v_1\rangle^2.$$

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Such a formula is applied to obtain the updated version of Lemma 5.4 in [1]:

Lemma 1. *Let Σ be a $[\varphi, \vec{e}_3]$-minimal immersion in $\mathbb{R}^3_\alpha$ with $k_1 < 0$, $H = k_1 + k_2 < 0$. If for any positive smooth function $\psi\colon \Sigma \to]0, +\infty[$ we consider the operator*

$$\mathcal{J}^{\psi} := \Delta^{\varphi + 2\log\psi},$$

then on $\Sigma \backslash \mathcal{U}$ we have

$$\mathcal{J}^{-k_1}\frac{\eta}{k_1} = \ddot{\varphi}\langle\nabla\mu, v_1\rangle^2\frac{\eta^2}{k_1^2} - \frac{\ddot{\varphi}\,\eta}{k_1}(1 - 2\langle\nabla\mu, v_1\rangle^2) - \frac{2\eta Q^2}{k_1^2(k_1 - k_2)}, \tag{5}$$

$$\mathcal{J}^{\eta}\frac{k_2}{\eta} = -\ddot{\varphi}\langle\nabla\mu, v_2\rangle^2 + \frac{\ddot{\varphi}\,k_2}{\eta}(1 - 2\langle\nabla\mu, v_2\rangle^2) - \frac{2Q^2}{\eta(k_1 - k_2)}. \tag{6}$$

In particular, if φ satisfies (1), then

$$\mathcal{J}^{\eta}\Big(\frac{k_2}{\eta}\Big) - \Big\langle\frac{4(K - \ddot{\varphi}\eta^2)\langle\nabla\mu, v_2\rangle}{(k_1 - k_2)\eta}v_2, \nabla\Big(\frac{k_2}{\eta}\Big)\Big\rangle \geq 0 \quad on\ \{p \in \Sigma : k_2(p) > 0\}. \tag{7}$$

Equations (5) and (6) are obtained as consequences of the formula (4), using the same arguments of the original proof. Inequality (7) is a consequence of

$$\begin{aligned}\mathcal{J}^{\eta}\frac{k_2}{\eta} &= -\ddot{\varphi}\langle\nabla\mu, v_2\rangle^2 + \frac{\ddot{\varphi}\,k_2}{\eta}\Big(1 + \frac{2H}{k_1 - k_2}\langle\nabla\mu, v_2\rangle^2\Big)\\ &\quad - \frac{2}{\eta(k_1 - k_2)}\Big(\frac{(K^2 + \ddot{\varphi}^2\eta^4)}{\eta^2}\langle\nabla\mu, v_2\rangle^2 + \eta^2\Big\langle\nabla\Big(\frac{k_2}{\eta}\Big), v_2\Big\rangle^2 + h_{22,1}^2\Big)\\ &\quad + \Big\langle\frac{4(K - \ddot{\varphi}\eta^2)\langle\nabla\mu, v_2\rangle}{(k_1 - k_2)\eta}v_2, \nabla\Big(\frac{k_2}{\eta}\Big)\Big\rangle,\end{aligned}$$

which is obtained when one develops the expression of Q^2 in (6). In fact, since we have that $H = -\dot{\varphi}\eta$, we conclude that $h_{11,2}$ is given by

$$h_{11,2} = -\eta\ddot{\varphi}\langle\nabla\mu, v_2\rangle + \frac{k_1}{\eta}\langle\nabla\eta, v_2\rangle - \eta\Big\langle\nabla\Big(\frac{k_2}{\eta}\Big), v_2\Big\rangle.$$

The updated proof of Theorem B in [1] follows the same steps as its original proof. From (7), we prove Claim 5.6 applying a strong maximum principle for elliptic operators, to conclude that the supremum of k_2/η does not attend in an interior point of $\Omega^+ = \{p \in \Sigma : k_2(p) > 0\}$. On the other hand, the generalized Omori–Yau maximum principle for Δ^φ holds in Σ and, taking the corresponding limits on the equations (5) and (6) in the same way it was done originally we prove Claims 5.8, 5.10, 5.12 and 5.13. Claims 5.7, 5.9, and 5.11 remain unchanged, and the result of Theorem B in [1] follows.

References

[1] Martínez, A., Martínez-Triviño, A. L. and dos Santos, J. P.: Mean convex properly embedded $[\varphi, \vec{e}_3]$-minimal surfaces in $\mathbb{R}^3$. *Rev. Mat. Iberoam.* **38** (2022), no. 4, 1349–1370.

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